

Peer Review

Review of: "Scrambling in Charged Hairy Black Holes and the Kasner Interior"

Run-Qiu Yang¹

1. Tianjin University, China

The paper systematically investigates the effects of the Einstein-Maxwell-Scalar (EMS) coupling constant, axion parameter, and charge density on the chaotic properties (e.g., quantum Lyapunov exponent, butterfly velocity) and the interior Kasner geometry of charged hairy black holes. While the connection between black hole chaos and Kasner geometry has been explored in previous works, this study introduces a more complex model with multiple parameter couplings, revealing non-monotonic behaviors in the relationship between boundary deformation and chaotic/Kasner exponents. The discovery that EMS coupling may lead to new behaviors in Kasner transition is particularly noteworthy. After carefully reading the manuscript, I have the following questions or comments.

- (1) The paper does not sufficiently differentiate its findings from recent similar studies (e.g., [38] [42]), especially in terms of the special properties of the parameter couplings used in this paper.
- (2) The physical mechanism behind "boundary deformation leading to enhanced chaos" is not thoroughly explained. A more detailed physical interpretation would strengthen the innovation claim. Though the authors used many sentences to describe how those parameters affect the chaos, these descriptions are more like "observation" rather than an explanation of the physical reason.
- (3) The difference between EMS coupling leading to Kasner transition (Figure 17) and the axion parameter causing inversion (Figure 14) should be more explicitly contrasted with existing studies.
- (4) In Section IV ("What Happens after Kasner Inversion/Transition"), the conclusion that "boundary parameters depend only on radii near the horizon" is not strongly supported by Figures 16-18. Further explanation is needed to clarify why the parameter effects appear "mirrored" after inversion.
- (5) Does $\lambda_L/\lambda_{LaRN} > 1$ imply stronger quantum chaos? What are the implications for black hole information loss or the AdS/CFT correspondence?

(6) While "rotating shock waves" and "wormhole traversability" are mentioned, they are not linked to the current results. It would be helpful to outline more concrete research goals (e.g., by proposing specific models).

Declarations

Potential competing interests: No potential competing interests to declare.