Research Article

Noise-Induced Decoherence-Free Zones for Anyons

Eric Bittner[1]

1. University of Houston, United States

We develop a stochastic framework for anyonic systems in which the exchange phase is promoted from a fixed parameter to a fluctuating quantity. Starting from the Stratonovich stochastic Liouville equation, we perform the Stratonovich–Itô conversion to obtain a Lindblad master equation that ties the dissipator directly to the distorted anyon algebra. This construction produces a statistics–dependent dephasing channel, with rates determined by the eigenstructure of the real symmetric correlation matrix D_{ab}. The eigenvectors of D select which collective exchange currents—equivalently, which irreducible representations of the system—are protected from stochastic dephasing, providing a natural mechanism for decoherence-free subspaces and noise-induced exceptional points. The key result of our analysis is the universality of the optimal statistical angle: in the minimal two-site model with balanced gain and loss, the protected mode always minimizes its dephasing at $\theta^\star = \pi/2$, independent of the specific form of D. This robustness highlights a simple design rule for optimizing coherence in noisy anyonic systems, with direct implications for ultracold atomic realizations and other emerging platforms for fractional statistics.

Corresponding author: Eric R. Bittner ebittner@central.uh.edu

I. Introduction

Quantum statistics provides the fundamental classification of indistinguishable particles. In three spatial dimensions, particle exchange produces only two consistent outcomes: bosons, which acquire a trivial phase $e^{i0} = +1$, and fermions, which acquire a minus sign $e^{i\pi} = -1$. This restriction follows from the topology of configuration space in 3D, where exchange paths can be continuously deformed into one another and only the two representations of the permutation group are allowed.

 doi.org/10.32388/95VF2I

In reduced dimensionality, the situation changes dramatically. In two dimensions the braid group replaces the permutation group, and continuous families of representations become possible. Leinaas and Myrheim first showed that indistinguishable particles in two dimensions can obey generalized exchange statistics interpolating between bosons and fermions [1], while Wilczek introduced the term *anyon* and provided explicit models for such particles [2]. Fractional quantum Hall states offered the first concrete physical setting in which such excitations arise, with Laughlin's wavefunction [3] and the Arovas–Schrieffer–Wilczek analysis of Berry phases [4] establishing the connection between fractional charge and fractional statistics. Extensions to one-dimensional systems further revealed that constrained motion can effectively realize anyon-like exchange [5][6]. Anyons are now recognized as central to our understanding of topological phases of matter, from spin liquids [7] to non-Abelian braiding proposals for quantum computation [8].

Experimental realizations have advanced dramatically. For many years, experimental evidence for anyons was restricted to indirect signatures in two-dimensional electron gases. Interferometric probes in the fractional quantum Hall regime have provided strong support for fractional statistics, while more recent work has advanced toward direct braiding experiments. A breakthrough came with the realization of one-dimensional anyons in ultracold atomic systems: Kwan *et al.* engineered a density-dependent Peierls phase in an optical lattice and demonstrated control of an arbitrary statistical angle, confirming anyonic behavior through quantum walks, Hanbury Brown–Twiss interference, and bound-state formation in the two-particle sector [9]. These advances establish that not only can the mean statistical phase be tuned, but anyons can be probed in platforms with high degrees of controllability.

In realistic settings, the statistical phase is never perfectly fixed. Environmental coupling, microscopic disorder, or engineered modulation can introduce fluctuations about the mean value of θ. These fluctuations render the effective exchange factor stochastic,

$$e^{i\theta} \longrightarrow e^{i(\theta+\phi(t))}, \tag{1}$$

with $\phi(t)$ encoding dynamical noise. Our central goal in this work is to develop a consistent framework for describing anyons subject to such fluctuating exchange phases.

Recent theoretical work has also shown that noise can play a constructive role in quantum dynamics. In particular, we have demonstrated that correlated noise can drive phase synchronization between otherwise independent quantum systems [10][11], and related studies have emphasized the broader importance of correlation structure in open-system dynamics [12]. These works highlight how the form of the noise correlation matrix can select protected collective modes and suppress decoherence. The present work extends this line of research into the realm of anyonic statistics, showing that fluctuations of the statistical phase lead to analogous protection mechanisms and, under suitable conditions, to noise-induced exceptional points. In this way, the framework developed here builds directly on our synchronization results while situating them within the emerging literature on correlated noise in quantum systems.

In this paper we formulate this problem by assigning each tunneling link between sites a stochastic phase variable $\phi_a(t)$. Starting from the Stratonovich stochastic Liouville equation, we perform the Stratonovich–Itô conversion to obtain a Lindblad master equation. This yields a statistics–dependent pure dephasing channel of the form $-\frac{\Gamma_\theta}{2}[K_\theta,[K_\theta,\rho]]$, where the exchange–current operator K_θ encodes the distorted anyon algebra. From this starting point, we obtain several new results. First, we demonstrate that stochastic exchange phases generate a dephasing channel whose rate depends explicitly on the statistical angle. Second, in the minimal two-site broken-$\mathcal{PT}$ model, we show that the protected eigenmode is universally stabilized at an optimal angle $\theta^\star = \pi/2$, where the dephasing channel vanishes in the absence of residual relaxation. Third, for multiple noisy links, correlations between phase fluctuations are captured by a correlation matrix D_{ab}. Real-symmetric correlations yield collective exchange currents and decoherence-free subspaces when D loses rank. Finally, we establish that noise-induced exceptional points cannot arise for real-symmetric D, but become possible when D carries complex or chiral correlations, which render the dissipator non-normal.

Taken together, these results provide a systematic framework for understanding how fluctuating statistical phases affect anyonic coherence and protection. By tying stochastic dephasing directly to the underlying anyon algebra, we offer a set of design rules for engineering "designer phases" in noisy anyonic systems, with relevance for ultracold atomic realizations and other platforms where fractional statistics are emerging as experimentally accessible degrees of freedom.

II. Theory

A. From Distorted Anyon Algebra to the Model Hamiltonian

To motivate our model Hamiltonian we begin with the distorted algebra that defines abelian anyons. In a fixed site ordering $(1 < 2 < \cdots)$, the annihilation operators obey

$$a_i a_j = e^{i\theta} a_j a_i, a_i a_j^\dagger = e^{-i\theta} a_j^\dagger a_i \quad (i < j), \tag{2}$$

doi.org/10.32388/95VF2I

with θ the anyonic statistical angle. This algebra can be realized by bosonic operators dressed with Jordan–Wigner strings,

$$a_j = b_j \exp\left(i\theta \sum_{k<j} n_k\right), n_k = b_k^\dagger b_k, \tag{3}$$

so that exchanging two particles produces the phase factor $e^{i\theta}$.

For two sites, labeled 1 and 2, the exchange operator that transfers an excitation from site 2 to site 1 carries this intrinsic anyonic phase,

$$\mathcal{T}_\theta = a_1^\dagger a_2 e^{i\theta}, \tag{4}$$

and the associated Hermitian exchange current is

$$K_\theta = i\left(\mathcal{T}_\theta - \mathcal{T}_\theta^\dagger\right) = i\left(a_1^\dagger a_2 e^{i\theta} - a_2^\dagger a_1 e^{-i\theta}\right). \tag{5}$$

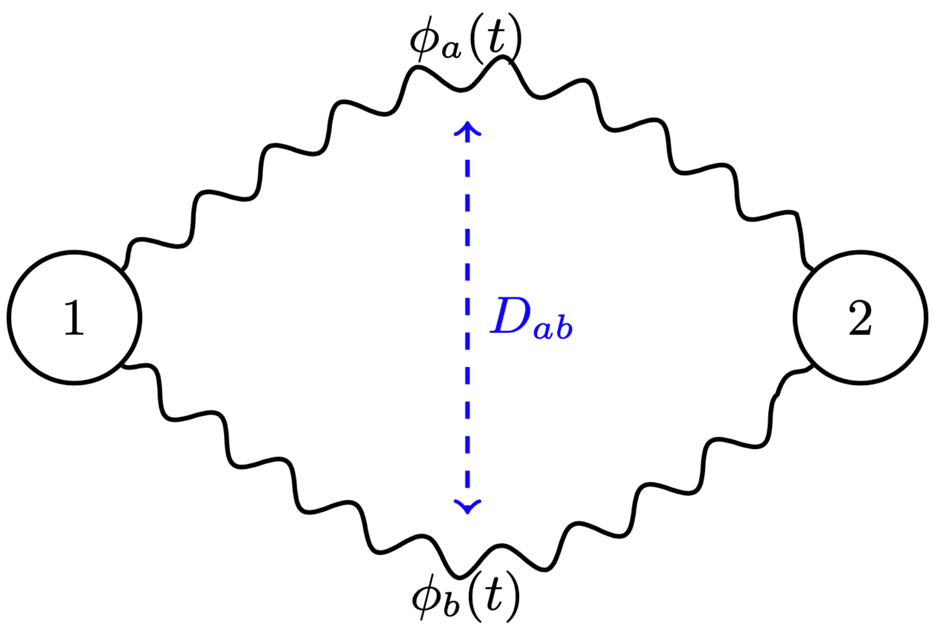

Figure 1. Two sites connected by fluctuating paths with random phases $\phi_a(t)$ and $\phi_b(t)$. A correlation D_{ab} between the two noise sources captures the strength of correlated environmental fluctuations.

In a realistic environment, the statistical phase imprinted on each tunneling link is not fixed. Each physical path may accumulate an additional stochastic phase $\phi_a(t)$ due to environmental fluctuations or

doi.org/10.32388/95VF2I

engineered modulation. This amounts to promoting the statistical factor $e^{i\theta}$ to a stochastic link variable $e^{i(\theta+\phi_a(t))}$,

$$\mathcal{T}_\theta \longrightarrow \mathcal{T}_\theta e^{i\phi_a(t)} = a_1^\dagger a_2 e^{i(\theta+\phi_a(t))}. \tag{6}$$

For multiple paths $a, b, \ldots$, each link carries its own $\phi_a(t)$, with correlations encoded by the diffusion (or quantum noise) matrix $\langle d\phi_a \, d\phi_b \rangle = 2D_{ab}dt$.

We therefore arrive at the physical picture illustrated schematically in Fig. 1 which illustrates two sites coupled by two noisy tunneling channels, each carrying both the intrinsic anyonic statistical phase θ and extrinsic stochastic components $\phi_a(t)$ and $\phi_b(t)$ which may be correlated via D_{ab}. If multiple tunneling paths $a, b, \cdots$ connect sites 1 and 2, each contributes its own exchange operator $\mathcal{T}_\theta^{(a)}$ and stochastic phase $\phi_a(t)$. The statistics of the stochastic phases are characterized by the correlation matrix

$$\langle d\phi_a \, d\phi_b \rangle = 2D_{ab}dt, \tag{7}$$

which encodes how noise on different links is correlated. The structure of D_{ab} will determine the collective dephasing modes of the system, as we analyze in the following sections.

The effective Hamiltonian is then

$$H(t) = H_0 - \sum_a J_a \left(\mathcal{T}_\theta^{(a)} e^{i\phi_a(t)} + \mathcal{T}_\theta^{(a)\dagger} e^{-i\phi_a(t)} \right), \tag{8}$$

where J_a is the tunneling amplitude along link a. Each link therefore carries both the intrinsic anyonic statistical phase $e^{i\theta}$ and an extrinsic stochastic modulation $e^{i\phi_a(t)}$.

For the case of a single exchange path, we have the effective Hamiltonian

$$H(t) = H_0 - J \left(\mathcal{T}_\theta e^{i\phi(t)} + \mathcal{T}_\theta^\dagger e^{-i\phi(t)} \right), \mathcal{T}_\theta = a_1^\dagger a_2 e^{i\theta}, \tag{9}$$

where H_0 contains the local mode energies and θ is the fixed anyonic statistical phase.

For small and rapidly fluctuating $\phi(t)$ we expand the exponential to linear order and collect terms into the Hermitian exchange–current operator. The Hamiltonian then takes the approximate form

$$H(t) \simeq H_0 - JK_\theta \phi(t). \tag{10}$$

We assume $\phi(t)$ undergoes phase diffusion according to a stochastic differential equation (SDE) such as

$$d\phi(t) = \sqrt{2D_\phi} dW_t, \tag{11}$$

where W_t is a standard Wiener process and D_ϕ is the phase–diffusion constant. The associated Stratonovich stochastic Liouville equation becomes

$$d\rho = -i[H_0, \rho]dt - iJ[K_\theta, \rho] \circ d\phi(t) \tag{12}$$

where the symbol $\circ$ denotes Stratonovich integration. We distinguish Itô from Stratonovich stochastic calculus by notation: $dX = Adt + BdW_t$ denotes the Itô form, while $dX = Adt + B \circ dW_t$ indicates the Stratonovich form, with the symbol $\circ$ specifying the Stratonovich interpretation.

To connect with ensemble-averaged dynamics we convert Eq. (12) into Itô form. For a Stratonovich stochastic differential equation

$$d\rho = \mathcal{A}(\rho)dt + \mathcal{G}(\rho) \circ dW_t, \tag{13}$$

the corresponding Itô form reads

$$d\rho = \left[\mathcal{A}(\rho) + \frac{1}{2}\mathcal{G}'(\rho) \cdot \mathcal{G}(\rho)\right] dt + \mathcal{G}(\rho)dW_t, \tag{14}$$

where $\mathcal{G}'$ is the Fréchet derivative. In our case, the noise superoperator is linear in ρ,

$$\mathcal{G}(\rho) = \sqrt{2D_\phi}(-iJ)[K_\theta, \rho]. \tag{15}$$

Hence $\mathcal{G}'(\rho) \cdot \mathcal{X} = \mathcal{G}(\mathcal{X})$ and

$$\frac{1}{2}\mathcal{G}(\mathcal{G}(\rho)) = -J^2 D_\phi[K_\theta, [K_\theta, \rho]]. \tag{16}$$

The full Itô equation of motion is therefore

$$\begin{aligned} d\rho = &\left(-i[H_0, \rho] - J^2 D_\phi[K_\theta, [K_\theta, \rho]]\right) dt \\ &+ \sqrt{2D_\phi}(-iJ)[K_\theta, \rho]dW_t. \end{aligned} \tag{17}$$

Averaging over the stochastic increments removes the explicit noise term in Eq. (17), leaving

$$\dot{\rho} = -i[H_0, \rho] - \frac{\Gamma_\theta}{2}[K_\theta, [K_\theta, \rho]], \Gamma_\theta = 2J^2 D_\phi. \tag{18}$$

This is a deterministic master equation in Lindblad form with a single Hermitian jump operator,

$$L_\theta = \sqrt{\Gamma_\theta} K_\theta. \tag{19}$$

Random exchange phases therefore produce pure dephasing in the eigenbasis of K_θ, with rates that depend explicitly on the anyonic statistical angle θ.

As an aside, the approach generalizes to other noise models. For instance, if the random phase follows an Ornstein–Uhlenbeck process with correlation function $C_\phi(\tau) = \sigma^2 e^{-|\tau|/\tau_c}$, the white-noise limit yields

$$\Gamma_\theta = 2J^2 \int_0^\infty C_\phi(\tau)d\tau = 2J^2\sigma^2\tau_c \tag{20}$$

where σ governs the gaussian width of fluctuations (classically proportional to temperature) and τ is the bath correlation time. We also note that the random phase can also be treated as an operator-valued phase generated by a quantum bath. In this case we still obtain the Lindblad form, however, with a rate proportional to the zero-frequency part of the noise-spectrum.

In practice, however, quasiparticles also undergo population relaxation due to coupling to thermal baths or lossy reservoirs. The combined dynamics can be written schematically as

$$\dot{\rho} = -i[H_0, \rho] - \frac{\Gamma_\theta}{2}[K_\theta, [K_\theta, \rho]] + \sum_\alpha \gamma_\alpha \mathcal{D}[L_\alpha]\rho, \tag{21}$$

where the last term collects Lindblad dissipators $\mathcal{D}[L]\rho = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$, which generate population relaxation (and, for thermal baths, detailed-balance thermalization). The essential physics is the *competition* between the statistics–dependent pure dephasing channel and the relaxation channel. The former damps coherences without altering populations, while the latter reshuffles populations and sets lifetime broadening. This same competition lies at the heart of the synchronization mechanism analyzed in our recent work on noisy anyonic systems: correlations in the stochastic phases can suppress one channel relative to the other, thereby stabilizing collective phase locking. In the $\mathcal{PT}$-symmetric regime this balance yields robust coherent oscillations, whereas in the broken phase the non-orthogonality of modes amplifies both mechanisms and accelerates decoherence. Pure dephasing suppresses coherences without altering populations, while relaxation reshuffles populations and can either restore or destabilize coherence depending upon the symmetry of the jump operators.

B. Decoherence free states

Having established that stochastic exchange phases give rise to a statistics–dependent dephasing channel in direct competition with population relaxation, we now turn to the conditions under which coherence can persist. Of particular interest are situations where the system supports modes that are insensitive to the noisy exchange phases. These decoherence-free states are selected by the structure of the exchange-current operator K_θ and by correlations among different tunneling paths. Identifying such protected modes allows us to quantify both their lifetimes and the statistical angles at which they are optimally stabilized.

To explore this, let us suppose there exists a single mode that is protected from the noise, $|u\rangle$, and that we initiate the system in that state as a pure state with density operator $\rho_u(0) = |u\rangle\langle u|$. Even when relaxation is suppressed, the stochastic exchange-phase channel derived above remains active and imposes a finite lifetime of this protected mode. For this, we evaluate the instantaneous decay of the survival probability $s_u(t) = tr\left(\rho_u(t)\rho_u(0)\right)$ at $t = 0$:

$$\begin{aligned} \frac{d}{dt}s_u(t)\Big|_{t=0} &= tr\left(\rho_u\dot{\rho}\right) = -\Gamma_\theta Var_{|u\rangle}\left(K_\theta\right), \\ Var_{|u\rangle}\left(K_\theta\right) &:= \langle u|\,K_\theta^2\,|u\rangle - \langle u|\,K_\theta|u\rangle^2. \end{aligned} \tag{22}$$

Thus, the *effective decoherence rate* of the mode is

$$\gamma_{\phi,u}(\theta) = \Gamma_\theta Var_{|u\rangle}\left(K_\theta\right), \tau(\theta) = \gamma_\phi(\theta)^{-1}, \tag{23}$$

which depends on the statistical angle exclusively through the exchange-current operator K_θ.

The statistics-dependent dephasing rate $\gamma_\phi(\theta)$ exhibits a simple and universal angular dependence. In Fig. 2, we plot the normalized rate $\gamma_\phi(\theta)/J$ as a function of the statistical phase θ for several values of the noise correlation coefficient ξ. Although the overall magnitude and curvature of $\gamma_\phi(\theta)$ depend on the degree of interlink correlation, the location of the minimum remains fixed at $\theta^\star = \pi/2$. This point corresponds to the half-fermionic statistics, where the exchange-current operator K_θ becomes orthogonal to the protected mode, minimizing the variance $Var_{|u\rangle}(K\theta)$ and hence the dephasing. The invariance of $\theta^\star$ with respect to ξ demonstrates that the protection mechanism is purely algebraic and not sensitive to the detailed structure of the noise correlation matrix D_{ab}. All rates are expressed in dimensionless units scaled by the exchange coupling J, with $J = 0.1$ in the simulations shown.

doi.org/10.32388/95VF2I

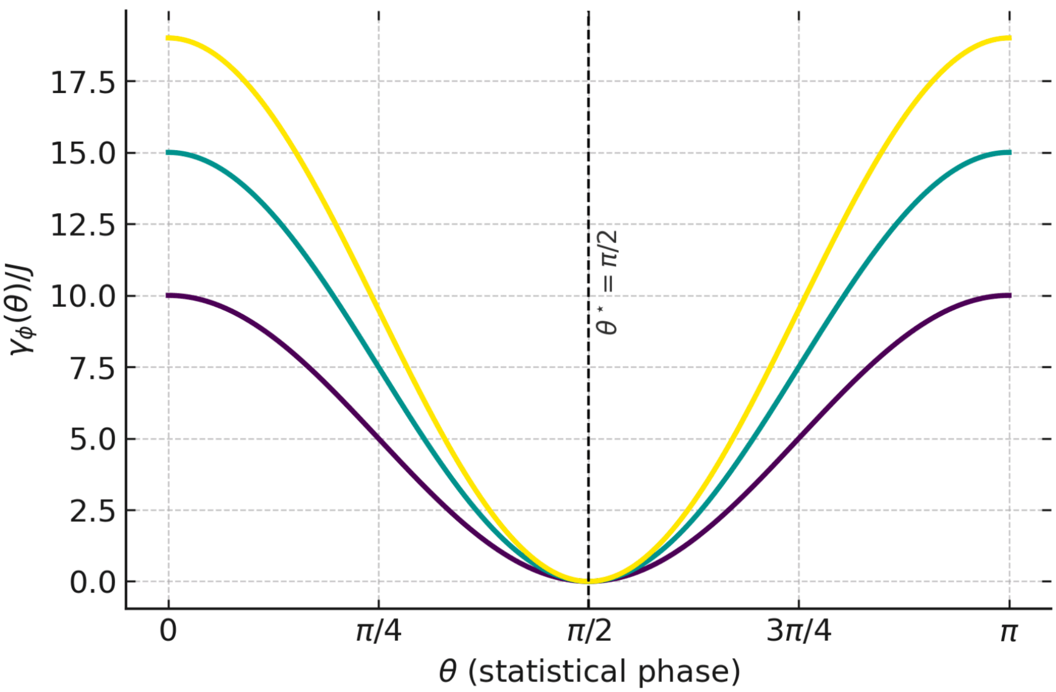

Figure 2. Dephasing rate scaled by exchange coupling, $\gamma_\phi(\theta)/J$, versus statistical phase θ for three representative correlation strengths: $\xi = 0$, $\xi = 0.5$, and $\xi = 0.9$. All curves exhibit a universal minimum at $\theta^\star = \pi/2$, independent of ξ, demonstrating the robustness of the half-fermionic protection point. Rates are expressed in dimensionless units normalized by the exchange coupling J (with $J = 0.1$ in the underlying simulation).

C. Two-mode single-excitation manifold

In the one-particle subspace spanned by $\{|1\rangle, |2\rangle\}$, the exchange-current operator takes the form

$$K_\theta = -\sin\theta\sigma_x + \cos\theta\sigma_y, K_\theta^2 = 1, \tag{24}$$

so that $Var_{|u\rangle}(K_\theta) = 1 - (\mathbf{n}(\theta)\cdot\mathbf{r}_u)^2$, where $\mathbf{n}(\theta) = (-\sin\theta, \cos\theta, 0)$ and $\mathbf{r}_u = (\langle u|\sigma_x|u\rangle, \langle u|\sigma_y|u\rangle, \langle u|\sigma_z|u\rangle)$ is the Bloch vector of the initial state. Substituting into Eq. (23) yields

$$\gamma_{\phi,u}(\theta) = \Gamma_\theta\left[1 - (\mathbf{n}(\theta)\cdot\mathbf{r}_u)^2\right]. \tag{25}$$

We therefore *maximize* the lifetime by maximizing $|\mathbf{n}(\theta)\cdot\mathbf{r}_u|$, i.e. by aligning $\mathbf{n}(\theta)$ with the in-plane projection of $\mathbf{r}_u$. Writing $\mathbf{r}_u^{\parallel} = (r_x, r_y, 0)$ and $\arg(r_x + ir_y) =: \varphi_u$, the optimal statistical angle is

doi.org/10.32388/95VF2I

$$\theta^\star = \frac{\pi}{2} - \varphi_u(mod\pi), \gamma_{\phi,u}^{\min} = \Gamma_\theta\left[1 - \|\mathbf{r}_u^\parallel\|^2\right]. \tag{26}$$

Physically, we choose θ so that $|u\rangle$ is *as nearly an eigenstate of K_θ as possible*. If $|u\rangle$ is exactly an eigenstate of K_θ (so $\|\mathbf{r}_u^\parallel\| = 1$ and $\mathbf{n}(\theta^\star) \parallel \mathbf{r}_u^\parallel$), then $Var_{|u\rangle}(K_{\theta^\star}) = 0$ and the stochastic exchange-phase channel does not decohere the protected mode. Thus, the lifetime of the mode becomes *infinite* in the ideal absence of relaxation.

This is a significant conclusion: *in the simplest two–mode broken-$\mathcal{PT}$ model, the optimal statistical angle collapses to a universal value of $\theta^\star = \pi/2$*. At this angle the exchange–current operator reduces to

$$K_{\pi/2} = -\sigma_x, \tag{27}$$

so that the protected mode is effectively an eigenstate of K_θ and the stochastic exchange–phase channel cannot induce decoherence. The corresponding lifetime $\tau_{\text{eff},u}(\theta^\star)$ diverges in the absence of residual relaxation.

This result is robust even in the presence of residual relaxation channels as we can see by simply adding an additional γ_{res} to the relaxation rate,

$$\gamma_{\text{eff},u}(\theta) \ = \gamma_{res} + \Gamma_\theta Var_{|u\rangle}\ (K_\theta). \tag{28}$$

Hence, the lifetime is given by

$$\tau_{\text{eff},u}(\theta) \ = \left[\gamma_{res} + \Gamma_\theta(1 - (\mathbf{n}\cdot\mathbf{r}_u)^2)\right]^{-1}. \tag{29}$$

Since our optimization maximizes $\Gamma_\theta Var_{|u\rangle}\ (K_\theta)$, the optimal statistical angle $\theta^\star$ is still given by Eq. (26).

The universality of $\theta^\star = \pi/2$ stems from the real structure of the protected eigenmode in the broken-$\mathcal{PT}$ regime: the absence of an intrinsic phase between site amplitudes forces alignment with the σ_x axis. Nontrivial dependence of $\theta^\star$ on system parameters requires a complex Bloch vector with $r_y \neq 0$, which can arise from asymmetric couplings or coupling to a chiral (complex–correlated) bath. Thus, while the universal angle $\pi/2$ reflects the robustness of the minimal model, it also highlights a route for engineering tunable protection through environmental chirality and correlation.

a. Correlated links / extended graphs

For systems with multiple links a carrying exchange-current operators $K_\theta^{(a)}$ and a positive semidefinite phase-noise matrix Γ_{ab}, the protected-mode dephasing generalizes to

$$\gamma_{\phi,u}(\theta) = \frac{1}{2}\sum_{a,b}\Gamma_{ab}Cov_{|u\rangle}\left(K_\theta^{(a)}, K_\theta^{(b)}\right), \tag{30}$$

doi.org/10.32388/95VF2I

in which the covariance between channels is given by

$$Cov_{|u\rangle}(A,B) := \langle u|\,\frac{1}{2}\{A,B\}\,|u\rangle - \langle u|\,A\,|u\rangle\,\langle u|\,B\,|u\rangle. \tag{31}$$

Equation (31) suggests that correlated phase noise can *reduce* the protected-mode dephasing when the covariances interfere destructively, providing an additional design knob complementary to the θ statistical angle.

D. Correlated Phase Noise on Two Links: Rates, DFS, and (the Absence of) Decoherence EPs

We now consider two links $a = 1, 2$ with exchange–current operators $K_\theta^{(1)}$ and $K_\theta^{(2)}$. Correlated stochastic exchange phases $\{\phi_a(t)\}$ obey

$$\langle d\phi_a\, d\phi_b\rangle = 2\mathsf{D}_{ab}\,dt, \mathsf{D} = \begin{pmatrix} 1 & \xi \\ \xi & 1 \end{pmatrix}, |\xi| \leq 1, \tag{32}$$

and we take equal tunneling amplitudes for clarity ($J_1 = J_2 = J$). Ignoring population relaxation, the ensemble–averaged Liouvillian reads

$$\dot{\rho} = -i[H_0, \rho] - \frac{1}{2}\sum_{a,b=1}^{2}\Gamma_{ab}[K_\theta^{(a)}, [K_\theta^{(b)}, \rho]], \tag{33}$$

with

$$\Gamma_{ab} = 2J^2\mathsf{D}_{ab}. \tag{34}$$

Define symmetric/antisymmetric combinations $K_\theta^{(\pm)} = (K_\theta^{(1)} \pm K_\theta^{(2)})/\sqrt{2}$. Since D is real symmetric and positive semidefinite, it diagonalizes as

$$\mathsf{D} = U diag(1+\xi,\, 1-\xi)U^\top, U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{35}$$

Equation (33) becomes a *sum of two independent dephasers*:

$$\dot{\rho} = -i[H_0, \rho] - \frac{\gamma_+}{2}[K_\theta^{(+)}, [K_\theta^{(+)}, \rho]] - \frac{\gamma_-}{2}[K_\theta^{(-)}, [K_\theta^{(-)}, \rho]], \tag{36}$$

with

$$\gamma_\pm = 2J^2(1 \pm \xi). \tag{37}$$

Thus correlated phases simply select two *collective* exchange currents with rates $\gamma_\pm$.

At $\xi = +1$ we have $\gamma_- = 0$ while $\gamma_+ = 4J^2$: the antisymmetric channel $K_\theta^{(-)}$ is noiseless, and any operator commuting with $K_\theta^{(+)}$ forms a decoherence–free subspace (DFS). At $\xi = -1$ the roles swap ($\gamma_+ = 0$, $\gamma_- = 4J^2$). These are *rank–deficiency* points of D (and Γ), where one stochastic eigenmode is completely suppressed. They are *not* exceptional points (EPs) in the spectral sense; the Liouvillian remains diagonalizable. This occurs because the generator (33) is a sum of *double commutators with Hermitian operators*. Equivalently, it is a sum of Lindblad dissipators with *Hermitian* jump operators:

$$\mathcal{L}[\rho] = -i[H_0, \rho] + \sum_{\nu=\pm} \gamma_\nu \left(K_\theta^{(\nu)} \rho K_\theta^{(\nu)} - \frac{1}{2} \{ (K_\theta^{(\nu)})^2, \rho \} \right). \tag{38}$$

Such a Liouvillian is *normal* with respect to the Hilbert–Schmidt inner product; its spectrum is real and it is diagonalizable by construction. Degeneracies of decay rates (e.g. $\gamma_+ = \gamma_-$ at $\xi = 0$) do not create nontrivial Jordan blocks. Hence, with real symmetric D and Hermitian K's, one does *not* obtain a spectral EP; instead, one encounters DFS formation when $rank(\mathsf{D})$ drops.

In contrast, when the environmental noise correlates the relaxation channels of distinct sites, the effective Lindblad operators couple collective annihilation modes rather than acting locally. A minimal realization is

$$L_\pm = \frac{1}{2} \sqrt{1 \pm \xi} \sqrt{\gamma} (a_1 \pm a_2),$$

where ξ quantifies the degree of correlation between the local dissipative baths. These non-local jump operators mix the two sites and render the full Liouvillian *non-normal* under the Hilbert–Schmidt inner product, even though each $L_\pm$ remains annihilation-like. As ξ is tuned, the Liouvillian eigenvalues can coalesce together with their corresponding eigenmodes, giving rise to *decoherence exceptional points* (EPs) that separate synchronized and desynchronized dynamical regimes. This mechanism was analyzed in detail in our recent works on noise-induced synchronization and Liouvillian spectral coalescence (Refs. [13][14]), where correlated relaxation was shown to induce spontaneous phase locking and non-Hermitian degeneracies in two coupled dissipative oscillators.

III. Quantum Phase Noise: From Microscopic Baths to QSDE

Up to this point we treated $d\phi$ as a classical (commuting) increment and obtained a statistics–dependent pure dephasing channel. We now upgrade $\phi(t)$ to an *operator–valued* phase generated by a quantum bath. Let $F(t)$ be a Hermitian bath force and define the (Heisenberg) phase operator $\phi(t) = \int_0^t F(\tau) d\tau$. The intermode coupling reads

$$H(t) \simeq H_0 - JK_\theta \phi(t) \quad \Rightarrow \quad H_{int}(t) = -JK_\theta \otimes F(t), \tag{39}$$

with $K_\theta = i(\mathcal{T}_\theta - \mathcal{T}_\theta^\dagger)$ as before.

doi.org/10.32388/95VF2I

A. Born–Markov (Gaussian) derivation.

Assume a stationary Gaussian bath with correlation and response

$$C(\tau) = \frac{1}{2}\langle\{F(\tau), F(0)\}\rangle, \chi(\tau) = \frac{i}{\hbar}\Theta(\tau)\langle[F(\tau), F(0)]\rangle, \tag{40}$$

and spectra $S_{FF}(\omega) = \int_{-\infty}^{\infty} C(\tau)e^{i\omega\tau}d\tau$, $\widetilde{\chi}(\omega) = \int_0^{\infty}\chi(\tau)e^{i\omega\tau}d\tau$. To second order in J (cumulant/Born expansion) and within the Markov approximation one obtains

$$\dot{\rho} = -i\,[H_0 + H_{LS}, \rho] - \frac{\Gamma_\theta}{2}[K_\theta, [K_\theta, \rho]], \tag{41}$$

with the *Lamb shift*

$$H_{LS} = J^2 \Xi K_\theta^2, \Xi = P\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{S_{FF}(\omega)}{\omega} = \int_0^{\infty}\chi(\tau)d\tau, \tag{42}$$

and the pure dephasing rate set by the *symmetrized* zero–frequency noise,

$$\Gamma_\theta = \ 2J^2 S_{FF}(0) \tag{43}$$

(we set $\hbar = 1$). Equation (41) is identical in form to the classical result, with the replacement $D_\phi \mapsto S_{FF}(0)$. At finite temperature T, $S_{FF}(0)$ carries the usual thermal factor (e.g. $\coth(\omega/2T)$ in an Ohmic bath), while squeezed or nonclassical baths modify $S_{FF}(0)$ accordingly.

B. QSDE (Hudson–Parthasarathy) route.

Alternatively, couple the system to a bosonic input field and write a quantum stochastic differential equation for the joint unitary U_t:

$$dU_t = \left\{L dB_t^\dagger - L^\dagger dB_t - \left(\frac{1}{2}L^\dagger L + iH_0\right)dt\right\}U_t, L = \sqrt{\gamma}K_\theta, \tag{44}$$

with Itô table (thermal occupancy n_{th}) $dB_t dB_t^\dagger = (n_{th}+1)dt$, $dB_t^\dagger dB_t = n_{th}\,dt$, others $= 0$. Tracing out the field gives the Lindblad master equation

$$\dot{\rho} = -i[H_0, \rho] + \gamma\left(K_\theta \rho K_\theta - \frac{1}{2}\{K_\theta^2, \rho\}\right), \tag{45}$$

i.e. $-\frac{\Gamma_\theta}{2}[K_\theta, [K_\theta, \rho]]$ with $\Gamma_\theta = 2\gamma$. For a thermal (or squeezed) input field, γ inherits the appropriate quantum noise prefactors and frequency dependence. In this language the "quantum Stratonovich" symbol is not used; one works directly with the quantum Itô rules.

doi.org/10.32388/95VF2I

a. Multiple links and quantum correlations.

For links a, b with operators $K_\theta^{(a)}$ and bath forces $F_a(t)$,

$$H_{int} = -\sum_a J_a K_\theta^{(a)} \otimes F_a(t), S_{ab}(\omega) = \int d\tau \frac{1}{2} \langle \{F_a(\tau), F_b(0)\} \rangle e^{i\omega\tau}. \tag{46}$$

In the white–noise Markov limit,

$$\dot{\rho} = -i[H_0 + H_{LS}, \rho] - \frac{1}{2} \sum_{a,b} \Gamma_{ab} [K_\theta^{(a)}, [K_\theta^{(b)}, \rho]], \Gamma_{ab} = 2 J_a J_b S_{ab}(0), \tag{47}$$

with $H_{LS} \propto \sum_{a,b} J_a J_b \Xi_{ab} \{K_\theta^{(a)}, K_\theta^{(b)}\}$ and Ξ_{ab} the principal–value transforms of the bath susceptibilities. Complete positivity requires the *matrix* $S_{ab}(0)$ to be positive semidefinite (quantum Bochner theorem). If $S_{ab}(0)$ is real symmetric, the dissipator is a sum of Hermitian–jump channels and remains normal (no spectral EP). Complex/chiral $S_{ab}(0)$ (nonreciprocal baths, feedback, or squeezed–phase correlations) render the dissipator non–normal and can produce genuine decoherence exceptional points.

C. Summary

Treating $d\phi$ as quantum noise does not alter the *structure* of the statistics–dependent dephasing channel: we still obtain $-\frac{\Gamma_\theta}{2}[K_\theta, [K_\theta, \rho]]$, but with a rate fixed by the *symmetrized quantum* zero–frequency noise $S_{FF}(0)$ and with a Lamb shift from the antisymmetric (susceptibility) part. In multi–link settings the quantum cross–spectrum $S_{ab}(0)$ is the fundamental object that controls rates, correlations, and the possibility of decoherence EPs.

IV. Discussion

Our analysis shows that the impact of stochastic exchange phases on anyonic coherence is governed by the structure of the correlation matrix D_{ab}. Because D is real and symmetric, it can always be diagonalized into a set of orthogonal eigenmodes. Each eigenvector corresponds to a collective exchange current—or, equivalently, an irreducible representation of the system—that couples to the environment with a rate given by the corresponding eigenvalue. Modes associated with vanishing eigenvalues are protected from dephasing and form decoherence-free subspaces. This establishes a clear design principle: by engineering correlations so that D develops null modes, one can guarantee that specific collective excitations remain robust against stochastic decoherence.

doi.org/10.32388/95VF2I

The role of D is therefore to select which irreducible representations of the anyon system couple to noise. Exceptional points in the Liouvillian spectrum emerge when these noise-selected modes coalesce with relaxation channels or Hamiltonian couplings, producing spectral degeneracies accompanied by non-orthogonal eigenvectors. This mechanism is consistent with our earlier work on noise-induced synchronization, where the eigenstructure of the correlation matrix dictated which collective phases became locked and which decayed.

The most striking result of the present analysis is the universality of the optimal statistical angle. In the minimal two-site model with balanced gain and loss, we find that the protected mode always minimizes its dephasing at $\theta^{\star} = \pi/2$, independent of the specific form of D. In other words, the alignment between the protected mode and the eigenbasis of the exchange-current operator K_{θ} occurs universally at half-fermionic statistics. This insensitivity to noise correlations highlights a robust design principle: regardless of how the stochastic phases are correlated, the half-fermion point remains optimal for coherence protection.

Extensions of this framework include colored noise, such as Ornstein–Uhlenbeck processes, which endow the rates with frequency dependence, and more general graph geometries, where D becomes the Laplacian of a correlation network. In all cases the essential features persist: the eigenvectors of D dictate which modes are protected, while the universality of $\theta^{\star} = \pi/2$ provides a simple and powerful rule for optimizing anyonic coherence.

Other recent theoretical advances have underscored the fundamental role of statistics in determining coherence and dynamical distinguishability in low-dimensional quantum systems. Mackel, Yang, and del Campo [15] demonstrated a universal orthogonality catastrophe for one-dimensional anyons, showing that the overlap between states with different statistical parameters decays in a manner governed solely by the exchange statistics. Our present framework extends this notion from a static geometric effect to a fully dynamical one: by promoting the statistical phase θ to a stochastic variable, we derive the resulting Lindblad dephasing channel and identify the universal half-fermionic protection point at $\theta^{\star} = \pi/2$. More generally, our treatment connects to recent studies of decoherence and spectral universality in open quantum systems. del Campo and collaborators [16] analyzed non-exponential survival probabilities and demonstrated universal long-time scaling laws for decoherence in complex environments. Zhao *et al.* [17] further showed that decoherence in generic Markovian processes can be characterized spectrally by the eigenvalue distribution of the Liouvillian superoperator. Within our stochastic anyon framework, the correlation matrix D plays an analogous spectral role: its eigenstructure selects protected and dissipative modes, giving rise to decoherence-free subspaces and, when correlations become complex or chiral, genuine Liouvillian exceptional points. Taken together, these developments situate the present work within a growing effort to unify the geometric and statistical origins of decoherence in quantum many-body systems.

Statements and Declarations

Conflicts of Interest

The author declares no competing interests.

Data Accessibility

Computational details. All symbolic and numerical routines (e.g., evaluation of $Var_{|u\rangle}(K_\theta)$, search for $\theta^\star$, and lifetime estimates) are provided in the Supplemental Information (SI).

Author Contribution

ERB conceived the project, developed the theoretical framework, and carried out all analytical and numerical calculations. He wrote the manuscript, prepared the figures, and approved the final version for submission.

Acknowledgements

The work at the University of Houston was supported by the National Science Foundation under CHE-2404788 and the Robert A. Welch Foundation (E-1337).

References

1. ^Leinaas JM, Myrheim J (1977). "On the Theory of Identical Particles" [К Теории Тождественных Частиц]. Il Nuovo Cimento B. **37**(1):1–23. doi:10.1007/bf02727953.
2. ^Wilczek F (1982). "Quantum Mechanics of Fractional-Spin Particles." Phys. Rev. Lett. **49**(14):957–959. doi:10.1103/physrevlett.49.957.
3. ^Laughlin RB (1983). "Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations." Phys. Rev. Lett. **50**(18):1395–1398. doi:10.1103/physrevlett.50.1395.

4. ^Arovas D, Schrieffer JR, Wilczek F (1984). "Fractional Statistics and the Quantum Hall Effect." Phys. Rev. Lett. **53**(7):722–723. doi:10.1103/physrevlett.53.722.
5. ^Haldane FDM (1991). "''Fractional Statistics'' in Arbitrary Dimensions: A Generalization of the Pauli Principle." Phys. Rev. Lett. **67**(8):937–940. doi:10.1103/physrevlett.67.937.
6. ^Kundu A (1999). "Exact Solution of Double δ Function Bose Gas through an Interacting Anyon Gas." Phys. Rev. Lett. **83**(7):1275–1278. doi:10.1103/physrevlett.83.1275.
7. ^Kitaev AY (2003). "Fault-Tolerant Quantum Computation by Anyons." Annals of Physics. **303**:2–30. doi:10.1016/S0003-4916(02)00018-0.
8. ^Nayak C, Simon SH, Stern A, Freedman M, Das Sarma S (2008). "Non-Abelian Anyons and Topological Quantum Computation." Rev. Mod. Phys. **80**(3):1083–1159. doi:10.1103/revmodphys.80.1083.
9. ^Kwan J, Segura P, Li Y, Kim S, Gorshkov AV, Eckardt A, Bakkali-Hassani B, Greiner M (2024). "Realization of One-Dimensional Anyons with Arbitrary Statistical Phase." Science. **386**(6725):1055–1060. doi:10.1126/science.adi3252.
10. ^Bittner ER, Tyagi B (2025). "Noise-Induced Synchronization in Coupled Quantum Oscillators." The Journal of Chemical Physics. **162**(10):104116. doi:10.1063/5.0246275.
11. ^Bittner ER, Li H, Shah SA, Silva-Acuña C, Piryatinski A (2024). "Correlated Noise Enhancement of Coherence and Fidelity in Coupled Qubits." Philosophical Magazine. **104**(13-14):630–646. doi:10.1080/14786435.2024.2341011.
12. ^Hou YB, Ai X, You R, Zhong C (2025). "Correlated Noise Can Be Beneficial to Quantum Transducers." Physical Review Research. **7**:L032042. doi:10.1103/PhysRevResearch.7.L032042.
13. ^Bittner ER, Tyagi B (2025). "Noise-Induced Synchronization in Coupled Quantum Oscillators." The Journal of Chemical Physics. **162**(10):104116. doi:10.1063/5.0246275. https://pubs.aip.org/aip/jcp/article-pdf/doi/10.1063/5.0246275/20435238/104116_1_5.0246275.pdf.
14. ^Tyagi B, Li H, Bittner ER, Piryatinski A, Silva-Acuña C (2024). "Noise-Induced Quantum Synchronization and Entanglement in a Quantum Analogue of Huygens' Clock." J. Phys. Chem. Lett. **15**(43):10896–10902. doi:10.1021/acs.jpclett.4c02313.
15. ^Mackel V, Yang W, del Campo A (2023). "Quantum Alchemy and Universal Orthogonality Catastrophe in One-Dimensional Anyons." Quantum. **7**:1044. doi:10.22331/q-2023-10-16-1044. arXiv:2210.10776 [quant-ph/].
16. ^del Campo A, García-Calderón G, Muga JG (2017). "Universality of the Quantum Zeno and Anti-Zeno Effects." Phys. Rev. Lett. **119**(13):130401. doi:10.1103/PhysRevLett.119.130401.

17. ^ *Zhao P, Xu Y, Xiang Y, Ramezani H, Gong J, del Campo A (2019). "Extreme Decoherence and Quantum Chaos." Phys. Rev. Lett.* ***122****(1):014103. doi:10.1103/PhysRevLett.122.014103.*

Declarations

Funding: NSF, Welch foundation.

Potential competing interests: No potential competing interests to declare.