Review Article

# Measurement Problem from the Perspective of Wittgenstein's Philosophy

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**We analyze the measurement problem in quantum mechanics from two aspects of Wittgenstein's philosophy.**
**First, Wittgenstein emphasizes how the first false step can be voided so that a philosophical problem is dissolved, rather than solved. In the unsuccessful theories of the measurement problem, the postulate of classicality of apparatus (PCA) turned out to be the first false step implicitly adopted. By discarding the PCA and by adopting the microscopic quantum jump interpretation simultaneously, the measurement problem itself disappeared completely.**
**Second, from Wittgenstein's view of languages, the microscopic world is described by quantum mechanics, whose language is the Schrödinger equation, while the macroscopic world is described by classical physics, whose language is either calculus for Newtonian mechanics or vector analysis for electromagnetism. Both worlds are described by mathematical languages, but they are distinct. The measurement problem deals with the transition from microscopic to macroscopic. There is no single physical theory to explain this transition, and there is no corresponding mathematical language to describe it. This shows why the measurement problem must be described by an ordinary language in use.**
**As concrete examples of the unsuccessful theories, we discuss the works of Niels Bohr and John von Neumann. This is a unique situation in which philosophy is used to assess the validity of physical theories.**
**Finally, we show the classicality of a one-particle state with an extreme number of bosons. Therefore, there is no change in language in a macroscopic quantum jump.**

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# I. Introduction

This is the third in a series of papers discussing the measurement problem.

In the first paper (Paper I[1]), we gave an interpretation of single-particle quantum mechanics, which elucidates a quantum jump (QJ) as a jump from microscopic to microscopic. We call this process a microscopic quantum jump (MIJ).

In the second paper (Paper II[2]), we discuss quantum jumps (QJs) and wave function collapse (WFC). The MIJ interpretation presented in Paper I better defines the QJ and allows a clearer distinction between QJ and WFC. We treat single-particle quantum mechanics and many-particle quantum mechanics separately and show that WFC does not occur in single-particle quantum mechanics. We also show that WFC can occur for macroscopic quantum states such as many-photon states or Bose-Einstein condensates. This is a QJ from macroscopic to macroscopic, and it is effectively a measurement of a classical observable.

In this third paper, we plan to discuss two aspects of the measurement problem that are not covered in the previous papers.

In Paper I, the first step to develop the MIJ interpretation is to discard the postulate of classicality of apparatus (PCA) implicitly adopted by existing theories. We realized that this procedure is what Wittgenstein, the greatest philosopher of the 20th century, called voiding of the first false step in a philosophical problem[3]. By this procedure, the problem is dissolved rather than solved. This implies that the measurement problem was a pseudo-problem instead of a real problem. One half of Section II covers this discussion.

Another question Wittgenstein's philosophy can possibly answer is why the measurement process cannot be described mathematically and must be described by an ordinary language. The second point is as interesting as the first because it explains why all the sophisticated treatments developed by experts who were proficient in mathematics turned out to be in vain. The first and second points are not necessarily independent, and the first point includes the second. The rest of Section II covers this subject. We consider that presenting a couple of concrete examples is important, and we discuss the unsuccessful efforts by Niels Bohr and John von Neumann in light of Wittgenstein's philosophy in Section III.

Finally, we consider a macroscopic quantum jump, a jump from macroscopic to macroscopic, from the perspective of a language in Wittgenstein's philosophy. We show the equivalence of a quantum state with a large number of bosons and a classical wave function in Section IV.

## II. Two aspects of Wittgenstein's philosophy

### *A. Part of "Philosophical Investigations" relevant to the measurement problem*

"Philosophical Investigations"[3] was published after Wittgenstein's death, and what is written in this book is close to his final thoughts. Here we quote some portion of this book most relevant to our analysis of the measurement problem from his philosophical perspective below.

According to Wittgenstein, philosophical problems arise when language is forced from its proper home into a metaphysical environment, where all the familiar and necessary landmarks and contextual clues are removed. He describes this metaphysical environment as like being on frictionless ice: where the conditions are apparently perfect for a philosophically and logically perfect language, all philosophical problems can be solved without the muddying effects of everyday contexts; but where, precisely because of the lack of friction, language can in fact do no work at all. Wittgenstein argues that philosophers must leave the frictionless ice and return to the "rough ground" of ordinary language in use. Much of the Investigations consists of examples of how the first false steps can be voided, so that philosophical problems are dissolved, rather than solved: "The clarity we are aiming at is indeed complete clarity. But this simply means that the philosophical problems should completely disappear."

### *B. First false step in the measurement problem*

Since the days of Bohr and von Neumann, several measurement theories have been proposed, numerous papers have been published, and many books have been written, but the measurement problem has not been solved. We felt something fundamental was wrong, and we scrutinized the basic postulates underlying these measurement theories in Paper I. We consider an experimental setup in which an observed system is composed of a single particle (S) and an observing apparatus (A). Here, existing theories implicitly adopt a very basic postulate. We call it the postulate of classicality of apparatus (PCA). Omnès describes this PCA in the following manner[4].

We clearly understand what an observing apparatus is as long as it is described by classical physics and common sense, but we do not understand it otherwise. If the observing apparatus is considered to be a quantum object, it will become tremendously complicated, and it will not be wise to consider that we still understand it.

According to this PCA, the incident particle S interacts with enormously many degrees of freedom in the


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apparatus A. The PCA has been implicitly adopted by all the existing measurement theories, but now we regard this as what Wittgenstein calls the first false step.

In Paper I, we present counterexamples to the PCA in concrete experimental setups. We discarded the PCA and we adopted the postulate of microscopic quantum jump (MIJ). By discarding the PCA and by adopting the postulate of the MIJ, we constructed a measurement theory consistent with standard quantum mechanics. Discarding the PCA and adopting the MIJ interpretation happen together. In MIJ, the apparatus is at least partly quantum mechanical, and the postulate of MIJ is fully contradictory to the PCA. More precisely, the PCA and MIJ are two sides of a coin, and we chose the side of MIJ for our interpretation. By this choice, the measurement problem was dissolved rather than solved.

### *C. Mathematics as a logical language in Wittgenstein's philosophy*

Wittgenstein calls an ideal logical language "frictionless ice". In physical science, the most obvious logical language is mathematics.

The language for Newtonian mechanics is calculus, and that for electromagnetism is Maxwell's equations described by vector analysis. Classical physics requires at least two languages to describe it. The language for special relativity is tensor calculus, and that for general relativity is tensor analysis.

The language underlying quantum mechanics is more complex. The components of this language are Hilbert space, quantum conditions, and equations of motion. Different representations such as the Heisenberg representation and Schrödinger representation are more like dialects. The Schrödinger equation as the equation of motion in the Schrödinger representation is the most widely used mathematics in measurement theories.

The languages describing different physical theories are distinct. However, one common feature is that each of these mathematical languages deals with an idealized condition or what Wittgenstein calls "frictionless ice".

### *D. Unique situation of the measurement problem in terms of language*

The measurement problem deals with the transition from the microscopic world to the macroscopic world. The microscopic world is governed by quantum mechanics, whose language, for example, is the Schrödinger equation. The macroscopic world is governed by classical physics, which includes Newtonian mechanics and electromagnetism. The language for the former is calculus and that of the latter is vector analysis. What is important is that the languages describing both microscopic and macroscopic worlds are mathematical, but they are distinct. We should realize the fact that there is no single physical theory in this transition region. There is no mathematical language describing the gap between the two worlds whose mathematical languages are different. We again quote a sentence from "Philosophical Investigations" below.

Wittgenstein argues that philosophers must leave the frictionless ice and return to the "rough ground" of ordinary language in use.

The description of the transition from microscopic to macroscopic is exactly the case that we must return to the "rough ground" of ordinary language in use. The measurement problem belongs to a special subfield in physics called "interpretation" which already suggests the absence of mathematics to describe it. This also explains why all mathematical theories have failed to describe the measurement problem.

### *E. How do we proceed from here?*

From Wittgenstein's point of view, the measurement problem is like a rough ground described by an ordinary language in use. He does not say how to use this ordinary language. Later researchers of Wittgenstein called his attitude "practical holism".[3].

In Paper I, we investigated the measurement problem by interpreting the operating principles of modern detectors as apparatuses and discovered the MIJ interpretation. This practical approach turned out to be in agreement with Wittgenstein's philosophy of practical holism unconsciously.

## III. Concrete examples

### *A. Niels Bohr*

Bohr was the Godfather of modern quantum mechanics. One of his strong influences is seen in his theory of nuclear sanctuary, which means that the interior of an atomic nucleus is a sanctuary where quantum mechanics is not applicable. In "The Story of Spin"[5], Sin-itiro Tomonaga makes an interesting observation.

In Copenhagen, no one challenged Bohr, and the people there believed in nuclear sanctuary. Werner Heisenberg in Leipzig developed his theory of nuclear structure based on his insight that the nuclear force is the exchange force between protons and neutrons. In his theory, he used quantum mechanics at the level of phenomenology. The origin of the nuclear force was eventually explained by the meson theory of Hideki Yukawa in Osaka. The meson theory proved the full applicability of quantum mechanics

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to the interior of an atomic nucleus, and Bohr's idea of nuclear sanctuary was denied. Tomonaga finds that the influence of Bohr gradually diminished from Copenhagen to Leipzig and from Leipzig to Osaka according to the geographical distance from Copenhagen.

One of the mainstream measurement theories was the Copenhagen interpretation[6] and Bohr was the central figure to formulate it. The authority of Bohr might have helped the longevity of the Copenhagen interpretation.

Here we focus on some features of the Copenhagen interpretation in comparison with our interpretation in the light of Wittgenstein's philosophy.

1. Quantum descriptions are objective, in that they are independent of physicists' personal beliefs and other arbitrary mental factors.
2. The Copenhagen interpretation is described by an ordinary language, which is our observation.
3. A measuring device is described by classical physics.

The first two points are in agreement with our interpretation, but the third point is problematic in that Bohr adopted the PCA in his thought experiments. This might have been inevitable, if we take his year of birth, 1885, into account. He was too old to turn his attention to new electronic detectors as observing apparatuses. So the PCA was his first false step in the Copenhagen interpretation.

### *B. John von Neumann*

John von Neumann was a distinguished mathematician, physicist, and computer scientist.

He wrote "Mathematical Foundations of Quantum Mechanics"[7]. This book is regarded as a bible by those interested in the mathematical aspect of quantum mechanics. Quantum mechanics was treated from the perspective of mathematics for the first time.

A mathematical treatment of the measurement problem was also covered by this book for the first time. Since von Neumann, theorists of the measurement problem have rarely questioned the applicability of mathematics to the measurement problem. However, if we carefully read this book, we notice one questionable description at the very end where the measurement problem is covered. First, he assumes that the observed system and the observing apparatus are both described by Hamiltonians and that quantum mechanics is applicable to both the observed system and the apparatus. Second, he introduces a strange Hamiltonian describing the interaction between the observed system and the observing

 doi.org/10.32388/9WGGN8 

apparatus without giving a clear justification. The role of this interaction Hamiltonian appears to represent the transition.

Regardless of the form of the interaction Hamiltonian, it is apparent that he assumes that the transition from microscopic to macroscopic or quantum to classical is described by quantum mechanics and by mathematics. This violates our view that the transition must be described by an ordinary language, and it is also the first false step in the problem setting. The last part of this book is not as rigorous as the rest of it. It appears that this weakness of the book has been well recognized, and in that sense, it is no longer a major problem.

## IV. Macroscopic quantum jump

So far we have considered the traditional measurement problem concerning a microscopic quantum jump (MIJ). In Paper II, we have shown that a MIJ yields only one set of specific eigenvalues of an observable. Therefore, the probability distribution of eigenvalues is not obtained from one measurement, and wave function collapse (WFC) does not happen. We have analyzed this process from the perspective of Wittgenstein's philosophy.

What we have not discussed is a macroscopic quantum jump (MAJ), which is a jump from macroscopic to macroscopic. In Paper II, we treated this process from the framework of the occupation number of a one-particle state. Here we reconsider this process from the point of view of the languages of Wittgenstein's philosophy.

The main reference used in this section is "Introduction to Superconductivity" by Sadao Nakajima, available only in Japanese[8].

### *A. Languages of macroscopic quantum world and classical world*

The physical theory that describes a many-boson system is quantum field theory (QFT). We would like to see how this QFT behaves when the number of bosons increases from a finite number to an extremely large number and if a quantum state with an extremely large number of bosons can be regarded as a classical state. Since this transition is governed by QFT alone, QFT describes the transition seamlessly, and there is no change in the mathematical language from a finite number of bosons to an infinite number of bosons.


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Since the MAJ is effectively a measurement of a classical observable, the measurement itself does not affect the quantitative results before and after the measurement. Our goal is to show the equivalence of a many-particle quantum system with an extremely large number of bosons and a classical system.

### *B. Many-photon system and classical electromagnetic waves*

For a quantum state specified by the number of photons N and phase $\phi$, N and $\phi$ can be regarded as canonical conjugate variables. N and $\phi$ are related to the particle nature and wave nature of light, respectively. There is an uncertainty relation,

$$\Delta \mathrm{N} \Delta \phi \geq 1, \tag{1}$$

where $\Delta$N is the fluctuation of the photon number and $\Delta\phi$ is the phase fluctuation. If the fluctuation of N satisfies the condition,

$$\mathrm{N} \gg \Delta \mathrm{N} \gg 1 \tag{2}$$

the condition for phase fluctuation,

$$\Delta \phi \sim (\Delta \mathrm{N})^{-1} \ll 2\pi \tag{3}$$

becomes possible, and in this case, a classical monochromatic plane electromagnetic wave is realized. What we have done is a reverse process of field quantization, in which classical electromagnetic fields exist first and we quantize them to obtain photon fields by introducing commutation relations. At high light levels corresponding to classical waves, individual photons are not noticeable, while at low light levels, individual photons become obvious. Since a classical wave is obtained in one measurement, WFC occurs as a MAJ. Photons are simple to handle because there is no interaction between a pair of photons, unless vacuum polarization occurs at very high energy. They are also easy to handle because they are massless and their chemical potential is zero. Therefore, they are freely created and annihilated regardless of their energy.

The same argument is applicable to phonons in metals. The relation between phonons and sound waves is analogous to that between photons and classical electromagnetic waves.

### *C. Bose-Einstein condensation*

A $He^4$ atom is a boson with spin 0. Its major difference from a photon or phonon is that $He^4$ atoms are not created or annihilated under normal circumstances, and their total number is conserved. In real life, atomic interaction is not negligible in helium gas or helium liquid, but here we tentatively consider

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perfect helium gas without atomic interaction. In this situation, the total energy of the gas is the sum of the kinetic energies of all the atoms. The total energy takes the minimum when all the atoms are in the lowest one-particle state. This phenomenon of bosons is called Bose-Einstein condensation (BEC). Even in the presence of atomic interaction, BEC occurs if appropriate conditions are satisfied. In this case, a finite fraction of the total number of atoms condenses into the lowest one-particle state.

The system of $He^4$ atoms can be seen as waves in the sense of material waves of de Broglie. Of course, these are quantized waves and they should be regarded as linear operators. However, when BEC occurs, the number of bosons in the lowest state is huge and the phase is well determined. Therefore, the wave associated with the BEC system can be regarded as classical. On the macroscopic scale, BEC is seen in superfluidity, which is a flow without friction. This BEC system is specified by a complex wave function called an "order parameter," which quantifies the macroscopic quantum coherence and characterizes the superfluid phase.

### *D. Superconductivity*

The microscopic theory of superconductivity is the BCS theory[9]. Describing the BCS theory is beyond the scope of this paper, and here we only point out the similarity of a superconducting state to a BEC state and that of supercurrent to superfluid.

In a metal, between a pair of electrons, only Coulomb repulsion exists at normal temperature. However, at a certain low temperature, attraction by phonon exchange slightly overcomes the Coulomb repulsion, and a pair of electrons forms a bound state called a Cooper pair. The electron spins are antiparallel, and the spatial wave function is an S wave. So the Cooper pair is in a singlet state, and it behaves as a boson with spin 0. A pair of electrons in a triplet state is not formed as a bound state.

At zero temperature, in absence of attraction by phonon exchange, electrons would fill from the lowest one-particle energy state to the Fermi level. We note that in an ordinary energy-number diagram, the energy means the kinetic energy of an electron. In the presence of the attraction, Cooper pairs are formed and the total energy becomes lower. In the energy-number diagram, the BEC of Cooper pairs occurs at a one-particle state of electrons whose energy is slightly above the Fermi level.

Once the BEC of Cooper pairs occurs, as a macroscopic phenomenon, supercurrent flows, which is analogous to the superfluid of bosons. There is no dissipation in supercurrent, unlike in Ohm current. We can see the equivalence of superconductivity and classical electromagnetism from this phenomenon.

At a finite temperature, but below the superconducting transition temperature, a finite fraction of the total Cooper pairs still condenses into the superconducting state. This phenomenon is similar to that of superfluidity. For instance, this transition temperature is 1.2 K for aluminum.

## V. Conclusion

The measurement problem in quantum mechanics was analyzed from the perspective of Wittgenstein's philosophy. It was found that the previous unsuccessful theories were plagued with what Wittgenstein calls the first false steps. By discarding the postulate of the classicality of the apparatus and by adopting the microscopic quantum jump interpretation simultaneously, the first false step was voided and the measurement problem disappeared completely. Wittgenstein's philosophy suggests that the transition from microscopic to macroscopic should be described by an ordinary language in use. This is consistent with what we have found in practice in interpreting the operating principles of modern detectors as apparatuses. As for a macroscopic quantum jump, we show the equivalence of a one-particle state with a large number of bosons and a classical wave.

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## Statements and Declaration

### *Data Availability*

The data that support the findings of this study are available from the corresponding author upon reasonable request.

### *Author Contributions*

T.N. conceived the idea, performed the analysis, and wrote the manuscript.

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