

Peer Review

Review of: "A Complex Topological Phase in C-Spin Active Matter"

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The author reports a theoretical model on active spin agents that interact reciprocally within the framework of a modified Kuramoto model. The English in the article is satisfactory. Although the author shows sound knowledge in phase transitions and active condensed matter, my main concern with this work is that there are no comparisons against experimental data; thus, the results may not have a physical counterpart and then only be part of a mathematical model. Nevertheless, the research is at a good level in condensed matter, and therefore I am in favor of proceeding with the manuscript for publication. I advise the author to correct the following issues in this article or in following works, depending on the amount of work required.

Major concerns

- At the end of the introduction, you describe your current contribution. Although the mathematical model is interesting, I would like to know how this could be applied in biophysics and where these types of reciprocal interactions are actually observed in experiments? (*'spin agents that belonging to the same population repel each other positionally and attract orientationally; and when belonging to different populations they do the opposite'*). I understand you write about this in the conclusions, but I would like to know a particular example, and not the general idea, which is well established in biological physics, that far from thermodynamic equilibrium, living organisms are sustained by incoming and outgoing flows of matter and energy. Please specify the examples.
- On page 6, you write '*.. makes in fact the system active, because it provides the spins a phase drift which, due to variables coupling, can indirectly activate positional dynamics'*'. I understand that this effect makes

simulations look ‘alive’, but this is not really why your system is active; it is a consequence. And I agree, your functions $\cos()$ and $\sin()$ couple translational and rotational dynamics. Your system is active because equations (1) and (2) contain input energy that translates into rotational and translational motility, which in biological systems is supposed to come from, for instance, ATP in cells (internal degrees of freedom). In these equations, translational and rotational motile forces are normalized to 1 at the l.h.s. Please correct that phrase.

- I wonder how your numerical simulations were performed. On page 7, you mention ‘*I have numerically integrated, by means of a standard method..*’. This is not correct for a research article. The fact that the entire work relies only on numerical simulations with no theoretical analysis makes numerical simulations even more important in your article. Please state the numerical method employed, with references in the main text.
- On page 12, you write ‘*.. type 2 patterns are non-local solutions that cannot be understood in terms of a local stability analysis*’. These issues are well known in nonlinear physics. This happens, for instance, in the context of partial differential equations when nonlinear solutions appear, and the only way to analyze them is by performing a weak nonlinear development or normal forms. As your set of equations (1 and 2) is discrete, you will have to derive a continuum theory (in some specific limit) where the vectorial field will show the solutions of the discrete set, and then perform nonlinear analysis. With the latter, you will understand these solutions. Due to the amount of work required for this task, I think this should be part of another article.

Minor concerns

- At the introduction, you refer to the modified Navier-Stokes (NS) equation. As there are many different modifications to the NS equations, you should specify here, with citations to the research works.
- On page 9, the phrase ‘*The expansion rate is logarithmic; consequently, for practical purposes, it vanishes when it attains approximately three times the interaction length*’ is incorrect. I see also in the supplemental material what you mean, as this expansion is logarithmic, it continues very slowly. Please correct this phrase.

- Please do not formulate phrases in the first person such as '*I have investigated ...*', which are more suitable for research statements and such.
- On page 1, as you refer to A. Winfree in the preceding paragraph, it is better to change the phrase '*His Winfree model*' to 'His model'.

Declarations

Potential competing interests: No potential competing interests to declare.