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# [Commentary] Solving the Paradox of Material Implication

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## Abstract

The paradox of material implication has remained unresolved since antiquity because it was believed that the nature of implication was entailment. The article shows that this nature is *opposition* and therefore the name "implication" should be replaced with the name "competition". A solution to the paradox is provided along with appropriate changes in nomenclature, the addition of connectives and the postulate that the biconditional take over the role of the previous implication. In addition, changes to the nomenclature of logic gates reflecting the competition in electronics are proposed.

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## 1. Introduction

Of all the logical functions the most controversial since ancient times has been the implication. It represents an idealization of the conditional found in everyday language. The members of the Megarean School (5<sup>th</sup> BC) were the first to notice the special properties of this mode. Philo of Megara (4<sup>th</sup> BC) stated that the conditional is true except when the antecedent is true and the consequent is false. His opinion was taken over by Stoics. Diodorus of Kronos (3<sup>rd</sup> century BC) added that the existence of a true antecedent with a false consequent is impossible, both in the present and past tense. The position of Diodorus has now come to be known as a strict implication, and that of Philo – a material one. The term “material implication” was introduced by B. Russell (1872-1970). The system of material implication was introduced in logic by Ch. Peirce, G. Frege, B. Russell and A. N. Whitehead (Borkowski 1977, 72). The possibility of a true conclusion from false premises and, in some cases, a true conclusion from one true premise and one false, was also noted by Aristotle (*Analytics First*, II, 2, 15). Medieval logic, despite the resistance of some scholars<sup>1</sup>, put these observations in the theorem

of Pseudo-Scot: from falsehood, anything [follows]<sup>2</sup>. It was included in the popular formula *ex falso quodlibet*, also known as the “principle of explosion”, which survived the nineteenth-century logic reform and is still used today.

Conditional mode logic theory has become an integral part of Frege's new logic and now it can be found in every logic textbook. Specialist literature on the mutual relations between the conditional mode and material implication, both formal and strict, already covers hundreds of items and continues to grow, but the paradox of implications remains unresolved. Logicians, on the one hand, consider sentences in the logical sense to be only those that can be evaluated as true or false, and on the other hand, they say that from the point of view of logic, it is not important what these sentences say. One could be indignant at this strange *dictum*, which allows scholars to pass over obvious contradictions to which, for example, the implication with a false antecedent leads. Such indignation, however, would be as sterile as indignation at Aristotle for not building modern physics. It is rather appropriate to repeat the words of the *Prayer for Wisdom*: “We laboriously discover the things of this earth, we hardly find what we have at hand” (Wis 9, 16, 765). Logic, exhausted by idealism, of which remains an analytical skeleton in a shabby nominalist garment, has no way of resolving the paradox of implication. The problem of following a false conclusion from true premises is now reduced to a finding that there is no relevant interpretation. The Polish logician Marcin Tkaczyk poses a rhetorical question as to whether such a reduction is legitimate and raises this issue as “the most important philosophical problem of logic” (Tkaczyk 2010, 53).

## 2. The current state of affairs

The proposed solution to the problem of material implication is based on the principle of isomorphism of reality, thought and language related to realistic philosophy. According to this principle, there is a mapping between things, concepts and words, as well as between states of affairs, judgments and sentences which results in logical functions playing the role of mental representations of real relations between beings. In language, they are expressed by complex or subordinate sentences. Complex sentences are linked by paratactic conjunctions, which according to the *Dictionary of the Polish Language* are divided into:

1. connective conjunctions, e.g. *and*, *and at the same time*,
2. disjunctive conjunctions, e.g. *or*,
3. excluding conjunctions, e.g. *neither*, *nor*,
4. opposing conjunctions, e.g. *but*, *otherwise*, *however*, *only*, *yet*, *rather*, *in the meantime*, *nevertheless*, *for that*,
5. consequentiality conjunctions, e.g. *therefore*, *consequently*,
6. explanatory (synonymous) conjunctions, e.g. *i.e.*, *that is*, *in other words* (SJP 2021).

Of the six types mentioned, explanatory conjunctions are used to introduce equivalent descriptions of the same being, and not to describe relationships. Therefore, on the basis of the linguistic criterion, five types of relationships can be distinguished: connectivity, disjunctivity, exclusion, opposition, and consequence (also entailment). They are mapped to logical functions: connectivity to the conjunction, disjunctivity to the disjunction, exclusion to the non-disjunction, opposition to the non-implication (also called inhibition), and consequence to both the implication and biconditional. These

functions use conjunctions taken from the everyday language and, despite some idealization, retain their essential meanings.

Conjunctions of connectivity and disjunctivity can both form complex sentences and combine naming arguments in simple sentences, while the conjunctions of opposition and consequence are usually used only to connect sentences. In this they show similarity to hypotactical conjunctions, among which there are conjunctions forming subordinate clauses in the conditional mode, always associated with the material implication. As noted by the Polish logician Stanisław Kiczuk:

*"Many other authors inquired about the relationship between the sign of material implication and the connective of natural language 'if... then...'. Ajdukiewicz wrote that natural speech does not have any term that would agree in its meaning with the sign of material implication. Sometimes it is said that the sign of material implication represents only a truth-functional component in the sense of the conjunction 'if... then...'. It is also noted that the natural language conditional 'if p, then q' has several different meanings"<sup>3</sup> (Kiczuk 2006).*

These several different meanings seem to result from the fact that six categories of adverb clauses (of place, time, reason, purpose, condition, concession) analogously map various types of movement between beings, and the seventh category (of manner) – the way this movement occurs.

Table 1.			
Adverbial subordinate	initiation	realization	consequence
place	starting point=>	way	point of arrival
time	starting moment=>	course	ending moment
reason and purpose	reason=>	method of operation (determined, caused, enabled)	purpose
concession	cause<=		effect
condition	condition=>		result
manner		manner	

It can therefore be concluded that, in the sphere of thought, entailment has the character of an analogous notion that reflects the dependencies related to what the metaphysical tradition calls "change". In the classical approach, any real change occurs between two terms – the beginning and the end. Initiation reflects the state of affairs initiating the movement, consequence – the ending state, and realization – the state of movement between the terms. In the sphere of thought, any change is a transition of the mind to a new cognitive state. The role of initiation in reasoning is played by premises, the role of realization – by the method of reasoning, and the role of consequence – by the conclusion. In the sphere of language, initiation takes the form of a sentence that plays the role of an antecedent of an implication, traditionally denoted by the letter "p", and consequence – the form of a sentence that plays the role of a consequent, traditionally denoted by the letter "q". Depending on the direction of the change (from cause to effect or vice versa, from past to future or vice versa), what meant condition (if p, then q) can mean cause (p, therefore q) or reason (q because p).

All types of entailment are described in the Classical Propositional Calculus, which includes sixteen logical functions, also called Boolean functions because they were discovered by the English mathematician and philosopher George Boole<sup>4</sup> (1815-1864). Some of these functions are binary operators, some are unary operators, and some are constants, as shown in the table below.

Table 2.					
1	2	3	4	5	6
	$p \ q$	1 1	1 0	0 1	0 0
1	antilogy	0	0	0	0
2	conjunction ( $p \wedge q$ )	1	0	0	0
3	nonimplication ( $p \neq q$ )	0	1	0	0
4	variable $p$	1	1	0	0
5	converse nonimplication ( $p < \neq q$ )	0	0	1	0
6	variable $q$	1	0	1	0
7	exclusive disjunction ( $p < \neq q$ )	0	1	1	0
8	disjunction ( $p \vee q$ )	1	1	1	0
9	non-disjunction (joint denial) ( $p \downarrow q$ )	0	0	0	1
10	biconditional ( $p \Leftrightarrow q$ )	1	0	0	1
11	negation of $q$ ( $\neg q$ )	0	1	0	1
12	converse implication ( $p \Leftarrow q$ )	1	1	0	1
13	negation of $p$ ( $\neg p$ )	0	0	1	1
14	implication $p \Rightarrow q$	1	0	1	1
15	non-conjunction (alternative denial, Sheffer stroke) ( $p \uparrow q$ )	0	1	1	1
16	tautology	1	1	1	1

Entailment is described in terms of the implication and biconditional, and the connective "if..., then..." is sometimes used to express both implication and biconditional, although there is a tendency to express the biconditional only in terms of the phrase "if and only if". However, the mathematical precision of logical functions is not sufficient to solve the title paradox. Logicians invariably state that "the implication operator does not correspond to the concept of one sentence resulting from another" (Mostowski 1948, 15), and computer scientists cannot answer the question why "Unlike propositional calculus, implication does not play a major role in the theory of logical systems. The same applies to the operation of inhibition..." (Leszczyński 1990, 37). As a result, until recently, electronics engineers only constructed logic gates corresponding to the conjunction (AND), disjunction (OR), biconditional (XNOR), non-disjunction (NOR), non-conjunction (NAND) and exclusive disjunction (XOR). Due to the discovery of the memristor, which will be discussed later, the production of IMPLY and INHIB gates began, which perform the functions of implication and inhibition, but even this fact did not contribute to solving the paradox of material implication.

This is the general outline of the current state of scientific inquiry regarding the isomorphism of thought and language in

the field of adverbial sentences, conjunctions and logical functions. Now it is necessary to examine how the functions abstracted by logicians relate to actually existing beings.

### 3. State of affairs analysis

A particularly attractive object of scientific research for a logician is a being called "football pools", which accepts bets on the result of a football match between teams A and B. During such a match, each team either scores a goal or fails to score. Let the number one mean a goal scored and zero mean no goal scored. For simplicity, it can be assumed that each team scores one goal. Let the variable "p" mean "A scores a goal" and let the variable "q" mean "B scores a goal". There are four possible combinations: (1) first combination – both teams score goals, (2) second combination – team A scores a goal, team B does not score a goal, (3) third combination – team A does not score a goal, team B scores a goal, and (4) combination four – neither team scores a goal. Depending on which combination you take into account, you can place bets on ten different match results.

If both teams score goals, it will be a goal draw. Its equivalent in logic is a logical function called conjunction, which uses the conjunction "and". Since it connects two states in which a goal is scored, it should be called an inclusive connective conjunction.

Table 3.					
p		q		match result	$p \wedge q$
A score a goal	1	B score a goal	1	goal draw	1
A score a goal	1	B do not score a goal	0	not a goal draw	0
A do not score a goal	0	B score a goal	1	not a goal draw	0
A do not score a goal	0	B do not score a goal	0	not a goal draw	0

If it happens that one team wins and the other loses, or there is a goalless draw, then all these results together can be described as "not a goal draw". The non-conjunction function corresponds to this result, using the conjunction "either not... or not" or "not... or not". Since it separates two states and does not affirm the pair of states in which the goal is scored, it should be called an exclusive disjunctive conjunction.

Table 4.

p		q		match result	$p \uparrow q$
A score a goal	1	B score a goal	1	goal draw	0
A score a goal	1	B do not score a goal	0	not a goal draw	1
A do not score a goal	0	B score a goal	1	not a goal draw	1
A do not score a goal	0	B do not score a goal	0	not a goal draw	1

A goalless draw corresponds to the function called non-disjunction, which uses the conjunction "neither, nor". It connects two states in which no goal is scored, so it is an exclusive connective conjunction.

**Table 5.**

p		q		match result	$p \downarrow q$
A score a goal	1	B score a goal	1	not a goalless draw	0
A score a goal	1	B do not score a goal	0	not a goalless draw	0
A do not score a goal	0	B score a goal	1	not a goalless draw	0
A do not score a goal	0	B do not score a goal	0	goalless draw	1

However, if one team wins and the other loses, or there is a goal draw, the result "not a goalless draw" will appear in the football pools. In logic, the disjunction that uses the conjunction "or" corresponds to it. Because it separates two states and affirms a pair of states in which a goal is scored, it deserves to be called an inclusive disjunctive conjunction.

**Table 6.**

p		q		match result	$p \vee q$
A score a goal	1	B score a goal	1	not a goalless draw	1
A score a goal	1	B do not score a goal	0	not a goalless draw	1
A do not score a goal	0	B score a goal	1	not a goalless draw	1
A do not score a goal	0	B do not score a goal	0	goalless draw	0

A "goal draw" and a "goalless draw" taken together mean a "draw". In logic, it corresponds to the biconditional, which uses the conjunction "if, then". Modern logic tries to replace this conjunction with the phrase "if and only if", but it does not seem necessary. The conjunction "if, then" should be considered an inclusive consequential conjunction because it maps the entailment of states and affirms the pair of states in which goal-scoring occurs.

**Table 7.**

p		q		match result	$p \Leftrightarrow q$
A score a goal	1	B score a goal	1	a draw	1
A score a goal	1	B do not score a goal	0	not a draw	0
A do not score a goal	0	B score a goal	1	not a draw	0
A do not score a goal	0	B do not score a goal	0	a draw	1

The result "not a draw" corresponds in logic to the exclusive disjunction. Nowadays, logicians assign the conjunction "either...or" to this function, but without fear of making a mistake, you can stick to the conjunction "if, then not". This conjunction should be called an exclusive consequential conjunction because it maps the entailment of contradictory states and does not affirm the pair of states in which the goal is scored.

**Table 8.**

p		q		match result	$p \nrightarrow q$
A score a goal	1	B score a goal	1	a draw	0
A score a goal	1	B do not score a goal	0	not a draw	1
A do not score a goal	0	B score a goal	1	not a draw	1
A do not score a goal	0	B do not score a goal	0	a draw	0

The state of affairs in which one team wins and the other loses is described in logic by the non-implication (also called inhibition), using the conjunction "but not". It is an exclusive opposing conjunction because it contrasts two states of affairs and does not affirm the pair of states in which the goal is scored.

**Table 9.**

p		q		match result	$p \nrightarrow q$
A score a goal	1	B score a goal	1	not winning A, not losing B	0
A score a goal	1	B do not score a goal	0	winning A, losing B	1
A do not score a goal	0	B score a goal	1	not winning A, not losing B	0
A do not score a goal	0	B do not score a goal	0	not winning A, not losing B	0

The complement of the inhibition is the implication, which corresponds to the result "not winning A, not losing B". The state of affairs described by such a result involves a draw or a win for team B. The implication uses, like the biconditional, the inclusive consequential conjunction "if, then."

Table 10.					
p		q		match result	$p \Rightarrow q$
A score a goal	1	B score a goal	1	not winning A, not losing B	1
A score a goal	1	B do not score a goal	0	winning A, losing B	0
A do not score a goal	0	B score a goal	1	not winning A, not losing B	1
A do not score a goal	0	B do not score a goal	0	not winning A, not losing B	1

The results "winning B, losing A" and "not winning B, not losing A" have as their logical equivalents converse non-implication and converse implication.

The obtained match results, logical functions and types of connectives assigned to them reveal an interesting regularity of mutual relations. To better illustrate the topic, it is worth adding arithmetic operations and relations to them – as they were understood by the above-mentioned George Boole: product as the equivalent of the conjunction, sum as the equivalent of the disjunction, equality as the equivalent of the biconditional and inequality as the equivalent of what he called *subsumption*, i.e. non-conjunction. To this should be added the complements of the listed elements. The following summary is created:

Table 11.



<b>conjunction</b> a goal draw	x	inclusive connective conjunctions	exclusive disjunctive conjunctions	/	<b>non-conjunction</b> not a goal draw
<b>non-disjunction</b> a goalless draw	—	exclusive connective conjunctions	inclusive disjunctive conjunctions	+	<b>disjunction</b> not a goalless draw
<b>biconditional</b> a draw	=	inclusive consequential conjunctions	exclusive consequential conjunctions	≠	<b>exclusive disjunction</b> not a draw
<b>non-implication</b> winning A losing B	>	exclusive opposing conjunctions	inclusive consequential conjunctions	≤	<b>implication</b> not winning A A not losing B B
<b>converse non-implication</b> winning B losing A	<			≥	<b>converse implication</b> not winning B not losing A

As you can see, there are three types of conjunctions in the middle columns of the table. The column on the left contains connective, consequential and opposing conjunctions, while the column on the right contains disjunctive, consequential and – surprisingly! – consequential conjunctions for the second time. So there is some strange irregularity. In the first line, exclusive disjunctive conjunctions take part in operations that are the negation of operations involving inclusive connective conjunctions. In the second line, inclusive disjunctive conjunctions take part in operations that are the negation of operations involving exclusive connective conjunctions. In the third line, exclusive consequential conjunctions take part in operations that are the negation of operations involving inclusive consequential conjunctions. Therefore, in the fourth line there should appear on the right side inclusive opposing conjunctions, which are the negations of exclusive opposing conjunctions used by non-implication. However, consequential conjunctions appear instead. This happens even though both the corresponding mathematical weak inequality and the football pools results are – as in rows one through three – negations of their counterparts on the left side of the table. This is an inexplicable irregularity. When you add Mostowski's above-mentioned objection that the implication operator does not correspond to the concept of one sentence resulting from another, an obvious conclusion comes to mind that the implication is not what it has been recognized as for two and a half thousand years. In other words, the implication is not consequence (or entailment).

So what is it? What is its true nature, speaking in the language of realistic philosophy?

#### 4. The true nature of the implication

From the example with the football pools discussed, it can be seen that the logical values of truth and falsity in the implication and inhibition truth tables do not *result* from each other, but *compete with* or *oppose* each other. This leads to the conclusion that the system of logical values attributed so far to the implication is in fact a system of a *competition*, and that this function reflects the relation of opposition just as the conjunction reflects the relation of connectivity, the disjunction – the relation of disjunctivity, and the biconditional – the relation of mutual consequence (entailment). The name *implication* could therefore be changed to *competition*. However, since there are two related terms in Polish, derived from the Latin word *competitio* – "konkurencja" and "kompetycja", I propose to replace the current term "implication" with

the term “special competition” (the English equivalent of “kompetycja”), and use the term “general competition” (the English equivalent of “konkurencja”) to describe the entire genus of opposition, which includes two species – the inhibition and the competition. For simplicity, in the rest of this article, the term “competition” will be used in the sense of “special competition”.

The identification of the implication as an opposition has been recently made by the French-Italian logician Alessio Moretti, who conducts research on the logical square. In his monumental doctoral thesis entitled *The Geometry of Logical Opposition*, he states that “subalternation (i.e. logical implication) can (and should) be viewed as a type of opposition” (Moretti 2009, 413). Moretti developed *n-opposition theory (N.O.T.)*, which he considers “some kind of (maybe not bloody!) revolution inside logic.” According to him, spatial structures describing all possible oppositions ( $\beta n$ -structures) are “apparently fundamental to logic” and “totally new” (Moretti 2009, 414). His position is already shared by several other logicians. It is therefore only a matter of time before there is widespread recognition of the fact that the true nature of what has been called for centuries “implication” is in fact “competition”.

Transforming the implication into the competition allows us to redress Table 11 by replacing the incorrect entry “inclusive consequential conjunctions” with the correct entry “inclusive opposing conjunctions”. However, there are many inclusive opposing conjunctions. The question arises whether all of them can act as connectives of the competition or only one, and if so, which one.

The function that complements the competition – the inhibition – already has an established connective “but not”. Since the competition is the negation of the inhibition, its connective should be the negation of the connective of the inhibition, i.e. the conjunction “but”. And indeed, an ordinary competition in which sentences describe equivalent states of affairs – such as “I drink coffee for breakfast, *but* I drink tea for dinner” – functions correctly with the opposing conjunctions *but*, *nevertheless*, *however*, *while*, *although* or *while*. However, if the sentences indicate a difference in preferences for the described states of affairs, the competition requires the use of the conjunctions *alternatively* or *eventually* – for example in the statement “Don’t drink coffee; *alternatively*, drink coffee, *but* add milk.” As Elżbieta Magner from the University of Wrocław notes, “the word *alternatively* indicates that the possibility referred to in the sentence before it is more real, more probable, or more desirable than the one in the second sentence. The possibility referred to in the second sentence is taken into account only when the first one does not come to fruition or when it is considered unrealistic” (Magner 2016, 65). A similar role is played by the compound conjunction *rather... than*, which my Father once used to express his attitude towards the communist party in the sentence “I would *rather* go pushing the wagons *than* join the party.”

Consequently, it should be stated that the competition requires the use of more than one connective. The use of individual opposing conjunctions as connectives depends on the type of opposition being described and on the type of relationship between the atomic sentences. An exhaustive analysis of these issues is beyond the scope of this article and requires a separate study with the participation of linguists. It remains to be said that the number and selection of the competition connectives remain – despite preliminary arrangements – open issues.

In addition to introducing new connectives, replacing the implication with the competition requires some changes in

nomenclature. Instead of saying "p results in q" or "p entails q", we should now say "p competes with q". Since both the competition and inhibition have two varieties – direct ( $p \neq q$ ,  $p \Rightarrow q$ ) and converse ( $p < \neq q$ ,  $p \Leftarrow q$ ), they should be called:

1. simple competition  $p \Rightarrow q$  – "competition with an indication of q",
2. converse competition  $p \Leftarrow q$  – "competition with an indication of p",
3. simple inhibition – "inhibition with contraindication of q",
4. converse inhibition – "inhibition with contraindication of p".

Furthermore, the term "non-implication" for the inhibition should be abandoned.

At the same time, it should be recognized that consequence or entailment is described only by the biconditional and the exclusive disjunction. They alternately describe the movement between initiation and consequence, and the entailment always occurs in both directions. Saying "If the weather is nice, I will go for a walk" you can just as easily say "I will go for a walk if the weather is nice", etc. Therefore, the formulas of some rules of logic should be modified and their names should be changed, replacing the term "implication" with the terms "competition" or "biconditional" as in the table below.

Table 12.		
Logic rule	Form with the competition	Form with the biconditional
Rule of identity	$p \Rightarrow p$ ; $p \equiv p$	$p \Leftarrow p$ ; $p \equiv p$
First Clavius's rule	$(\neg p \Rightarrow p) \Rightarrow p$	$(\neg p \Rightarrow p) \Leftarrow p$
Second Clavius's rule	$(p \Rightarrow \neg p) \Rightarrow \neg p$	$(p \Rightarrow \neg p) \Leftarrow \neg p$
Rule of transitivity of the competition and biconditional	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$	$[(p \Leftarrow q) \Leftarrow (q \Leftarrow r)] \Leftarrow (p \Leftarrow r)$
<i>Modus ponendo ponens</i>	$[(p \Rightarrow q) \wedge p] \Rightarrow q$	$[(p \Leftarrow q) \wedge p] \Rightarrow q$
<i>Modus tollendo tollens</i>	$[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$	$[(p \Leftarrow q) \wedge \neg q] \Rightarrow \neg p$

Moreover, it is worth noting that the presented approach to both varieties of the competition is fully in harmony with Boole's approach to the analogy between logical functions and arithmetic operations and relations. Thanks to this, it can be concluded that the search for the foundation of mathematics in logic, undertaken a century ago, is finally crowned with success – despite the discouragement of many thinkers<sup>5</sup>. What's more, it seems that we can talk about the emergence of a common core of things, thought, language, logic, mathematics and computer science, and in connection with reports about the discoveries of equivalents of logic gates in living organisms – also biology and, probably, also other sciences. As if "by the way", the eternal problems of deriving falsehood from true premises and deriving anything from falsehood disappear once and for all.

## 5. The logical competition in computer science and electronics

The competition function is reproduced in electronic circuits by a logic gate previously called the implication gate and marked with the symbol IMPLY. Respect for logic requires calling it the competition gate and changing the symbol IMPLY

to COMP. Until this change is officially implemented, the names should be used collectively in the form COMP(IMPLY). In the literature, the inhibition gate is marked with the symbol INHIB, which should be retained, or NIMPLY, which should be abandoned.

The interest in the competition and inhibition logic gates has increased rapidly in the last dozen years due to the invention of the memristor<sup>6</sup>. It turned out that the competition and inhibition gates can be constructed using fewer memristors than the most comprehensive NAND gates to date. This suggests that the systems based on the COMP(IMPLY) and INHIB gates will prove to be more efficient than those used so far (Lavanya, Gopal 2015). On the other hand, specialists are of the opinion that simply replacing NAND and AND / OR gates with COMP (IMPLY) gates will not bring the optimal result (Bürger 2012). We still have to wait for the final result of the appearance of memristors in the world of digital circuits, but scientists have no doubts that a new era in electronics is just beginning (Cyganik 2016). The discovery of the memristor also triggered a whole series of discoveries in the field of science covering molecular biology and a wide range of related disciplines. There are numerous reports in scientific journals about the existence or creation of logic gates, especially the competition and inhibition gates, in living organisms.

## Footnotes

<sup>1</sup> For example, Peter Abelard (1079-1142) rejected the possibility of anything resulting from falsehood (Loffredo D'Ottaviano 2008).

<sup>2</sup> At the beginning of the 20<sup>th</sup> century, this phrase was assigned to Duns Scotus (1266-1308), because it was found in two of his commentaries, however, research proved (1936) that the author of both of these books was an unknown author, who was called Pseudo-Scot. The persistent tendency to assign the *ex falso* principle to the Subtle Doctor some associate with the influence of the authority of Jan Łukasiewicz (1878-1956) who gave it a formalized version and called it Duns Scotus Law (Loffredo D'Ottaviano 2008).

<sup>3</sup> Ajdukiewicz proposed solutions based on distinguishing two semantic functions of sentences – the function of stating and expressing (Ajdukiewicz 1956). It was questioned by Z. Czerwiński (Czerwiński 1958). J. J. Jadacki (Jadacki 1986; Jadacki 1996) provides a comprehensive overview of this discussion and the entire issue.

<sup>4</sup> George Boole's (1815-1864) binary algebra, operating in binary 1 as the equivalent of truth and zero as the equivalent of falsehood, combined mathematics with logical propositional calculus (Boole 1854). Logic and mathematics were combined with computer science thanks to the discoveries of Claude Elwood Shannon (1916-2001) (Shannon 1948).

<sup>5</sup> Seemingly, the problem of material implication has effectively discouraged mathematicians and logicians from searching for the logical foundations of mathematics. As a result, "the whole topic has already become marginalized in mathematical research, no one is particularly interested in it any more, except for a group of researchers of the so-called non-classical set theories, and... mathematical logic has already lost its status as a mainstream field. (...) no... natural foundations of mathematics in the sense of axiomatization simply do not exist;...we were chasing a chimera" (Kisielewicz 2018, 44).

Transl. JP.

<sup>6</sup> The memristor is the fourth elementary electronic circuit – next to the resistor, capacitor and inductor. Its existence was foreseen in 1971 by the American engineer Leon Chua (1936-). (Strukov D.B., Snider G.S., Stewart D.R., Williams R.S. 2008, Williams 2010).

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