Research Article

Value and Price Theories: Compatibility Between Classical and Neoclassical Economics

Jie Wu[1,2], Albert R. Bakhtizin[3], Valery Makarov[3], Wu Zili[2]

1. Institute of Shandong Development, Shandong University, China; 2. Economic Simulation Base, National Simulation and Control Engineering Research Center, Guangzhou Milestone Software Co. Ltd.; 3. Central Economics and Mathematics Institute of Russian Academy of Sciences (CEMI RAS), Russia

This paper explores the compatibility of value and price theories between classical and neoclassical economics. Through mathematical modeling, it proposes that the value of a commodity is jointly constituted by quality, quantity, labor value, and use value. Prices are expressed through Quadratic Dynamic-Static Simultaneous Equations (QDSSE). The study introduces the Weighted Ratio Scaled Function (WRS Function) to adapt to price changes under fiat currency circulation. It argues that the law of diminishing marginal utility in neoclassical economics is a theorem valid only in non-equilibrium states within the classical price theory framework, not applicable to all economic scenarios. The paper aims to establish a multi-level equilibrium theory system that integrates classical and neoclassical price theories.

I. Introduction

At first glance, classical economics and neoclassical economics might be perceived as two fundamentally opposing theoretical frameworks within the history of economic thought. However, this perception is misleading. In fact, classical economics and neoclassical economics are mutually compatible, or essentially consistent.

Firstly, human social wealth is composed of labor value and use value. Classical economics focuses on labor value, whereas neoclassical economics emphasizes use value[1]. They study two distinct facets of the same object, demonstrating complete compatibility.

Secondly, the law of diminishing marginal utility is a conclusion derived from the classical theory of value[1]. Accordingly, starting from the axiomatic system grounded in the classical theory of value, it can not only deduce various theoretical schools that are compatible with marginalism, including the main theoretical conclusions of general equilibrium theory and Keynesian theory, but also provide many new theoretical insights not articulated within neoclassical economics that are demonstrated to be compatible with the established tenets of neoclassical theory. However, if the law of diminishing marginal utility is regarded as a fundamental axiom, there must be an error of generalizing from a partial observation. This is because considering a partial theorem as a fundamental axiom would inherently impede the construction of a consistent economic theoretical system characterized by completeness and compatibility. Similarly, in the history of science, the geocentric theory, founded on the misconception that the sun rises in the east, failed to provide a convincing explanation for the general motion laws of the planets within the solar system.

Reviewing the history of economic thought, the qualitative conclusions of the classical theory of price were progressively formulated between the mid-17th and the latter half of the 18th centuries. However, the corresponding mathematical theoretical framework that corresponds to it, hereafter referred to as the new theory of value, has only emerged in recent years. Specifically, the new theory of value holds that the value of a commodity is determined by the amount of labor expended during its production[2]. Additionally, with quality and population held as constants, the proportional relationship of value per unit among disparate commodities determines their "value in exchange"[2][3] Moreover, the price of a commodity can be represented as the general equivalent form that compares the unit value of precious metal currency to the unit value of other commodities[2][4].

Specifically, the price of a commodity manifests in two forms: "natural price" and "market price"[2]. The former is determined by the amount of labor expended during its production. The latter, however, is influenced by the objective existing factor of "effectual demand"2[2] in the market. When the supply of a commodity is equal to the 'effectual demand', the 'market price' of the commodity is equal to its 'natural price'. If the supply of the commodity does not match the 'effectual demand', the 'market price' will deviate from the 'natural price'. In cases of short supply, the 'market price' is higher than the 'natural price'. Conversely, in cases of oversupply, the market price is lower than the natural price.

Of course, the composition of the commodity price not only includes value (cost), but also surplus value (profit). Smith[2] simply believed that the price of commodities is composed of wages, profits, and rent, leading to a contradiction between this definition and his labor theory of value. In response, although

Marx[4] did not explicitly formulate the fundamental axiom that the innate intelligence and wisdom of humans leads to the force of increasing dexterity through repeated labor, that is the labor gravitational force, he still persisted in his conviction regarding the general law[3] of continuous improvement in labor efficiency within the sustained labor process of workers. Accordingly, he deduced the conclusion that labor creates surplus value in the process of capital operation. Moreover, Marx[5] further deduced that in the process of total social production, there must be a law of average profit rates in the operation of capital across various industries. This means that the natural price of a commodity includes profit. When supply exceeds demand and prices are higher than value, producing products will yield excess profits; conversely, when there is an oversupply and market prices are lower than value, producing products will incur losses. Therefore, when individuals pursue profit maximization, market supply and demand tend to balance Smith[2]. Thus, Marx's theory of production prices, which is grounded in Smith's theory of labor value and Marx's theory of surplus value, and further built upon the theory of average profit rates across multiple industries, constitutes a compatible classical theory of price.

II. Mathematical Modeling for Compatibility

To be specific, the value of a commodity is jointly constituted by its quality, quantity, labor value, and use value under general conditions. However, assuming that product quality and use value remain constant, the movement process of the commodity value is composed of its quantity and labor value. In this case, the commodity price based on classical economics can be expressed as a globally weighted scale function (GWS). The core characteristic of this function is that, under the condition of constant product quality and population, the price function of each commodity in both dynamic and static states is expressed through a system of simultaneous equations derived from the intersection of three independent functions, referred to as the Quadratic Dynamic-Static Simultaneous Equations (abbreviated as QDSSE). Within this framework, the static expression is a linear proportional function $p = aq$ (p is price, p is a constant coefficient, q is the quantity variable of the commodity), the dynamic equilibrium expression is a quadratic function $p = q^2$ [4], and the dynamic disequilibrium expression is an inverse proportional function q_0/q (q^* is a positive constant).

By analyzing QDSSE[5] for the price of a commodity, it can be observed that three distinct functions establish varying equivalence relations between the price p and quantity q of a commodity through a singular intersection point, which can be interpreted as the fixed point of QDSSE for a particular commodity. Specifically, the intersection point corresponds to the values of p and q that fulfill all three

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functions, that is, $p = aq = q_0^{-2/3} \cdot q_0^{1/3} = 1$. The above conclusion assumes that q_0 is a positive constant and q is non-zero. On this basis, we can provide three figures[6] that shows the prices of n commodities, as follows:

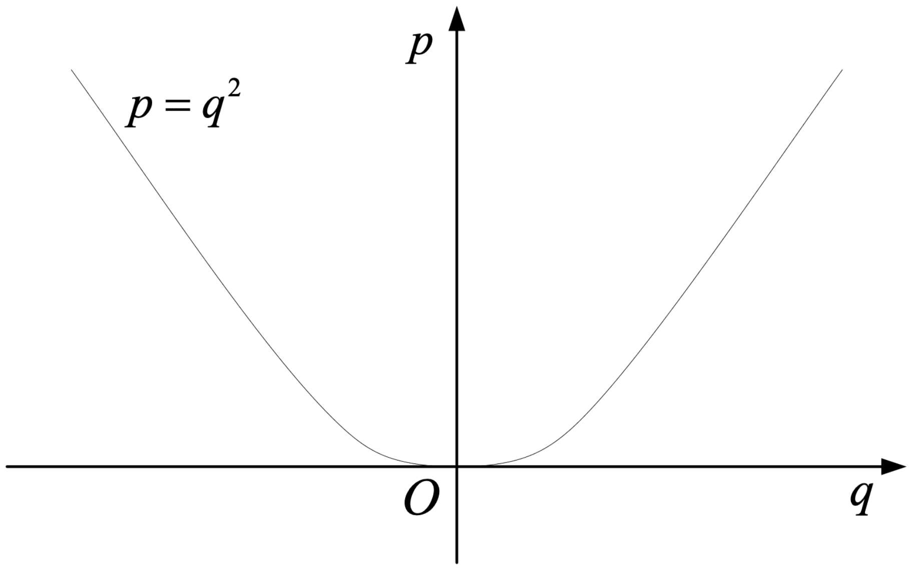

Figure 1. dynamic equilibrium of a quadratic function

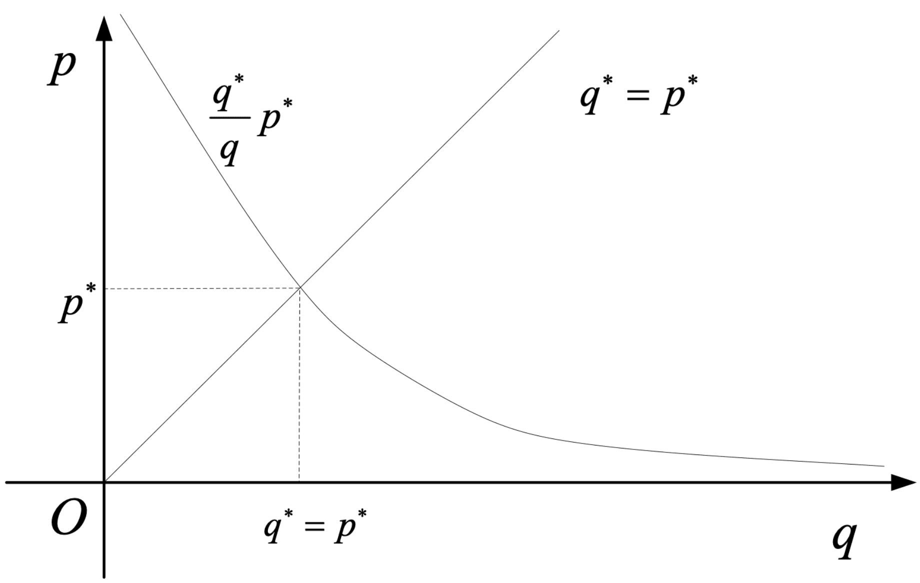

Figure 2. static equilibrium of a linear proportional function

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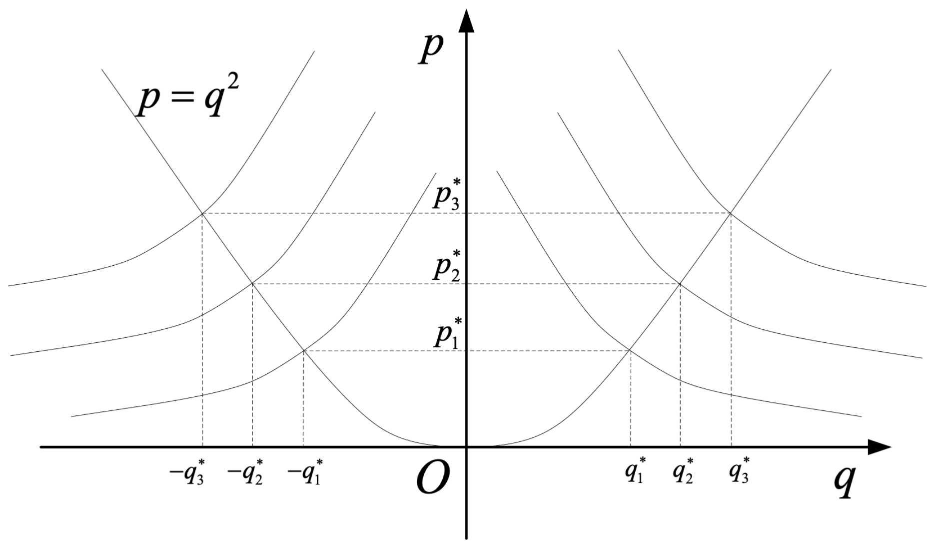

Figure 3. dynamic disequilibrium of an inverse proportional function

In general, we assume that there are n commodities in the market. Then, subject to the Law of Equivalent Exchange within classical economics, the prices of these n commodities are determined by a system of n QDSSEs that satisfy the Law of Equivalent Exchange (referred to as QDSSEs-EE). It is easy to demonstrate that QDSSEs-EE is a simple generalization of QDSSE for a single commodity, and to prove the existence of a unique fixed point of QDSSEs-EE.

It should be noted that the above discussion adopts an approach of abstracting economic problems into pure mathematical sense. In these mathematical expressions, we have not explained the economic meanings of different equations, independent variables, dependent variables, and parameters. Here we will provide the economic interpretation:

For the mathematical expressions of the above n simultaneous equations, it is assumed that the monetary flow per unit time $h \in [0, a) \subset R$ depends on a limited quantity of a precious metal currency - gold - which serves as the general equivalent among the n commodities, then the other n-1 commodities (excluding gold) are the equivalents represented and valued by the measure of value - gold. Let $D \subset Q^n$ be the production feasibility set within the vector space Q^n of commodity quantity, if commodity quality remains constant, a mapping from D to Q^n will be defined as:

$$\begin{gathered} f : D \subset Q^n \to P^{n-1} \\ (q_1, q_2, \cdots q_{n-1}, h) \mapsto (p_1, p_2, \cdots, p_{n-1}) \end{gathered}$$

which means that the prices of n-1 commodities depend on their exchange value.

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Specially, the labor value of the ith commodity, based on Marx's law of value[4], when quality is held constant, is transformed into an analytical expression of the price function of the jth commodity, which is represented as a piecewise function:

$$f_j(q_i) = \begin{cases} \frac{\frac{h \cdot w_j q_j^*}{\sum_{i=1}^n |q_j| q_j^*}}{q_j} = \frac{h \cdot \omega_{ij}}{\sum_{k=1}^n \omega_{kj}} \cdot \frac{1}{q_j} = \frac{h \cdot \lambda_j}{q_j} = h \cdot \bar{\lambda}_j = p_j, q_j \in R \\ \frac{\frac{h \cdot w_j q_j^*}{\sum_{i=1}^n |q_j| q_j^*}}{q_j^2} = \frac{h \cdot \omega_{ij}}{\sum_{k=1}^n \omega_{kj}} \cdot \frac{1}{q_j^2} = \frac{h \cdot \lambda_j}{q_j^2} = h \cdot \bar{\lambda}_j = p_j^*, \sum_{i=1}^{n-1} p_i q_i - \sum_{i=1}^{n-1} p_i^* q_i^* = 0 \end{cases} \quad (A.1.1)$$

where

- f_j is the price function of the jth commodity, $j = 1, 2, \ldots, n-1$;
- $q_i \in R_+$ is the quantity vector of the ith commodity, $i = 1, 2, \ldots, n$;
- m_i is the coefficient of quality standard for the ith commodity, with quantity as the physical bearer;
- h is defined as the norm of an n-dimensional vector, $h = |q_n|$, and $h/|q_n| = 1$ means that the norm of the quantity vector of a commodity whose quality is a constant, when expressed in terms of precious metal currency, is equal to 1;
- $w_j = |q_j|/q_j$ is the norm of the quantity vector of the jth commodity related to its price, that is, the amount of value per unit of the jth commodity;
- p_j is the price of the jth commodity;
- λ_j is the total exchange value related to the price of the jth commodity;
- $\bar{\lambda}_q$ is the exchange value for any commodity per unit related to the price of the jth commodity; and

$q^* = q_0 \in R_+$ means that the rational supply quantity of a commodity with constant quality in the first quadrant is a real number that exists within a closed interval from a positive value to a finite limit.

Thus, P^{n-1} is the space for market price of commodities, and $p \in P^{n-1}$ is the vector for market price of n-1 commodities.

From (A.1.1), we know that, according to the qualitative conclusions of the classical theory of price, the price of a commodity is a mapping from an n-dimensional vector space to an (n-1)-dimensional vector space that depends on the exchange value of the commodity. If $\bar{\lambda}_j = Q_j^* = \frac{\frac{w_j q_j^*}{\sum_{i=1}^n w_i q_i^*}}{q_j}$, then the mapping can be expressed as $f(Q_i^*) = p_j$.

Please note that the properties summarized above are based on a general discussion of Q_j^* and f. The price function of a commodity can be expressed in two different forms, one with the quantity of the commodity

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as the independent variable, and the other with the price of the commodity as the independent variable. Specifically, the former is a given function:

$$f(q_i) = m_j \cdot h \cdot \bar{\lambda}_j = p_j$$

However, when we consider the constraints of supply and demand balance, if the supply and demand relationship is dynamically changing, then the market price mapping for each of the n commodities will be represented as a quadratic function $f_j(q_i) = p_j = q_i^2$ [8] with q_i as the independent variable. We can equivalently construct the quantity-price function $f(Q_j^*)$ of a commodity as a self-mapping $f_j(p_i)$ of the commodity price, where the monetary flow k in the form of paper money[9] is integrated as a key parameter:

$$f_j(p_i) = \frac{m_{ij} \cdot k \cdot \omega_{ij}}{\sum_{k=1}^{n} \omega_{kj}} \cdot \frac{1}{\sqrt{p_i}} = p_j^0, p_i = q^2 \tag{A.1.2}$$

and $p_i = q_i^2$. When the monetary flow h is the ratio of the norm of the precious metal currency q_n to the sum of the norms of the other n-1 dimensional vectors, the commodity price system is a gold standard system. However, when the monetary flow k is in the form of paper money, the ratio of the issuance quantity of q_n to the sum of the norms of the other n-1 dimensional vectors becomes the commonly referred to as the price index[10]. At this point, the commodity price system is a national paper money credit system.

The properties of the price function in the form of paper money have changed, mainly by introducing the quantity of paper money k as a scaling factor to make it more flexible and expressive. However, this also requires more consideration of selecting and adjusting k to ensure that the behavior of the function meets expectations and is stable and effective in practical applications. During the design and implementation process, parameter tuning and sensitivity analysis may be necessary to optimize system performance. Considering the characteristics and composition of this function, we will name it the "Weighted Ratio Scaled Function" (hereafter WRS Function for short).

Therefore, the classical theory of price is compatible with the neoclassical theory of price. According to the classical theory of price, when the market supply and demand of a commodity deviates from the equilibrium point, there must be a function of diminishing marginal value that the price of the commodity deviates from its value, which is an inverse proportional function. It is a theorem deduced from the axiomatic system based on the classical theory of price. Clearly, various theoretical conclusions in

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neoclassical economics based on the law of diminishing marginal utility can also be deduced from the axiomatic system based on the classical theory of price, which will be further discussed below.

It is worth noting that the law of diminishing marginal utility is ultimately a theorem which is partially valid in non-equilibrium states, not for the entire theoretical system of price. If it is extended to all economic scenarios, there must be an error of generalizing from a partial situation. For example, according to the classical theory of price, in an equilibrium state, the price of a commodity is equal to its value. Then, the prices of n commodities are a convex polyhedron represented by n-1 hyperplanes. This is a crucial conclusion. If we ignore this conclusion, we will easily make a mistake that as long as $p_i = q_i^2$, the market supply and demand of the commodity will be in equilibrium. In fact, although the market price $p_i = q_i^2$ of a commodity satisfies Walras' law, unless it also satisfies $\sum_{i=1}^{n-1} p_i q_i - \sum_{i=1}^{n-1} p_i^* q_i^* = 0$ at the same time, otherwise it is impossible to be in a state of supply and demand equilibrium. It is easy to confuse the essential difference between market prices and equilibrium prices, without understanding this conclusion.

In addition, it is also wrong for mistaking that as long as Arrow Debreu's theorem[6] that the differential demand is always equal to zero proves the existence of fixed points, then the supply and demand of a commodity in the vicinity of fixed points must be balanced, that is, Walras' law $\sum_{i=1}^{n-1} p_i q_{iD} - \sum_{i=1}^{n-1} p_i q_{iS} = 0$ means this is the equilibrium point of market supply and demand. Subject to the law of diminishing marginal utility, the market price in an non-equilibrium system also has a constant fixed point, $\sum_{i=1}^{n-1} p_i \frac{q_i^*}{q_i} \equiv \sum_{i=1}^{n-1} p_i^* q_i^*$. In fact, except for the supply-demand equilibrium of commodities in the virtual work equilibrium system, commodities cannot be in a real supply-demand equilibrium state in redundant equilibrium and game equilibrium systems.

3. Conclusion

In summary, the classical theory of price is a theoretical system of that conforms to human common sense, with objective existence, and mathematical expression of WRS function, while the neoclassical theory of price is a theoretical system with many imperfections and compatibility defects. However, the classical theory of price and neoclassical equilibrium theory study the same objective economic problems. With the development of scientific theories and the elimination of errors, they will eventually integrate into one. In this article, we will examine the virtual work equilibrium system under pure and ideal conditions, the redundant equilibrium system with rational errors, and the game equilibrium system dominated by irrational economic behavior based on the classical theory of price. We will also analyze the Walrasian general equilibrium theory, Pareto optimal equilibrium theory, and Nash equilibrium theory based on neoclassical economics. We hope to establish a multi-level equilibrium theory system that is compatible with the classical theory of price and the neoclassical theory of price.

Footnotes

[1] Under normal circumstances, neoclassical economics studies the relationship between utility and price. It may seem that there is no direct connection between utility value and use value. However, the utility of an object determines its use value[2], and concurrently, use value is an essential related factor within the unity of opposites of labor value. Therefore, utility is interconnected with both use value, and labor value. For clarity and ease of understanding, this paper shall refer to utility value directly as use value.

[2] In this paper, we interpret Smith's "effectual demand"[2] as "rational demand".

[3] This axiom was first proposed by Smith[2]. However, Marx overlooked this point.

[4] The dynamic equilibrium function for the supply and demand of a commodity shows axial symmetry. Generally speaking, there is no universal explicit that can represent all functions with axial symmetry. Among these functions, we choose merely $p = q^2$ and $p = q^{2n}$ for discussion in this paper, primarily due to the simplicity and convexity of the former, as well as the continuity and convexity of the latter.

[5] The neoclassical theory of price generally assumes that prices are non-negative. In fact, this is a one-sided understanding. This is because, in the standard double-entry bookkeeping method, the price of any commodity is a dual form of positive and negative numbers in the real number field.

[6] In the neoclassical theory of price, there are also three common graphs: Marshall's supply and demand (S-D) model, Hicks indifference curve, and Edgeworth box. In our view, they are all imaginative transformations of subjective conjecture based on the combinations of graphs from the classical theory of price, and mere manifestations that lack analytical expressions and objective existence. For specific explanations, please refer to Appendix 1.

[7] According to Marx's Law, when the difference between supply and demand for any commodity is equal to zero, the price of the commodity is equal to its value. Specifically, Marx's Law reveals the fixed point of the price function within classical economics.

[8] Here, we modify the exponent of the quadratic function from 2 to 2n, making it more general.

[9] Precious metal currency can also be silver or other commodities with stable unit value, such as oil.

[10] Strictly speaking, the mathematical expressions of Ricardo's quantity theory of money and Fisher's monetary equation can both be represented in the form of the GWS function when k is paper money.

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