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Unpacking the Complexities of Cryptocurrency Prices Volatility in Times of Crisis: A Time Series Data with Long-term Memory or Long-range Dependence

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Abstract

This article explores the complexities of cryptocurrency price volatility during times of crisis. We analyze time series data with long-term memory or long-range dependence to understand the impacts of crises on cryptocurrency prices. Specifically, we examine the effects of the Covid-19 pandemic and the Russo-Ukrainian war on cryptocurrency markets, as well as the role of investor sentiment in price fluctuations during periods of uncertainty. To do so, we use fractionally integrated models to analyze the short- and long-term effects of these external factors on cryptocurrency prices. Our study mainly focuses on Bitcoin returns volatility using specific fractionally integrated models during four sub-period of historical crises from 2014. It assesses and compares the fractionally integrated models of the GARCH, the FIGARCH-BBM, the FIGARCH-CHUNG, FIEGARCH, and the FIAPARCH-BBM during the sub-periods of the pre-Covid-19, of the Covid-19 situation, between the Covid-19 and the Russo-Ukrainian War, and of the Russo-Ukrainian War. Conditional volatility models' parameters are first estimated from the four sub-sample data series BTC/USD exchange rate returns and it is calculated. Estimated conditional volatilities are then compared to specific volatilities relying on information criteria, after which the models are ranked. Finally, we test the specifics fractionally integrated volatility models with the normality test, the Q-Statistics on Standardized Residuals Test, the ARCH Test, and the graphic analysis. The specific volatility model of the first sub-period pre-Covid-19 is FIAPARCH-BBM(2,1). BTC/USD returns evolution during the Covid-19 crisis indicates that the FIEGARCH(2,2) is the appropriate volatility model. In addition, our results find that the FIEGARCH(2,1) is the appropriate model of volatility over the third sub-period and during the Russo-Ukrainian War period. By extrapolating the results of the four events, the study showed that the series of BTC/USD returns sampled over the four sub-periods were not immune to risk leading to historical crisis situations. The fluctuations of Bitcoin data during a political or economic event influence the choice of volatility models and their coefficients. More specifically, the parameters of the determined models of conditional volatility show that a war will make cryptocurrency more important on the exchange market even than an epidemic in the example of Covid-19. Our results suggest that the pandemic and geopolitical tensions have had a significant impact on cryptocurrency prices, but investor sentiment has played a crucial role in exacerbating price volatility. Additionally, we demonstrate the effectiveness of fractionally integrated models in predicting cryptocurrency prices during times of crisis. In summary, this study provides important insights into the dynamics of cryptocurrency markets during global crises, highlighting the need for sophisticated modeling techniques to effectively capture the complexities of these markets.

Keywords: Cryptocurrencies, Bitcoin, Conditional Volatility, Fractionally Integrated Models, Covid-19 Crisis, Russo-Ukrainian War.

JEL Classification: C22, C20, C52, C58, E42, H12

1. Introduction

Cryptocurrencies are recently emerging financial markets that are growing rapidly due to their ability to facilitate direct, transparent, and secure blockchain-based electronic payments between individuals all around the world (Foroutan and Lahmiri, 2022).

Bitcoin is the first known decentralized cryptocurrency that was founded in 2009 by a pseudonymous programmer Satoshi Nakamoto (www.investopedia.com). Aras (2021) indicate that Bitcoin exchange rate can generally be regarded as fulfilling two functions in the world economy. One of them is to provide an alternative payment method between seller and buyer as a currency or medium of exchange without needing any regulator. The second function is to act as a new kind of asset that allows hedging against uncertainty, stocks markzt, and the exchange rate of US dollar (Dyhrberg, 2016; Bouri and al., 2017; Demir, 2018; Shaikh, 2020) and to present an opportunity to build well-diversified portfolios (Lee and al., 2018). Therefore, the analysis of Bitcoin volatility and the factors that drive this variability is very important for investment and portfolio diversification (Bakas and al., 2022). As of November 15, 2021, the global cryptocurrency market capitalization is $2809.5 billion with 7381 cryptocurrencies (www.coinmarketcap.com) among which Bitcoin has the highest market capitalization of $1210 billion (43.09% of the total cryptocurrency market capitalization). In 2020, bitcoin gained 28% over the first 6 months of the year and Ether 77%. Since 2011, the leader of cryptocurrencies or cryptos tends to rise in July. In August, it fell in two out of three cases, on average by 16.4%. Bitcoin turmoil could resume after a lull since early June. Its annual volatility is still 84% but betting on bitcoin's fall currently seems just as dangerous. As of February 2021, there are 299 cryptocurrency exchanges globally with a market capitalization of $1.16 trillion and over 4,010 cryptocurrencies being traded. Initially, they were designed as secure payment methods utilizing blockchain technology, but with the potential for high returns, they have evolved into speculative investment tools. The low correlation with traditional investments, lower transaction costs, and safe-haven status during economic uncertainty have contributed to their increasing popularity. (Baur and al., 2018; Bouri and al., 2017 and Corbet and al., 2018; Stavroyiannis and Babalos, 2019; Bouri and al. 2019).

Bitcoin, the world's largest cryptocurrency, was the world's best-performing currency (in returns) in 2013, 2015, and 2016. It was also the world's worst-performing currency in 2014 (Desjardins, 2017), only to experience extreme movements again in 2017 and 2018. "Exceptional price volatility" is one of the main undesirable features of blockchains (Saleh, 2019); excessive volatility may be problematic to investors and users of cryptocurrencies. Understanding cryptocurrency price volatility has emerged as an important research topic, especially as cryptocurrencies gain more ground as an investment vehicle and asset class. Particularly, an understanding of cryptocurrency volatility is relevant to portfolio management and risk management. This paper aims to study Bitcoin exchange returns volatility by examining the (causal) impact of investor attention on returns volatility, the empirical analyses use Fractionally Integrated models. The work of Andrei and Hasler (2015) can be explained by the theory of investor attention and market returns volatility, that volatility increases with attention.

Volatility has a key role for decision-makers when making their decisions regarding risk management and trading, and even making monetary policy sounder. Volatility is a key and recurrent topic in finance. Many areas such as risk management, trading, security pricing, and monetary policy rely on volatility as the main input in the decision-making process. Also, volatility in the sense of uncertainty is a major player in the state of the economy where it represents an indicator of investors' and consumers' confidence. Volatility, on the other hand, is contagious among markets and their participants. From this perspective, volatility forecasting appears to be an important field of research with many theoretical and practical implications. Most volatility forecasting models that have been developed over the years fall into three broad categories: historical, stochastic, and implied volatility models (Naimy and Hayel, 2018). Bitcoin is a very volatile asset and therefore it's risky for investors to trade it.

Foroutan and Lahmiri (2022) use the EGARCH-M model to study the effect of volatility on cryptocurrency returns and the VAR model and Granger causality tests to study the return-volume relationships. They find that the return-volatility relationships for Tether, Ethereum, Ripple, Bitcoin Cash, EOS, and Monero are significant during the Covid-19 pandemic, while the same relationship is not significant prior to the pandemic for any of the studied cryptocurrencies. Their findings of the return-volume relationship support the availability of causal relations from returns to trading volume changes for Chainlink and Monero in the preCovid-19 period and for Ethereum, Ripple, Litecoin, EOS, and Cardano during the Covid-19 period.

Umarac and al. (2021) used the time-varying parameter-VAR of daily data series to explore the impact of Covid-19-related media coverage on the dynamic return and volatility connectedness of the three dominant cryptocurrencies (Bitcoin, Ethereum/USD, and Ripple/USD) and the fiat currencies of the euro, GBP, and Chinese yuan. Their empirical results with the return connectedness analysis indicate that the media coverage index (only before the first wave) and the cryptocurrencies are the net transmitters of shocks while the fiat currencies are the net receivers of shocks.

Gradojevic and Tsiakas (2021) used the wavelet Hidden Markov Tree model to examine the volatility cascades across multiple trading horizons in cryptocurrency markets (one-minute data on Bitcoin, Ethereum, and XRP/USD). They suggest that when moving from long to short horizons, volatility cascades tend to be symmetric. In this context, low volatility at long horizons is likely to be followed by low volatility at short horizons, and high volatility is likely to be followed by high volatility. In contrast, they find that when moving from short to long horizons, volatility cascades are strongly asymmetric. In this context, high volatility at short horizons is now likely to be followed by low volatility at long horizons.

Naeem and al. (2021) follow a social constructivist approach to examine the asymmetric efficiency of four cryptocurrencies and suggest that significant amounts of market inefficiency can appear in periods of a global epidemic crisis. Cryptocurrencies can be a potential safe-haven asset for the stock market, commodity market, and forex market during periods of financial market crisis due to the Covid-19 pandemic while framework empirical evidence results that Ethereum is a better safe-haven than Bitcoin (Mariana and al., 2021; Melki and al., 2021; Corbet, 2022).

Likewise, evidence from the EGARCH model indicates that the leverage impact is significant for three cryptocurrencies (Litecoin, Ripple, and Ethereum), but not for Bitcoin (Yousuf-Khan and al., 2021) and that Bitcoin returns volatility are highly unstable in speculative crises periods compared to stable ones (Lopez-Cabarcos and al., 2021; Kumar and Anandarao, 2019). Moreover, the impact of the news on the predictability of returns volatility of the cryptocurrencies market during the Covid-19 pandemic is studied by a GARCH-MIDAS modeling which suggests that the returns volatility of cryptocurrencies is riskier during the pandemic (Salisu and Ogbonna, 2021). Fang and al. (2020) indicate that economic policy uncertainty has appeared as a crucial predictor of volatility in the cryptocurrency market. They used the GARCH-MIDAS model to educate the impacts of News-based Implied Volatility on the long-term volatility of five cryptocurrencies (Bitcoin, Ethereum, Ripple, Litecoin and New Economy Movement). They also assessed the hedging effectiveness of cryptocurrencies against the stock market. Their results show that News-based Implied Volatility has a negative and significant impact on the long-term volatility of five cryptocurrencies.

Fractionally integrated models play an important role in financial econometrics and time series analysis. These models help to capture long memory properties in financial data, which can be observed in many financial time series such as stock prices, exchange rates, and interest rates. The fractional integration concept allows for the modeling of persistent, long-range dependence in the data, which is not captured by traditional linear models like ARIMA. The models also enable the estimation of long-term dependencies and the capturing of non-linear relationships in the data. In financial applications, fractionally integrated models have been used to forecast stock returns, volatility, and to detect structural breaks in financial time series. Overall, fractionally integrated models are an important tool for analyzing financial data and making more accurate predictions. The autoregressive conditional heteroscedasticity (ARCH) model, introduced by Engle in 1982, is a key representative of the fractionally integrated model class. It has been extended to include the generalized ARCH model (Bollerslev, 1986) and the fractionally integrated generalized autoregressive conditional heteroscedasticity (FIGARCH) model (Baillie, 1996; Baillie and al., 1996). These models differentiate between the conditional variance and the unconditional (or long-term) variance, which can vary over time and remain constant, respectively. Cryptocurrencies are in general highly volatile and are subject to sudden, massive price swings. The pandemic Covid-19, the Russo-Ukrainian war and historical crises have generated substantial global financial market volatility and have driven investors to seek alternative assets to preserve their cryptocurrencies. Bitcoin has received a large amount of attention since its introduction by Nakamoto (2008).

To evaluate the impact of historical crises and the Russo-Ukrainian war on the specification models of conditional volatility, the paper determines and compares the volatility models between the six Univariate GARCH models (fractionally integrated models) resulting from the four sub-periods samples of Bitcoin returns. The results of our study suggest that the choice of a specific model among the GARCH, FIGARCH-BBM, FIGARCH-CHUNG, FIEGARCH, FIAPARCH-BBM, and HYGARCH models mainly relies on the parameters of the appropriate Fractionally Integrated Models of the conditional volatility, especially since the negative asymmetry effect exists for the Bitcoin series during the four sub-periods. Our study also conducted tests on the volatility of Bitcoin returns during the four sub-periods historical crises using the normal test, the Q-Statistics on Standardized Residuals test, and the ARCH-LM test.

The results for the conditional daily volatility of BTC/USD returns for the different time periods and models used show that the FIAPARCH(2,1) model was used for the pre-covid-19 crisis period, and the FIEGARCH(2,2) model was used for the Covid-19 period. The skewness, excess kurtosis, and Jarque-Bera test statistic are reported for each period, with t-test statistic and p-value for each statistic. Q-statistic measures the significance of the residuals of the model by comparing them with a theoretical distribution that represents random fluctuations. The results suggest that there is evidence of autocorrelation in the residuals of the FIAPARCH(2,1) model used in the pre-covid-19 crisis period, while there is weak evidence of autocorrelation in the residuals of the FIEGARCH(2,2) model used in the Covid-19 crisis period. However, there is no evidence of autocorrelation in the residuals of the FIEGARCH(2,1) model used for the period between the pre-covid-19 crisis and the Russo-Ukrainian war and the Russo-Ukrainian war period.

The results suggest that the GARCH models used in the analysis are appropriate for capturing the ARCH effects in the data, as evidenced by the non-significant ARCH-LM tests for all models during the different periods studied. This implies that the models are able to adequately capture the conditional volatility of Bitcoin returns. Our findings suggest that peaks in both returns and conditional volatility were observed during the 2014-2019 period, which coincided with several of these events. During the Covid-19 crisis period, the authors found that Bitcoin returns were weakly varied, except for a peak on 17 March 2020, which could be attributed to market uncertainty and panic caused by the pandemic. Between the height of the pandemic and the start of the Russo-Ukrainian war, Bitcoin returns were highly volatile, as indicated by the FIEGARCH(2,1) model. Interestingly, our study also notes an increase in fluctuations in Bitcoin returns volatility during the period of the Russo-Ukrainian war, as indicated by the FIEGARCH(2,1) model. This suggests that geopolitical events can have a significant impact on the Bitcoin market and its volatility. In summary, the results of the study provide insights into the volatility of the Bitcoin market during different periods and the events that can influence it. The use of GARCH models allows for a better understanding of the conditional volatility of Bitcoin returns, which can help investors and policymakers make more informed decisions.

The rest of the paper is organized as follows. Section 2 describes the data used and their fluctuations during our historical selection crises. Section 3 presents the fractionally integrated volatility methodology models. Section 4 presents the main results of the paper. It also includes the determination of appropriate and specific volatility models during our selection events and a discussion on the difference between conditional volatility models during the Covid-19 crisis and the Russo-Ukrainian war. Some concluding remarks appear in Section 4.

2. Data and summary statistics

These are the daily returns of the Bitcoin during four sub-periods defined in table 1. The covered period is 31/12/2014 - 15/09/2022, which represents 2921 daily observations. Daily returns (in %) of Bitcoin prices (BTC/USD) are defined as

$$y_t = 100 * \ln(P_t / P_{t-1})$$

where P_t is the price series at time t.

Table 1: Sub-sample and historical Events

Sub-periods	Events type
Pre-Covid-19 Crisis	17/09/2014 - 30/12/2019
High Covid-19 situation	31/12/2019 - 31/12/2020
Between the Covid-19 Crisis and the Russo-Ukrainian War	01/01/2021 - 23/02/2022
Russo-Ukrainian War	24/02/2022 - 15/09/2022

Other than historical cryptocurrency events, historical crises such as the ruble crisis (2014), the stock market crash in China (2015), the Turkish lira crisis (2018), the economic crisis of Coronavirus (2020), the global energy crisis (2021), and the Russo-Ukrainian war (February 2022) are influencing Bitcoin prices. It must be said that the performance of cryptocurrencies during events makes you dizzy. Figure 1 shows daily BTC/USD prices, and it consists of four parts indicating the sub-periods indicated in Table 1.

War is almost always largely financed by a budget deficit (just as we have also seen with the Covid-19 pandemic). Hence usually the inflation that accompanies it and the illusions about future budget revenues or the payment of reparations by the defeated country. However, bitcoin is by nature a financial object whose mode of the issue is opposed to indebtedness. It is therefore inadequate in this regard.

During 2014, the price of Bitcoin soared and passed above $1150 for the first time. This year also sees the appearance of the first businesses accepting payments in Bitcoin. Towards the end of 2017, bitcoin reached $20,000, but in 2018, this price lost nearly 80% of its value. It then trades at $3,200. In 2019, the price of Bitcoin begins to soar again, reaching $10,000 in June, before falling again, and ending the year at around $7,500. Bitcoin is taking advantage of Covid-19 to soar in a gradual and continuous way. The price of bitcoin, for example, soared by around 130% over the first ten and a half months of 2020.

The war in Ukraine is causing strong price movements of cryptocurrencies. Especially, Bitcoin was quoted at $47,000 on March 28, 2022, while it was at $34,782 on February 26, 2022. But these data are above all to be compared to its price of 67,566 dollars on November 8, 2021 (after the high period of the Covid-19 pandemic and pre-Russo-Ukrainian War), a maximum that he has never found since. The Covid-19 crisis and the Russo-Ukrainian war have renewed the appeal of crypto-assets as safe-haven or portfolio diversification assets, particularly among institutional investors.

These crypto variations reflect the financial turmoil caused by rapidly rising inflation from historic crises in recent years as investors seek to shed their "risky" assets, such as cryptocurrencies. Bitcoin's supply and demand make its prices more volatile.

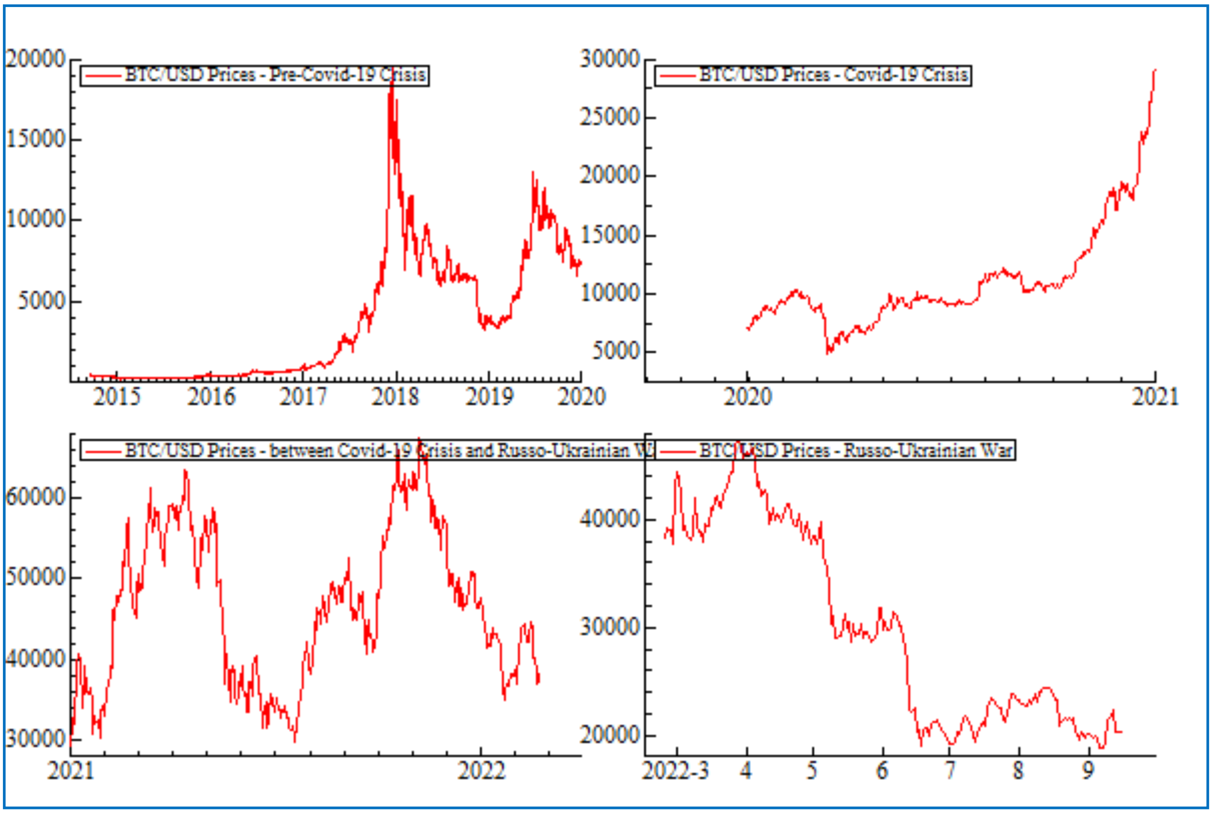

Figure 1: BTC/USD exchange rate evolution

Table 2 provided summarizes the statistical properties of cryptocurrency price volatility during different time periods. The data shows the mean, median, maximum, minimum, standard deviation, skewness, kurtosis, and Jarque-Bera statistic for each time period. The four time periods studied are: *Pre-Covid-19 Pandemic; Covid-19 Crisis; Between the Covid-19 Crisis and the Russo-Ukrainian War and Russo-Ukrainian War.* The data in Table 2 suggests that cryptocurrency price volatility was highest during the Covid-19 crisis, with a mean value of 0.3799 and a maximum value of 16.7104. The period between the Covid-19 crisis and the Russo-Ukrainian war had the lowest mean and median values of price volatility, indicating a relatively stable period for cryptocurrency prices. The data also shows a wide range of price volatility during each time period, with the maximum values ranging from 13.5764 to 22.5119 and the minimum values ranging from -46.4730 to -23.7558. This suggests that cryptocurrency prices are highly volatile and can experience large fluctuations in value in a short period of time.

The standard deviation measures the amount of variability or dispersion in the data. A higher standard deviation indicates that the data points are more spread out from the mean, suggesting a greater degree of volatility in cryptocurrency prices during that time period. The skewness measures the degree of asymmetry in the distribution. Negative skewness means that the distribution is skewed to the left, while positive skewness indicates skewness to the right. The data shows that during the Covid-19 crisis, cryptocurrency price volatility had a highly negative skewness value of -4.0780, indicating that the distribution was heavily skewed to the left, with many extreme negative values. Kurtosis measures the degree of peakedness or flatness of the distribution. Higher kurtosis values indicate a more peaked distribution. The data shows that during the Covid-19 crisis, cryptocurrency price volatility had a very high kurtosis value of 54.0080, indicating a very sharp and peaked distribution. The Jarque-Bera statistic is a test of normality. Higher values of Jarque-Bera indicate a greater deviation from a normal distribution. The data shows that during the Covid-19 crisis, the Jarque-Bera value was significantly higher than the other time periods, indicating a greater departure from a normal distribution. In addition the descriptive test suggest that the Covid-19 crisis had the highest degree of volatility, asymmetry, peakedness, and deviation from normality in cryptocurrency price volatility compared to the other time periods studied.

The data suggest that cryptocurrency price volatility was highest during the Covid-19 crisis and lowest during the period between the Covid-19 crisis and the Russo-Ukrainian war. The Covid-19 crisis also had the highest skewness and kurtosis values, indicating a highly skewed and peaked distribution of price volatility.

Table 2: Descriptive statistics of daily returns of BTC/USD exchange rate

	Pre-Covid-19 Pandemic	Covid-19 Crisis	Between the Covid-19 Crisis and the Russo-Ukrainian War	Russo-Ukrainian War
Mean	0.1434	0.3799	0.0570	-0.3139
Median	0.1940	0.2544	0.0313	-0.2229
Maximum	22.5119	16.7104	17.1821	13.5764
Minimum	-23.7558	-46.4730	-14.8107	-17.4053
Std. Dev.	3.8589	4.0018	4.0924	3.6937
Skewness	-0.2780	-4.0780	-0.0842	-0.4607
Kurtosis	8.2558	54.0080	4.6643	6.29386
Jarque-Bera	2247.414	40803.38	48.8521	99.4346

3. Methodology

3.1. ARCH Model

Engle (1982) proposed the ARCH model (Auto-regressive Conditional Heteroskedastic Model). This is given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 \quad (1)$$

3.2. GARCH Model

The Generalized $ARCH$ ($GARCH$) model of Bollerslev (1986) is based on an infinite ARCH specification and it allows to reduce the number of estimated parameters by imposing nonlinear restrictions on them. The $GARCH(p,q)$ model can be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \quad (2)$$

Using the lag (or backshift) operator L, the $GARCH(p,q)$ model becomes:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \quad (3)$$

With $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + \ldots + \beta_p L^p$.

3.3. EGARCH Model

Nelson (1991) devoted to research on the asymmetric evolutions of the variance, this research gave birth to the Exponential GARCH (EGARCH) model. It is re-expressed in Bollerslev and Mikkelsen (1996) as follows:

$$\log(\sigma_t^2) = \omega + [1-\beta(L)]^{-1}[1+\alpha(L)]g(z_{t-1}) \quad (4)$$

The EGARCH model is useful to better capture the phenomenon of asymmetry in the dynamics of volatility. This model makes the conditional variance of time t depend on that of the previous period, i.e. the standardized shocks in $(t-1)$ and the difference between the absolute value of the standardized shocks and their expectation in $(t-1)$.

3.4. IGARCH Model

The $GARCH(p,q)$ model can be expressed as an $ARMA$ process. Using the lag operator L, we can rearrange Equation (2) as:

$$[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2) \quad (5)$$

When the $[1-\alpha(L)-\beta(L)]$ polynomial contains a unit root, i.e. the sum of all the α_i and the β_j is one, we have the $IGARCH(p,q)$ model of Engle and Bollerslev (1986). It can then be written as:

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2) \quad (6)$$

Where $\phi(L) = [1-\alpha(L)-\beta(L)](1-L)^{-1}$ is of order $\max\{p,q\}-1$.

We can rearrange Equation (6) to express the conditional variance as a function of the squared residuals. After some manipulations, we have its $ARCH(\infty)$ representation:

$$\sigma_t^2 = \frac{\omega}{[1-\beta(L)]} + \left\{1-\phi(L)(1-L)[1-\beta(L)]^{-1}\right\}\varepsilon_t^2 \quad (7)$$

3.5. FIGARCH Model

To mimic the behavior of the correlogram of the observed volatility, Baillie and al. (1996) (hereafter denoted BBM) introduce the Fractionally Integrated GARCH (FIGARCH) model by replacing the first difference operator of Equation (7) by $(1-L)^d$.

The conditional variance of the $FIGARCH(p,d,q)$ is given by:

$$\sigma_t^2 = \underbrace{\omega[1-\beta(L)]^{-1}}_{\omega^*} + \underbrace{\left\{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\right\}\varepsilon_t^2}_{\lambda(L)} \quad (8)$$

FIEGARCH (Fractionally Integrated Exponential Generalized Autoregressive Conditional Heteroscedastic) is an extension of the GARCH model that allows for fractional integration of the conditional variance. This means it captures long-range dependencies in volatility and can model persistent volatility patterns.

3.6. HYGARCH Model

The HYGARCH is given by Equation (8), when $\lambda(L)$ is replaced by $1-\left[1-\beta(L)\right]^{-1}\phi(L)\left\{1+\alpha(1-L)^{d}\right\}$. HYGARCH (Hybrid Generalized Autoregressive Conditional Heteroscedasticity) is a hybrid volatility model that combines aspects of both the GARCH and Stochastic Volatility (SV) models. The HYGARCH model aims to improve upon the limitations of the GARCH and SV models by combining the advantages of both models in one framework.

3.7. FIEGARCH and FIAPARCH Models

The idea of fractional integration has been extended to other $GARCH$ types of models, including the Fractionally Integrated $EGARCH$ ($FIEGARCH$) of Bollerslev and Mikkelsen (1996) and the Fractionally Integrated $APARCH$ ($FIAPARCH$) of Tse (1998).

Similarly to the $GARCH(p,q)$ process, the $EGARCH(p,q)$ of Equation (4) can be extended to account for long memory by factorizing the autoregressive polynomial $\left[1-\beta(L)\right]=\phi(L)(1-L)^{d}$ where all the roots of $\phi(z)=0$ lie outside the unit circle. The $FIEGARCH(p,d,q)$ is specified as follows:

$$\log(\sigma_t^2)=\omega+\phi(L)^{-1}(1-L)^{-d}\left[1+\alpha(L)\right]g(z_{t-1}) \quad (9)$$

Finally, the $FIAPARCH(p,d,q)$ model can be written as:

$$\sigma_t^{\delta}=\omega+\left\{1-\left[1-\beta(L)\right]^{-1}\phi(L)(1-L)^{d}\right\}(\left|\varepsilon_t\right|-\gamma\varepsilon_t)^{\delta} \quad (10)$$

FIAPARCH-BBM (Fractionally Integrated Asymmetric Power Autoregressive Conditional Heteroscedastic with Bivariate and Multivariate Components) is an extension of the FIEGARCH model that allows for asymmetric volatility responses and considers multiple time series in the volatility modeling process. Thirdly, FIGARCH (Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic) is an extension of the GARCH model that allows for fractional integration of the conditional variance. This means it can model long-range persistence in volatility and is suitable for modeling financial time series with persistent volatility patterns. Finally,

In summary, the FIEGARCH, FIAPARCH-BBM, and FIGARCH models are all variants of the fractionally integrated GARCH model, each adding different features to capture different aspects of volatility. The HYGARCH model is a hybrid model that combines the advantages of both the GARCH and SV models.

4. Empirical results

4.1. Bitcoin returns and volatility specifics models and its changes

For each of the Bitcoin, the choice of a specific model among the GARCH, FIGARCH-BBM, FIGARCH-CHUNG, FIEGARCH, FIAPARCH-BBM and HYGARCH models mainly relied on information criterion (Akaike, Shibata, Schwarz, and Hannan-Quinn). Specifically, to check the change of volatility models following of the historical crises, the Covid-19, and the Russo-Ukrainian War of the models, we created four sub-sample over the period of 17/9/2014-15/9/2022.

Table 3: Criteria information for Fractionally Integrated volatility models selection

	Volatility Models	Akaike	Shibata	Schwarz	H.-Quinn
Pre-Covid-19	GARCH(2,1)	5.318567	5.318554	5.332979	5.323868
	FIGARCH-BBM(2,1)	5.314565	5.314546	5.331859	5.320926
	FIGARCH-CHUNG(2,1)	5.314598	5.314579	5.331892	5.320959
	FIEGARCH(2,1)	5.312473	5.312439	5.335531	5.320954
	FIAPARCH-BBM(2,1)*	**5.311212**	**5.311178**	**5.334271**	**5.319694**
	HYGARCH(2,1)	5.319076	5.319050	5.339252	5.326498
Covid-19 Crisis	GARCH(2,2)	5.424682	5.424159	5.488530	5.450051
	FIGARCH-BBM(2,2)	5.449585	5.448875	5.524074	5.479182
	FIGARCH-CHUNG(2,2)	102.945124	102.944415	103.019614	102.974721
	FIEGARCH(2,2)*	**5.271016**	**5.269851**	**5.366787**	**5.309069**
	FIAPARCH-BBM(2,2)	5.407201	5.406036	5.502973	5.445254
	HYGARCH(2,2)	5.422191	5.421267	5.507322	5.456016
Between Covid-19 Crisis and Russo-Ukrainian War	GARCH(2,1)	5.614214	5.613934	5.662399	5.633261
	FIGARCH-BBM(2,1)	5.654157	5.653755	5.711979	5.677013
	FIGARCH-CHUNG(2,1)	5.631788	5.631385	5.689609	5.654644
	FIEGARCH(2,1)*	**5.570812**	**5.570101**	**5.647908**	**5.601287**
	FIAPARCH-BBM(2,1)	5.657093	5.656382	5.734188	5.687567
	HYGARCH(2,1)	5.656586	5.656040	5.724045	5.683252
Russo-Ukrainian War	GARCH(2,1)	5.430484	5.429320	5.511810	5.463382
	FIGARCH-BBM(2,1)	5.481480	5.479815	5.579072	5.520958
	FIGARCH-CHUNG(2,1)	5.481296	5.479631	5.578888	5.520774
	FIEGARCH(2,1)*	**5.390327**	**5.387403**	**5.520449**	**5.442963**
	FIAPARCH-BBM(2,1)	5.404444	5.401520	5.534566	5.457081
	HYGARCH(2,1)	5.485942	5.483690	5.599799	5.531999

On the basis of the results of Table 3, we estimated the FIAPARCH-BBM(2,1) model in the case of the sub-period Pre-Covid-19, while the specific models estimated at the sub-period of the Covid-19 pandemic was FIEGARCH(2,2) and between at the sub-period between the Covid-19 pandemic and the Russo-Ukrainian War was FIEGARCH(2,1). In the case of the sub-period of Russo-Ukrainian War, we followed FIEGARCH(2,1) model. All fractionally integrated models were employed based on estimates for the sample sub-periods, and then the results were used to compare the conditional variance of BTC/USD returns in Table 4.

Table 4: Estimated parameters vector of Bitcoin appropriate Fractionally Integrated Volatility Models

Pre-covid-19 crisis: FIAPARCH(2,1) Model	Covid-19 period : FIEGARCH(2,2) Model	Between the pre-covid-19 crisis and the Russo-Ukrainian war period: FIEGARCH(2,1) Model	Russo-Ukrainian war period: FIEGARCH(2,1) Model
$Cst(M)$ = 0.127	$Cst(M)$ = 0.421	$Cst(M)$ = -0.322	$Cst(M)$ = -0.634
$Cst(V)$ = 0.508	$Cst(V)$ = 2.564	$Cst(V)$ = 3.5536	$Cst(V)$ = 2.603
$d-FIGARCH$ =0.649	$d-FIGARCH$ = 0.462	$d-FIGARCH$ = 1.186	$d-FIGARCH$ = 0.721
$ARCH(\phi_1)$ = 0.092	$ARCH(\phi_1)$ = -7.5883	$ARCH(\phi_1)$ = -0.873883	$ARCH(\phi_1)$ = 1.458
$GARCH(\beta_1)$ = 0.545	$ARCH(\phi_2)$ = 20.884	$GARCH(\beta_1)$ = 0.010	$GARCH(\beta_1)$ = 0.609
$GARCH(\beta_2)$ = 0.097	$GARCH(\beta_1)$ = 0.730	$GARCH(\beta_2)$ = 0.271	$GARCH(\beta_2)$ = -0.452
$APARCH(\gamma_1)$ = 0.017	$GARCH(\beta_2)$ = -0.778	$EGARCH(\theta_1)$ = -0.118	$EGARCH(\theta_1)$ = -0.010
$APARCH(\delta)$ = 1.192	$EGARCH(\theta_1)$ = -0.013	$EGARCH(\theta_2)$ = -0.132	$EGARCH(\theta_2)$ = -0.094
---	$EGARCH(\theta_2)$ = 0.018	---	---

The $d-FIGARCH$ and the $GARCH$ coefficients are positively associated with Bitcoin returns during the four sub-periods. Further, the $ARCH$ coefficient (ϕ_1) is positively related to exchange rate of Bitcoin returns pre-Covid-19 epidemic and during the Russo-Ukrainian war and is negatively associated with BTC/USD returns during the two sub-periods of Covid-19 crisis and pre-Russo-Ukrainian war. From Covid-19 and during the Russo-Ukrainian war, the $GARCH$ coefficient (β_2) are negatively related with daily Bitcoin returns, unlike the pre-covid sub-period ($GARCH(\beta_2) \succ 0$). The $APARCH$ coefficients (γ_1 and δ) are positively associated with Bitcoin returns pre-Covid-19 crisis, but the $EGARCH$ coefficients are negatively related to the daily variable during the three other sub-periods samples.

The parameters of the appropriate Fractionally Integrated Models of the conditional volatility are almost significant, especially since the negative asymmetry effect exists for the Bitcoin series during the four sub-periods. This means that for all the BTC/USD exchange rate, past returns show a lot of persistence in its shift to negatives, so they increase volatility more strongly than positive past returns.

4.2. Tests of daily returns volatility of BTC/USD exchange rate

We test conditional volatility of Bitcoin returns during the four sub-periods historical crises. with the normal test, the Q-Statistics on Standardized Residuals test, and the ARCH-LM test (See Tables 5, 6 and 7).

Table 5 presents the results of tests on the volatility of Bitcoin returns. The tests were conducted for different time periods and models. The first row of the table shows the normality test results for the conditional daily volatility of BTC/USD returns for two time periods: pre-covid-19 crisis and Covid-19 period. The FIAPARCH(2,1) model was used for the pre-covid-19 crisis period, and the FIEGARCH(2,2) model was used for the Covid-19 period. The table reports the following statistics for each period: skewness, excess kurtosis, and Jarque-Bera test statistic. The skewness measures the degree of asymmetry in the distribution of the data. The excess kurtosis measures the degree of peakedness in the distribution of the data. The Jarque-Bera test statistic tests the null hypothesis that the data is normally distributed. The second row of the table shows the normality test results for the conditional daily volatility of BTC/USD returns for the period between the pre-covid-19 crisis and the Russo-Ukrainian war period, and the Russo-Ukrainian war period. The FIEGARCH(2,1) model was used for both periods. The table reports the same statistics for each period as in the first row. For each time period, Table 5 also reports the t-test statistic and p-value for each statistic. The t-test statistic measures the difference between the sample mean and the null hypothesis. The p-value represents the probability of observing a test statistic as extreme as the one computed, assuming that the null hypothesis is true.

Table 5: Normality test of conditional daily volatility of BTC/USD returns

	Pre-covid-19 crisis: FIAPARCH(2,1) Model			**Covid-19 period: FIEGARCH(2,2) Model**		
	Statistic	t-Test	P-Value	Statistic	t-Test	P-Value
Skewness	-0.41620	7.4723	7.8809e-014	-0.11589	0.91003	0.36281
Excess Kurtosis	7.9711	71.592	0.00000	3.3337	13.124	2.3853e-039
Jarque-Bera	5167.9	---	0.00000	170.77	---	8.2754e-038
	Between the pre-covid-19 crisis and the Russo-Ukrainian war period: FIEGARCH(2,1) Model			**Russo-Ukrainian war period: FIEGARCH(2,1) Model**		
	Statistic	t-Test	P-Value	Statistic	t-Test	P-Value
Skewness	-0.24949	2.0923	0.036408	-0.13639	0.80108	0.42309
Excess Kurtosis	0.83773	3.5210	0.00042991	1.5318	4.5198	6.1902e-006
Jarque-Bera	16.599	---	0.00024866	20.577	---	3.4022e-005

The Q-Statistics on Standardized Residuals test of conditional daily volatility of BTC/USD returns assesses whether there is evidence of autocorrelation in the residuals of the volatility models. The Q-statistic measures the significance of the residuals of the model by comparing them with a theoretical distribution that represents random fluctuations. A smaller p-value for a Q-statistic indicates a stronger rejection of the null hypothesis that the residuals are independent and identically distributed. Looking at the Table 6, we can observe that for the pre-covid-19 crisis period, the Q-statistics for all lags up to 50 are statistically significant (p-values < 0.05), except for Q(20). This suggests that there is evidence of autocorrelation in the residuals of the FIAPARCH(2,1) model. For the Covid-19 crisis period, only the Q(5) statistic is significant, indicating weak evidence of autocorrelation in the residuals of the FIEGARCH(2,2) model.

Table 6 shows that during the period between the pre-covid-19 crisis and the Russo-Ukrainian war and the Russo-Ukrainian war period, none of the Q-statistics are statistically significant, suggesting that there is no evidence of autocorrelation in the residuals of the FIEGARCH(2,1) model. The Q-Statistics on Standardized Residuals test provides evidence that there is some degree of autocorrelation in the residuals of the FIAPARCH(2,1) model used in the pre-covid-19 crisis period, while there is weak evidence of autocorrelation in the residuals of the FIEGARCH(2,2) model used in the Covid-19 crisis period. However, there is no evidence of autocorrelation in the residuals of the FIEGARCH(2,1) model used for the period between the pre-covid-19 crisis and the Russo-Ukrainian war and the Russo-Ukrainian war period.

Table 6: Q-Statistics on Standardized Residuals test of conditional daily volatility of BTC/USD returns:

	Pre-covid-19 Crisis: FIAPARCH(2,1) Model	**Covid-19 Crisis: FIEGARCH(2,2) Model**	**Between the pre-covid-19 Crisis and the Russo-Ukrainian War: FIEGARCH(2,1) Model**	**Russo-Ukrainian war: FIEGARCH(2,1) Model**
Q(5)	11.5482 [0.04153]*	5.8943 [0.3166]	2.8227 [0.7273]	1.82590 [0.8727]
Q(10)	38.6049 [0.00003]**	6.3779 [0.7826]	8.1850 [0.6108]	11.7021 [0.3055]
Q(20)	49.3501 [0.00027]**	20.4878 [0.4278]	16.1610 [0.7066]	16.0897 [0.7110]
Q(50)	72.3173 [0.02115]*	47.4691 [0.5755]	57.9167 [0.2063]	43.4580 [0.7316]

The ARCH-LM test is used to test for the presence of ARCH effects in the residuals of a GARCH model. A significant result indicates that there is evidence of ARCH effects and the GARCH model may not be appropriate. Looking at Table 7, we can see that for the pre-covid-19 crisis period, none of the ARCH-LM tests are statistically significant for the FIAPARCH(2,1) model. This suggests that the model adequately captures the ARCH effects in the data. For the Covid-19 period, the ARCH-LM tests are also not statistically significant for the FIEGARCH(2,2) model, indicating that the model is appropriate for this period as well. For the period between the pre-covid-19 crisis and the Russo-Ukrainian war, and the Russo-Ukrainian war period, the ARCH-LM tests are not statistically significant for the FIEGARCH(2,1) model, suggesting that the model is adequate for these periods as well.

In summary, the results suggest that the GARCH models used in this analysis adequately capture the ARCH effects in the data, and there is no strong evidence of residual ARCH effects.

Table 7: The ARCH-LM test of conditional daily volatility of BTC/USD returns

	Pre-covid-19 crisis: FIAPARCH(2,1) Model	**Covid-19 period: FIEGARCH(2,2) Model**
ARCH 1-2 test	F(2,1923) = 0.16521 [0.8477]	F(2,358) = 1.3027 [0.2731]
ARCH 1-5 test	F(5,1917) = 0.92912 [0.4609]	F(5,352) = 0.87546 [0.4976]
ARCH 1-10	F(10,1907)= 0.82670 [0.6028]	F(10,342) = 0.83278 [0.5972]
	Between the pre-covid-19 crisis and the Russo-Ukrainian war period: FIEGARCH(2,1) Model	**Russo-Ukrainian war period: FIEGARCH(2,1) Model**
ARCH 1-2 test	F(2,411) = 1.0760 [0.3419]	F(2,196) = 0.31178 [0.7325]
ARCH 1-5 test	F(5,405) = 1.1693 [0.3235]	F(5,190) = 0.90832 [0.4768]
ARCH 1-10	F(10,395) = 0.88996 [0.5426]	F(10,180) = 1.1168 [0.3518]

4.3. Graphic Analysis

Figure 2 exhibits the evolution of the returns and the conditional volatility series during all sub-periods. There is a distinct episode, and this is related to the history events of Bitcoin and historical crises from 2014 to 2022, where conditional volatility returns series have reached unprecedented levels.

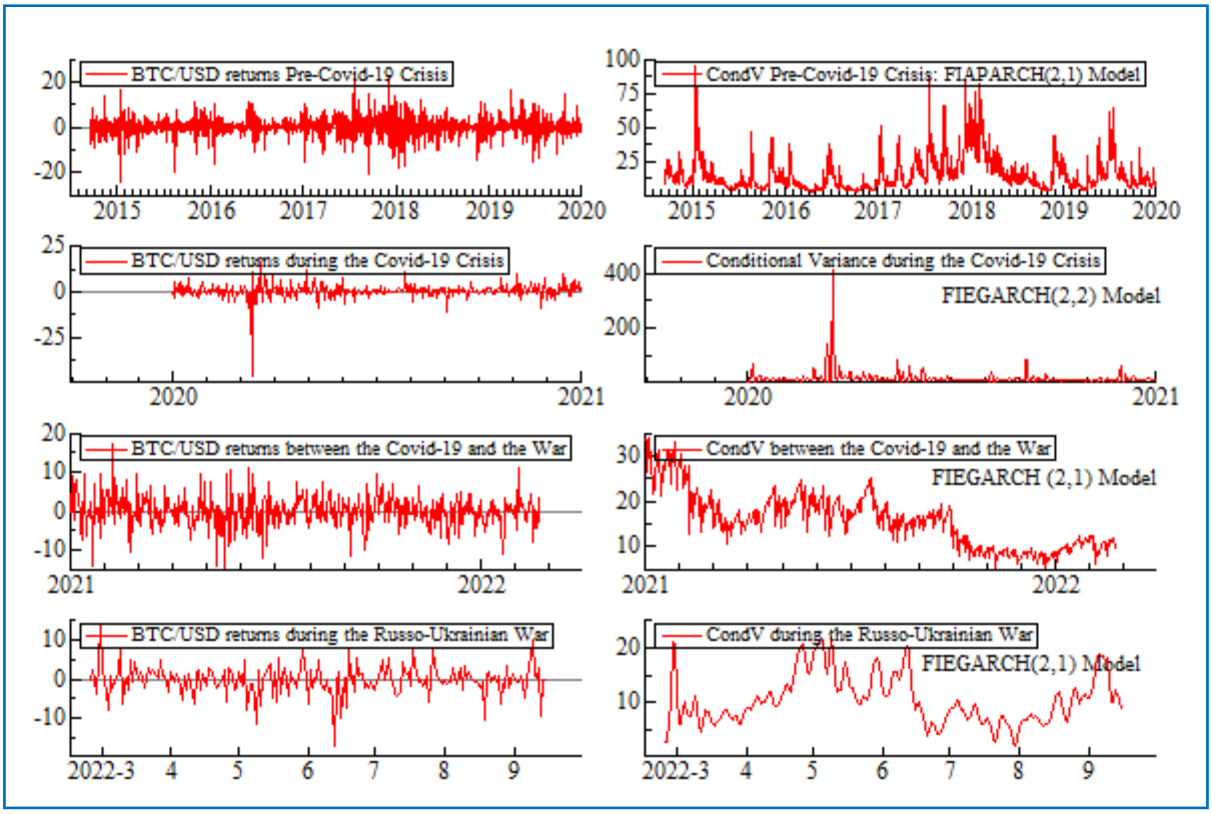

Figure 2: Returns and conditional volatility fluctuations for BTC/USD exchange rate

There is a distinct episode, and this is related to the event history of Bitcoin and historical crises from 2014 to 2022, where conditional volatility returns series have reached unprecedented levels. Furthermore, peaks can be observed during 2014-2019 period for all returns and conditional volatility (measured by FIAPARCH(2,1) model) series, which can be related to the Bitstamp (04 January 2015), Bitfinex (02 August 2016), when the Chinese authorities decided to immediately close all Bitcoin and cryptocurrency exchanges (15 September 2017), CoinRail (10 June 2018), and others cryptocurrencies events. In addition, during the sub-period of Covid-19 crisis, conditional variance with FIEGARCH(2,2) model indicates that BTC/USD returns are weakly varied, except for a peak indicated on 17 March 2020. Between the height of the Covid-19 pandemic and the start of the Russo-Ukrainian war, Bitcoin returns are highly volatile. This interpretation is shown by the movements of the conditional variance using the specified FIEGARCH(2,1) model. Interestingly enough, we do observe more fluctuations of the bitcoin returns volatility (indicated by FIEGARCH(2,1) model) during the period 24/02/2022 - 15/09/2022 when the Russo-Ukrainian War occurred.

5. Conclusion

Our research in this paper focuses on modeling the volatility of the four samples during the four sub-periods including the Covid-19 crisis and the Russo-Ukrainian war. The objective is to apply new techniques to identify and estimate conditional volatility at daily frequencies during the period of an historical event. In our case, the variable is the Bitcoin. In addition, the results obtained in our paper make it possible to highlight a certain number of empirical interpretations concerning the volatility of cryptocurrencies during the Covid-19 crisis and the Russo-Ukrainian war.

We find that the main models of the volatility of Bitcoin returns are the FIAPARCH(2,1) model during the pre-Covid-19 sub-period, the FIEGARCH(2,2) model during the Covid-19 crisis and the model FIEGARCH(2,1) during the two sub-periods pre and during the Russo-Ukrainian war. In most cases, the estimation of the reference asymmetric univariate GARCH models (Fractionally Integrated Models) made it possible to eliminate the conditional heteroscedasticity, the autocorrelation and the long memory of the standardized innovations. By extrapolating the results of the four events, the study showed that the series of BTC/USD returns sampled over the four sub-periods were not immune to risk leading to historical crisis situations.

In general, it is necessary to use a model that takes into account the effects of asymmetries, i.e. allowing a differentiation of the impact of shocks on volatility according to their nature (positive or negative). Indeed, a negative shock systematically results in a greater increase in volatility than a positive shock on all Cryptocurrencies. The fluctuations of Bitcoin data during a political or economic event influence the choice of volatility models and their coefficients. More specifically, the parameters of the determined models of conditional volatility show that a war will make a cryptocurrency more important on the exchange market even than an epidemic in the example of Covid-19.

Bitcoin is a virtual currency and not an electronic currency. It resembles electronic currencies because of its support, but an electronic currency is a currency which has a legal tender, and which allows, by its payment, to free oneself from a debt. In all countries, money in general and its dramatic disturbances have a decisive influence on the evolution of business law and financial economics. Bitcoin is no exception to the rule.

There are all the risks inherent in the design and nature of a new currency. In their short history, bitcoins have already experienced many crises, crises of success which resemble the Tulipomania crisis of 1633 in Holland, or crisis of defiance, which resembles the American crisis of 1907 or the explosion of many financial bubbles and the birth of modern flash crashes. Half of humanity, around 4 billion people, are deprived of banking services, and in the future, bitcoin or a virtual currency of this type may be their only hope of doing business and financial transactions.

Overall, the study's findings suggest that the use of GARCH models can provide a better understanding of the conditional volatility of Bitcoin returns. The negative asymmetry effect identified in the study implies that investors need to exercise caution when investing in Bitcoin. Policymakers need to consider the impact of their actions on market sentiment and volatility in the Bitcoin market. The study's identification of specific episodes of high volatility in the Bitcoin market can also be useful for investors and policymakers in making informed decisions.

The study's findings that negative past returns of BTC/USD exchange rate cause more significant volatility than positive past returns have significant economic implications for investors and policymakers. This negative asymmetry effect implies that investors need to exercise caution when investing in Bitcoin as negative returns are more likely to result in greater volatility, potentially leading to significant losses. Policymakers need to take note of the effect of negative past returns on Bitcoin volatility, as it suggests that regulatory actions that result in negative market sentiment could lead to significant volatility in the market. The study's identification of specific episodes of high volatility in the Bitcoin market also has implications for investors and policymakers. Understanding the events that cause peaks in returns and volatility can help investors make better-informed decisions regarding the timing and size of their investments. Policymakers can use this information to craft regulatory policies that minimize market disruptions during periods of high volatility.

The question that arises in the future is by what reasons we can explain the evolution of cryptocurrencies in place of fiat currencies or their failure to become fully safe haven and means of payment in the event of economic and financial collapse of a country in connection with an armed conflict or a pandemic situation?

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