

## Peer Review

# Review of: "Tutorial on Fourier and Hankel Transforms for Ultrafast Optics"

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I appreciate the author preparing the manuscript titled "Tutorial on Fourier and Hankel Transforms for Ultrafast Optics" that can be useful to the broad range of students and scientists, especially those working in the field of ultrafast optics. Below are some comments/suggestions which I hope will contribute to the manuscript's further improvement.

1. Abstract. It is not clear what the author refers to in the sentence "Most important of all, there have been misuse of Fourier transform from my observations, which cannot be easily detected by checking the smoothness of the result of a numerical implementation or by seeing if the simulation duplicates the "overall physics."" I think the sentence needs precision, which will also make it clear what problem(s) the next sentence refers to.

2. Abstract. "...which is the core element of fast radially-symmetric full-field ultrafast propagation." There are two adjectives "fast" and "ultrafast"; which one? Maybe the author also wants to consider defining what is fast/ultrafast here.

3. Abstract. "Feel me to send me ..." sounds a bit strange. Also, I would leave this part of the Abstract to the end of the manuscript.

4. Intro. "These two equations make intuitive sense if the field is a simple sinusoidal wave ..." It feels like it is defined for the reader what is intuitive (and what is not). I would just start as "If the field is a simple sinusoidal wave... then the amplitude  $E(t)$  in Eq. (S1) is ..."

5. Intro. "...the coefficient  $E(t)$  in Eq. (S1) is just  $-1/2i$ ". Should it be just  $-1/i$  because  $1/2$  is already present in (S1)?

6. Intro. "...infinite possible options for complex-valued  $E(t)$  that satisfies Eq. (S1)" Are there any that do not satisfy Eq. (S1)?

7. Using the arXiv reference [2] from 2013 is not ideal because it has not been peer-reviewed for more than 11 years, unless the author is really convinced that it can be used.

8. "In addition, this tells us that the Fourier transform of the real-valued field  $E(t)$  is different ..." Here a symbol for  $E$  is used that was never defined earlier, making it inconsistent.

9. The author should comment that some ultrafast spectroscopy experiments measure complex-valued electric fields such as in coherent Raman spectroscopy (10.1063/1.5090585) and two-dimensional electronic spectroscopy (10.1063/5.0186915). In these manuscripts, it was demonstrated that positive and negative spectral components can be generally different and complementary. Of course, these works are not the first demonstrating this, and the author is welcome to look for older works.

10. Spectral Fourier transform. "...Various conventions out there in the world ..." I feel it should be rephrased.

10.1. Section "2.1. Definition" should probably read as "A. Definition" for consistency with earlier notations. Same comment for other sections.

11. A. Definition. "...we sloppily treat the result ..." could also be rephrased/made precise.

12. A. Definition. "... the inverse Fourier transform in mathematics becomes the Fourier transform in physics..." - this does not make much sense. A Fourier transform is a Fourier transform, and it is the same in both math and physics. What the author probably wanted to say is that FT "in physics" is rather FT applicable to the cases with discrete sampling? or the one that operates on spaces with certain units (such as time and frequency)?

13. A. Definition. "... here I introduce "cross correlation" that is less-frequently used. ...". It would be good to briefly delineate why cross-correlations are interesting in the ultrafast optics field; otherwise, it sounds like "it is not important, but I will throw it at you anyways".

14. 2. Spectral Fourier transform. Certain statements are made, which could be supported by visual demonstrations. For example, one of them is "Since the analytic signal, or its envelope, is generally complex-valued, the spectral signal transformed with mathematical fft is not a complex conjugate of the spectral signal transformed with mathematical ifft."

15. Subsection B. "... mathematical (MATLAB's) ifft for Fourier transform into the spectral domain and use mathematical (MATLAB's) fft for ...". Here, "mathematical" is probably meant to indicate that the spaces of direct and inverse FT are unitless, in contrast to FT "in physics" where these are often in

units of time and frequency. Fundamentally, it is still the same mathematical FT, where certain meanings are assigned to the corresponding abstract quantities. It is probably worth clarifying better what "mathematical" means in the manuscript.

## **Declarations**

**Potential competing interests:** No potential competing interests to declare.