

Consistent Interpretation of Quantum and Classical Mechanics

Andrew Das Arulsamy
Condensed Matter Group,
Thiruvalluvar Institute of Interdisciplinary Science,
No. 24, level-4, Block C, Lorong Bahagia, Pandamaran,
42000 Port Klang, Selangor DE, Malaysia.

e-mail: sadwerdna@gmail.com

Tel.: 6010-4653041

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Abstract

Interpreting quantum mechanics is a hard problem basically because it means explaining why and how the mathematics exploited to formulate wave-particle duality are related to observations or reality in classical physics (due to Newtonian mechanics, special relativity, gravity and entropy). Here, we first prove the limitation of the Heisenberg uncertainty principle by invoking the wave-particle duality and apply it to revise and/or extend the Copenhagen postulates. In particular, even though the said uncertainty exists due to non-commuting operators, but these operators do not commute simply because one of the operators is well-defined in classical physics (particle-like), while the other is wave-like (well-defined in quantum mechanics), or the eigenvalue is not at all defined in classical physics. We also construct a new quantum mechanical postulate (Postulate 9) to deduce why the forbidden gap between discrete energy levels should exist in atoms, molecules and condensed matter phases. In fact, we tackle all the problems arising from the Copenhagen interpretation without violating established experiments and without proposing ideas that violate physical reality.

Keywords: Observables in quantum and classical physics; Wave-particle duality; Heisenberg uncertainty principle; Electron's orbital; Copenhagen interpretation.

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1 Introduction

Due to unsettled issues within the foundations of quantum mechanics, we have three main options for scientists to choose from when asked about the position of a 'quantum' particle before measurement [1]. A quantum particle here usually means an electron or a photon, which can be detected as free particles. The first is the **realist**

position—After the measurement, it is found that the particle was at some point A . But quantum mechanics is incomplete because it clearly lacks the ability to predict that the particle was at point A before the measurement. This realist position was advocated by Einstein himself, but he was unable to defend it in an unambiguous manner where one such attempt is the well-known Einstein-Podolsky-Rosen (EPR) paradox [2]. However, the realist position exposed here does not imply wavefunction realism, but implies observable or eigenvalue realism. The second position is known as the **orthodox** position—Before the measurement, the particle was not really anywhere such that quantum mechanics can only predict the probability distribution of the particle’s possible position. This probability distribution is unique in a sense that it implies the act of measurement forced the particle to take a stand and exposed its position at point A . Here, ‘taking a stand’ refers to the notion of wavefunction collapse (whatever the mechanism for this collapse is). This is the core Copenhagen-interpretation that was advocated, and enforced to be the truth by von Neumann and Dirac [3, 4]. Bohr [5] on the other hand, advocated that the quantum property did not exist and the measurement produces it somehow, but not due to wavefunction collapse. This wavefunction collapse is theoretically ambiguous because it is manually activated [1], and this collapse remains experimentally not observable. Consequently, we do not rely nor invoke such a concept here where we relied on the so-called wavefunction transformation theory proposed in Ref. [6].

Anyway, the third view is called the **agnostic** position. This is also a valid position because where is the point in taking a side when quantum mechanics is unable to predict the position of that particle. Raman was an advocate of this position. This is the current situation with the foundations of quantum mechanics, and in the absence of additional technical arguments, we are not entirely sure which position is viable, let alone the truth. Even though Ogilvie [7] has argued that quantum mechanics is largely irrelevant in the general practice of chemistry, but we will expose the existence of two new postulates (New Postulates 9 and 13) that are directly relevant to the time-irreversible processes and the foundations of photochemistry and biochemistry (photosynthesis, vision and formation of vitamin D). In particular, we provide the physical meaning of discrete energy levels and/or electron orbitals in atoms that is valid for molecules and condensed matter (including solids) based on quantum mechanics that is technically well-defined and experimentally observable.

The primary aim of this work is to expose the new rules contained in this consistent interpretation systematically, including the proper and correct physics and mathematics needed to take a firm stand on only one of the positions listed above. This will lead us to the application of these postulates in both physics and chemistry. We begin by revisiting the primary notion that is related to the wave-particle duality, which is the core quantum mechanical idea embedded in the Copenhagen interpretation that is responsible for the Heisenberg uncertainty principle. However, we shall not revisit the well-known Copenhagen postulates one-by-one, which can be obtained from Refs. [1, 8, 9], instead, we shall re-evaluate and completely reconstruct the Copenhagen postulates with additional physics. Here, Copenhagen postulates 1, 4, 5, 6 and 8 are replaced by the revised postulates, while the remaining Copenhagen postulates, 2, 3 and 7 stay unchanged. There are five additional (and new) postulates from the classical physics (four) and quantum mechanics (one), which give a total of 13 postulates to interpret the foundations of physics and chemistry

in a straightforward manner.

Before embarking on constructing the postulates, we shall first start elaborating the Heisenberg uncertainty principle. Physically, the Heisenberg uncertainty principle is a consequence of wave-particle duality. Here, we should note that the Heisenberg uncertainty principle only applies for $r \leq \Delta\mathbf{r}$, or when the observation scale, r is within the probability distribution of a quantum particle, $\Delta\mathbf{r}$. Therefore, if $r > \Delta\mathbf{r}$, then the Heisenberg uncertainty principle is inapplicable because we can identify the position of the particle or the position of the wave packet or the position of the particle's or wave packet's distribution function is well-defined. Here, we elaborate and apply this Heisenberg uncertainty principle and describe how it is actually related to one's ability to know both the momentum and the position of a quantum particle. However, unlike the momentum–position Heisenberg uncertainty principle discussed above and elsewhere, we cannot determine or predict all the spin components of a quantum particle for all or any scales or range of observation, either for $r \leq \Delta\mathbf{r}$ or for $r > \Delta\mathbf{r}$.

For example, r is the range of observation or the region of observation that is defined within $|_l$ and $|_r$. In particular, $|_l \odot |_r$ implies \odot is an atomic hydrogen with its outer circle (with radius \mathbf{r}) representing an electron wave packet surrounding the proton, \cdot in the center. In this case, $\Delta\mathbf{r} = \pi\mathbf{r}^2$, while r denotes the region between $|_{\text{left}}$ and $|_{\text{right}}$. For a photon, $\Delta\mathbf{r}$ denotes the size of a single photon. Here, the size of a single photon should be defined by its wave packet. Therefore, for $r > \Delta\mathbf{r}$, we can know the position of an electron, which is defined by the Bohr radius, or the position of a photon, but these positions are not known at a given Cartesian coordinate (x,y,z) because such a point-like position does not exist for any electron nor photon. The above new interpretation of Heisenberg uncertainty principle, which limits its validity, belongs to the revised interpretation that shall be properly exposed after the summary. The above limitation of the Heisenberg uncertainty principle is easily understood by noting the fact that $r \leq \Delta\mathbf{r}$ implies the quantum state cannot be defined, and only for $r > \Delta\mathbf{r}$ the quantum state is properly defined by a wavefunction or distribution function. If the quantum state or the wavefunction cannot be defined, then the probability distribution is not defined as well.

The quantum mechanical observables (calculated from the wavefunctions) do exist in a real physical sense, but they are subject to these two physical conditions, C1 and C2.

(C1) The Heisenberg uncertainty principle.

(C2) Whether the said quantum mechanical observables are also observable in classical physics. Observable in classical physics means a quantum mechanical observable is well-defined at any given time, t_n .

These two conditions exist entirely due to wave-particle duality of quantum particles. All quantum mechanical observables, regardless of whether they are also observable in classical physics or not, are guaranteed to satisfy the Heisenberg uncertainty principle if and only if $r \leq \Delta\mathbf{r}$. For those quantum mechanical observables **that are also observable in classical physics**, are not subject to the Heisenberg uncertainty principle for $r > \Delta\mathbf{r}$. In this range of observation, the quantum-classical observables are subject to classical uncertainties. On the other hand, for all quantum mechanical observables **that are not observable in classical physics**,

the Heisenberg uncertainty principle is always valid for all range of observation, $0 \leq r \leq \infty$. Therefore, C2 limits the applicability or validity of the Heisenberg uncertainty principle.

2 Revised and new postulates

The interpretation presented here is complete because it incorporates both classical and quantum mechanics consistently, which is another unique feature that is absent in the Copenhagen interpretation. The special relativity and the second law of thermodynamics within classical physics have been worked out in Refs. [10, 11]. The core problem resides in the interpretation of wave-particle duality, whether the particles are massive or massless, and whether a physical quantity is observable (measurable) or not, in classical physics.

Let us now provide further details on this wave-particle duality so as to properly expose the limitation of the Heisenberg uncertainty principle. We can calculate the probability of finding the ground state electron ($\mathbf{n} = 1, l = 0$) at $r/a = 1$ and again at $r/a = 4$ using the suitable spherical harmonics. For example [9],

$$a[R_{10}]^2 r^2 = 4\left(\frac{r}{a}\right)^2 \exp\left(-2\frac{r}{a}\right) = 0.54 \Big|_{\frac{r}{a}=1}, \quad (1)$$

$$= 0.02 \Big|_{\frac{r}{a}=4}. \quad (2)$$

Apparently, 0.54 and 0.02 are the probabilities of finding the electron at radii, $r/a = 1$ and $r/a = 4$, respectively, for the ground state electron ($\mathbf{n} = 1, l = 0$).

Now comes the interesting part, the interpretation. What do we make of these probabilities? Does this probability for each r/a means that we could also make the measurement for each r/a as in a Cartesian coordinate, (x, y, z) ? The answer is definitely NO because each r/a refers to a radius, not a point in (x, y, z) . As a matter of fact, even though we can calculate the probability of an electron for a given point-like position, r/a from the radial distribution function, but we cannot measure the position confined to a point, r/a . What we can measure is the whole radial distribution function curve (for a given radius r) in one go, not as individual r/a points as in Cartesian coordinate points, (x, y, z) .

The above measurement of the whole distribution function curve is supposed to mean that if we were to pin-point the detector at a particular Cartesian point, (x_1^a, y_1^a, z_1^a) on an atomic hydrogen Bohr radius, then the intensity of detection (for that ground state electron) is maximum. This does not mean the electron was indeed at that particular (x_1^a, y_1^a, z_1^a) point at the moment of detection. If we repeat this measurement for eternity and for whatever shortest time interval as allowed by technology, we would obtain the same intensity for (x_1^a, y_1^a, z_1^a) , (x_2^a, y_2^a, z_2^a) , and so on. Here, measuring the same intensity at a given cartesian point implies with certain classical uncertainty about the values of (x_1^a, y_1^a, z_1^a) , (x_2^a, y_2^a, z_2^a) , and so on. If we now move a little away from the Bohr radius to $k > a$ (at point, (x_1^k, y_1^k, z_1^k)) or $q < a$ (at point, (x_1^q, y_1^q, z_1^q)), then the intensities would become smaller as determined by the electron distribution function for an atomic hydrogen. Again, the intensities shall stay the same for all (x_1^k, y_1^k, z_1^k) , (x_2^k, y_2^k, z_2^k) , \dots , and for all (x_1^q, y_1^q, z_1^q) ,

$(x_2^q, y_2^q, z_2^q), \dots$, respectively. We would never encounter a zero or negligible intensity of detecting the electron for any $(x_{1,2,\dots}^a, y_{1,2,\dots}^a, z_{1,2,\dots}^a)$, $(x_{1,2,\dots}^k, y_{1,2,\dots}^k, z_{1,2,\dots}^k)$ and $(x_{1,2,\dots}^q, y_{1,2,\dots}^q, z_{1,2,\dots}^q)$ for as long as k , a and q are within the electron distribution function. Therefore, the electron is not a point-like particle that moves around a nucleus, or a proton in the case of atomic hydrogen.

This whole-curve measurement (for a given radius) ‘in one go’ that represents the electron’s position as a radial function would reproduce the probability distribution of that electron, depending on the electron’s orbital. Hence, our new interpretation here is that the electron’s charge, e and mass, m_{el} are distributed in the shape of the probability distribution curve such that the mass and charge of this ground state or excited-state electron is never ‘fractionalized’ nor can it be broken into smaller pieces or parts. This interpretation is invoked from the measurement elaborated above with respect to point-like particle that cannot be detected at a Cartesian (x, y, z) point. In this case, the mass and charge of an electron remain the same and their respective values, m_{el} and e are definite. However, the distribution of charge and mass of an electron does not imply they depend on the electron distribution function. In fact, the mass and charge of an electron stay the same for different electron distribution functions, for example, when the electron makes a transition from the ground to an excited state. Consequently, there should not be any relationship between mass or charge with the shape and size of the electron distribution function. In other words, the electron has a certain size, and its size and shape can change due to interaction with its surrounding such that the electron’s definite mass and charge are distributed within the electron’s size and shape (for whatever size and shape). This does not modify either the definite mass nor the definite charge of that electron. Therefore, the electron remains as an indivisible fundamental particle. The group velocity of an electron, that is bound to a given principal quantum number, \mathbf{n} cannot be defined if $r \leq \Delta\mathbf{r}$. The reason for this is that the mass and charge, as well as the group and phase velocities of an electron are always defined by the ‘whole probability distribution curve’, which is then defined by the wavefunction (see Revised Postulate 5).

However, the charge (not the mass) is the one that gives rise to wave-like property for an electron with Pancharatnam phase velocity that is adjusted upon interaction as we have learned earlier [12]. The group velocity of an electron bound to a given principal quantum number, \mathbf{n} cannot be defined if $r \leq \Delta\mathbf{r}$ due to its relation with mass distribution according to the probability distribution curve or function. The probability distribution function or the wave function is not properly defined for $r \leq \Delta\mathbf{r}$, and definitely undefined for $r < \Delta\mathbf{r}$. The phrase, ‘cannot be defined’ here and elsewhere does not mean ‘do not exist’ or ‘does not exist’. Here, $r \leq \Delta\mathbf{r}$ refers to the range of observation (r) with respect to the size of the electron or electron’s distribution function ($\Delta\mathbf{r}$). Thus, the electron cannot be observed (as a point particle) if $r \leq \Delta\mathbf{r}$ because the electron is never a point-like particle, but is defined by the distribution function.

Note this, we do not know (and we can only guess) the physics that is in play within or inside an electron, and this particular ‘hidden’ or ‘unknown’ physics also stays unknown for all other elementary quantum particles like quarks and photons. I have been careful with these two words, ‘hidden’ (unknown and cannot be observed even indirectly) and ‘unknown’ (exists but cannot be observed directly) because we

have to admit that we do not even know what we do not know about such physics. Here, hidden physics does not imply the existence of hidden variables. Hidden physics means that we do not know the physics of charge within an electron such that we do not invoke any hidden variables (that cannot be measured or determined even indirectly) to define the charge. In view of this revised interpretation, we have a situation that the Born's statistical interpretation cannot be entirely correct and needs to be replaced with the one that is compatible with the revised interpretation.

Before revising the Born's statistical interpretation, let us understand what it says first. The Born's original interpretation states that the position of an electron can only be measured for each r/a with a probability, and therefore, we cannot predict the electron's position, r/a with certainty. This uncertainty is due to Heisenberg's uncertainty principle. There are two physical problems with Born's interpretation. The first problem is related to the assumption that the electron can be confined within a point-like position, r/a , and the second problem is that even though the position of the electron cannot be determined with certainty due to uncertainty principle, but this uncertainty is assumed to be true for all range of observation, namely, for $0 < r < \infty$. Both of these assumptions are problematic that are derived from Born's own interpretation, which are false. The standard quantum formalism accepts these two false assumptions by activating the notion of particle for all r .

In this revised interpretation, the notion of point-like particle makes sense only when $r > \Delta\mathbf{r}$ due to finite range of distribution function for an electron, while for $r < \Delta\mathbf{r}$ the electron as a particle or as a wave cannot be defined because wavefunction or the electron distribution function is undefined. On the other hand, the defined distribution function does not and cannot locate the electron as a particle as in a (x, y, z) coordinate for $r < \Delta\mathbf{r}$. Therefore, whether an electron or a photon is a wave or particle does not make sense for all r . What we can postulate affirmatively is that an electron has particle-like property for $r > \Delta\mathbf{r}$ and a wave-like property for $r < \Delta\mathbf{r}$, while for a photon, it has both particle-like and wave-like properties for $r > \Delta\mathbf{r}$. The photons are undefined for $r < \Delta\mathbf{r}$ and relevant postulates cannot be constructed.

This revised interpretation also states that the mass and charge of an electron is distributed following the probability distribution function such that the point-like position of an electron, r/a cannot be defined properly for all $r \leq \Delta\mathbf{r}$ and within this range, Heisenberg's uncertainty principle is valid. Here, $\Delta\mathbf{r}$ is defined as the range for $a[R_{\mathbf{n}}]^2 r^2 \neq 0$, while $a[R_{\mathbf{n}}]^2 r^2 = 0$ is for $r > \Delta\mathbf{r}$. Moreover, the revised interpretation implies that the said uncertainty principle is inapplicable for $r > \Delta\mathbf{r}$. For an electron in the free space or vacuum, $\Delta\mathbf{r}$ refers to the size of the electron's distribution function. The electron's path (defined by $\Delta\mathbf{r}$) that was detected in a cathode-ray tube (CRT) or in the bubble-chamber experiment [13, 14], respectively, are possible simply because $r > \Delta\mathbf{r}$ where r here refers to the size of the CRT tube and its screen or the size of the bubble chamber that detects the electron's path. In other words, the range of measurement (or observation) is much larger than that of the electron's size or the space occupied by the 'shape-shifting' electron. This shape-shifting notion refers to different orbitals for an electron in an atomic hydrogen, which is true for all electrons, regardless of whether the electron is bounded (in atoms) or free (in vacuum).

Therefore, we should be able to identify the fact that the electron is correctly represented by a probability-distribution-shifting (or shape-shifting) function that can be determined from the wavefunction that could change due to wavefunction transformation. For example, the wavefunction transformation is needed when the electron in an atomic hydrogen is excited from the ground state ($\psi_{\mathbf{n}=1,l=0}$) to an excited state ($\psi_{\mathbf{n}=2,l=1}$). We should be able to deduce that the revised interpretation also automatically rules out the existence of Feynman's nonclassical paths. We have given unequivocal exposition with quantitative analysis on this matter in Ref. [15].

Anyway, if $\Delta \mathbf{r}$ is the size of the crystal, then $r \leq \Delta \mathbf{r}$ means our range of observation is smaller than the size of the crystal and therefore, the electron's position is subject to Heisenberg uncertainty principle. Of course the situation is infinitely complex to determine the position of any single electron in a crystal because there are at least of the order of 10^{23} interacting valence electrons in a crystal. Whereas, for larger observation range, $r > \Delta \mathbf{r}$, we are certain that the electrons can always be found within the crystal, while the probability to find any electron beyond the surface of the crystal is zero.

Let us now begin constructing the revised postulates as listed below. We first recall the first Copenhagen Postulate and reconstruct it as given below.

Revised Postulate 1: The state function, $\Psi(\mathbf{r}, t)$, which is a function of time and space coordinates, is also known as the wavefunction. However, $\Psi(\mathbf{r}, t)$ for massive and charged quantum particles does contain all the observable or measurable information about a system except for the particle's mass, charge and the interactions in that system. We also demand that $\Psi(\mathbf{r}, t)$ to be single-valued (uniquely determines the eigenvalue), continuous and quadratically integrable. For continuum states, quadratic integrability should be excluded. The Hamilton operators contain the remaining information about charge, mass and interactions. For massless and chargeless photons however, we cannot invoke the state function to represent photons where photons themselves are treated as waving wave packets [15], which are subject to wave equation.

The complete information about a system can be obtained from the Schrödinger equation,

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = H\Psi(\mathbf{r}, t), \quad (3)$$

where we also need to know the state function, $\Psi(\mathbf{r}, t)$ and H is the Hamiltonian that is composed of the relevant kinetic (K), potential (V) and other interaction (H_{int}) energy operators. Here, $\Psi(\mathbf{r}, t)$ alone does not contain all or the complete information about a system.

On the other hand, the electric and magnetic field components of an electromagnetic wave do contain all the information about photons and these components are subject to the wave equation,

$$\frac{\partial^2(\mathbf{E}, \mathbf{B})}{\partial t^2} = c^2 \frac{\partial^2(\mathbf{E}, \mathbf{B})}{\partial x^2}, \quad (4)$$

where c is the speed of light. Therefore, the rest of the postulates do not apply for photons primarily because $V = 0 = H_{\text{int}}$ and its energy is defined by,

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda}, \quad (5)$$

where h is Planck constant, ν denotes the frequency, λ is the wavelength of an electromagnetic wave and E_{photon} is 100% kinetic energy.

Here, the state function cannot be a physical wave for two reasons—(i) $\Psi(\mathbf{r}, t)$ is the linear combination of many eigenfunctions, ψ_i and (ii) the particle-property cannot be incorporated into $\Psi(\mathbf{r}, t)$ nor into the eigenfunctions, ψ_i . We need the constants (m_{el} and $-e$) attached to the relevant physical operators (kinetic energy and potential energy operators, respectively) to do that, to give particle-property such as mass and charge to an electron. Note this, acoustic or mechanical waves with waving mass do not satisfy or cannot represent the wave function of any quantum particles because the wave function should be independent of mass. If mass can be incorporated into the wave function, then Heisenberg uncertainty principle cannot exist. The wave function or the state function in the Broglie-Bohm theory [16] also does not incorporate the particle property (the mass). As for the Feynman non-classical paths, regardless of whether there is a protocol-to-observe [17] or not, it has never been observed or confirmed. The existence of this protocol does not imply the observation has been made and/or confirmed. In fact, the physics of the double-slit experiment exposed in Ref. [15] with new experimental observation already demonstrated that the Feynman paths are physically false. Whereas, the electric field (\mathbf{E}) nor the magnetic field (\mathbf{B}) component of an electromagnetic wave cannot be a physical wave for the second reason, (ii). In particular, the particle-property of a photon cannot be incorporated into the said wave-property of the magnetic- and electric-field components.

The second and third Copenhagen Postulates remain the same in the revised interpretation. Therefore, we just reproduce them below.

Copenhagen Postulate 2: Each linear Hermitian operator corresponds to a physical observable. To find this operator, we can exploit the correspondence rules, $i\hbar \frac{\partial}{\partial t} \longrightarrow E$, $\frac{\hbar}{i} \frac{\partial}{\partial x} \longrightarrow p$ and $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \longrightarrow \frac{p^2}{2m} = K$.

In Copenhagen Postulate 2, we did not specify whether the total energy, E , the momentum, p nor the kinetic energy K refers to a particle or a wave or both. The proper specification is given in the revised Postulate 6 and in the discussion after the revised Postulate 8.

Copenhagen Postulate 3: The physically observable property, A produces measurable eigenvalues, a_i subject to the following eigenvalue equation, $\hat{A}\psi_i = a_i\psi_i$ where \hat{A} is the Hermitian operator corresponding to the observable property, A and eigenvalues, a_i . The eigenfunctions, ψ_i are of course required to be well-behaved as stated in the first revised Postulate.

The fourth Copenhagen Postulate [9] is *ad hoc* and is limited in its validity. Therefore, we reconstruct it so as to make it suitable for the revised interpretation.

Revised **Postulate 4**: The Hermitian operator, \hat{A} of an eigenvalue equation, $\hat{A}\psi_i = a_i\psi_i$ is also a linear operator, and is responsible for a physically observable property, A . Its eigenvalues, a_i are all real and their respective eigenfunctions, ψ_i transform to a new set of eigenfunctions upon measurement of the physical property A . This transformation is subject to the new Postulate 9, and the paragraphs that come after the new Postulate 9.

Here, we do not require the eigenfunctions to form a complete set because during each measurement, there is this thing called the wavefunction transformation such that the complete set for the eigenfunctions before the measurement is no longer valid during the measurement. Therefore, it is more meaningful to evaluate the wavefunction transformation during a measurement than to assume completeness before and after a measurement. Moreover, in quantum mechanical calculations, the guessed wavefunctions or eigenfunctions are constructed and adjusted to obtain eigenvalues that are comparable to experimental values, without any regard as to whether the constructed and adjusted eigenfunctions form a complete set or not.

Revised **Postulate 5**: If $\Psi(\mathbf{r}, t)$ is orthonormalized and well-behaved, and if it is the state function of a system, then the physical observable, A at time t is,

$$\langle A \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{A} \Psi d^3\mathbf{r}, \quad (6)$$

where $\Psi(\mathbf{r}, t) = \Psi$. Here the physical property, A of an electron can only be measured for a range of r with a probability such that we cannot predict A with certainty if the range of observation, $(-\infty, +\infty)$ is replaced with $[-a, +a]$ and $\infty \gg |2a|$. This uncertainty is due to Heisenberg's uncertainty principle.

The Born's modified statistical interpretation reads,

$$\int_{-b}^{+b} \Psi^* \Psi d^3\mathbf{r} \approx 1 = \int_{-\infty}^{+\infty} \Psi^* \Psi d^3\mathbf{r}, \quad (7)$$

where $\infty \gg |2b| > \Delta\mathbf{r} > |2a|$ such that the following integral,

$$\int_{-a}^{+a} \Psi^* \Psi d^3\mathbf{r}, \quad (8)$$

cannot be defined if $2a < \Delta\mathbf{r}$ and $\Delta\mathbf{r}$ refers to the size of the electron's distribution function or the size of the wave packet determined by the wavefunction. Therefore, the physical property A (including position, momentum or any other observable physical properties) of an electron cannot be defined if the range of observation $2a < \Delta\mathbf{r}$.

Here, the average value, $\langle A \rangle$ of an arbitrary physical property, A of an electron cannot be defined for $r = 2a < \Delta\mathbf{r}$ because the electron does not have a point-like position, (x, y, z) . Therefore, for $r = 2a < \Delta\mathbf{r}$, the position of the electron

cannot be determined with certainty due to Heisenberg uncertainty principle, while for $r = 2b \geq \Delta\mathbf{r}$, the position is not subject to Heisenberg uncertainty principle, especially when $r = 2b > \Delta\mathbf{r}$, and definitely when $r = 2b \gg \Delta\mathbf{r}$. However, the intrinsic spin (or spin angular momentum) of a quantum particle is not subject to the revised Postulate 5 because the spin is classically undefined for all r . In fact, even within the standard quantum mechanics, we do not exploit Eq. (6) to calculate the probability of finding $\pm\frac{1}{2}$.

The original Copenhagen Postulate 6 is redundant for the revised interpretation because it has been incorporated in the revised Postulate 1. Therefore, our next postulate is related to the applicability of Heisenberg uncertainty principle for certain physical observables that are also captured or observable in classical physics.

Revised Postulate 6: The state function, $\Psi(\mathbf{r}, t)$ that is subject to time-dependent Schrödinger equation captures the time evolution of a quantum mechanical system that is free from any external disturbances. This equation is given in Revised Postulate 1. The state function is separable into products, which means,

$$\Psi = \Psi(\mathbf{r}, t) = R(\mathbf{r})Y_l^{\mathbf{m}_l}(\theta, \phi)\chi(\mathbf{m}_s)\exp\left(-\frac{iEt}{\hbar}\right), \quad (9)$$

where $R(\mathbf{r})$ is the radial wavefunction, $Y_l^{\mathbf{m}_l}$ is the orbital angular momentum eigenfunction or angular wavefunction or sometimes known as the spherical harmonics, $\chi(\mathbf{m}_s) = |s\mathbf{m}_s\rangle$ is the spin eigenstate (not an eigenfunction), and finally, $\exp(-iEt/\hbar)$ is the time-dependent phase factor that takes the effect of time evolution of the state function subject to the Schrödinger equation. Note this, $\mathbf{m}_s = \pm\frac{1}{2}$. The above separable product solution implies that the quantum mechanical physical observables are physically separable from the observables in classical physics. Therefore, the Heisenberg uncertainty principle has limited application such that the said uncertainty is not and cannot always be true for all quantum mechanical observables and for all range of observation.

In Ref. [18], you can find the details on how the usual Hamilton operators operate on Eq. (9), as well as to learn why the spherical harmonics and the spin eigenstates can be taken as constants. Let us now elaborate on this Postulate 6 further. The momentum eigenvalue in quantum mechanics is physically different from the momentum in classical mechanics to the extent that the quantum mechanical momentum refers to the wave-property of an electron, while in classical physics, the momentum refers to the particle-property of the same electron. In addition, there are quantum mechanical observables such as orbital angular momentum states, $\mathbf{m}_l = 0, \pm l$ and spin angular momentum states, $\mathbf{m}_s = 0, \pm s$, which are not observable in classical physics entirely because orbital and spin angular momenta states in quantum mechanics refer to the wave-property of an electron and there are no particle-property for these angular momenta states in classical physics.

The group velocity of an electron, that is bound to a given principal quantum number, \mathbf{n} cannot be defined if $r \leq \Delta\mathbf{r}$ due to its relation with mass distribution according to the probability distribution curve defined by the wavefunction (see the Revised Postulate 5).

The original Heisenberg uncertainty principle is activated for all quantum mechanical physical observables if and only if the pairs of the physical operators are non-commuting. However, the said uncertainty has limited application for certain quantum mechanical physical observables even if the pairs of the relevant operators are non-commuting. In particular, Heisenberg uncertainty principle has limited applicability for quantum mechanical observables that are also observable in classical physics. Since, $\mathbf{m}_l = 0, \pm l$ and $\mathbf{m}_s = 0, \pm s$ are not observable in classical physics, then the Heisenberg uncertainty principle is always valid for all range of observation, r for these observables. Whereas, for the momentum, p and position, x , the same uncertainty principle has limited application because both x and p are also observable in classical physics. What this means is that for $r > \Delta \mathbf{r}$, the Heisenberg uncertainty principle does not apply because the electron can be considered to be a particle, while for $r < \Delta \mathbf{r}$, one has to invoke the said principle because x and p refer to the wave-property of the electron such that x is always undefined, while p is the momentum of the wave-property, which is well-defined.

For example, the position operator, \hat{r} , even though it does not commute with the momentum operator, \hat{p} , but both the position and momentum eigenvalues can be observed simultaneously if the range of observation, $r > \Delta \mathbf{r}$. Here, r and p refer to particle-like position and momentum of a massive electron in classical physics. However, the following eigenvalues, $\mathbf{m}_l = 0, \pm l$ and $\mathbf{m}_s = 0, \pm s$ are not observable in classical physics, and therefore, $\mathbf{m}_l = 0, \pm l$ and $\mathbf{m}_s = 0, \pm s$ are subject to Heisenberg uncertainty principle for all range of observation, $0 \leq r \leq \infty$.

Consequently, the Heisenberg uncertainty principle is always true only for physical observables that are not observable in classical physics, and if the pairs of the operators for those observables are non-commuting. Whereas, for quantum mechanical physical observables that can also be observed in classical physics are not subject to Heisenberg uncertainty principle if $r > \Delta \mathbf{r}$, even if the pairs of quantum mechanical operators are non-commuting.

Let us now apply the revised Postulate 6. This postulate means that for $r \leq \Delta \mathbf{r}$, the quantum mechanical momentum operator refers to the momentum of a wave-like electron, while for $r > \Delta \mathbf{r}$ the observable momentum refers to classical momentum of a particle-like electron with mass, m_{el} and velocity, v . Subsequently, we can apply the revised Postulate 6 to understand why the wave-like momentum of a photon remains wave-like for all range of observation ($0 \leq r \leq \infty$), which is because photons are not subject to Schrödinger equation, instead, they obey the massless wave equation. For example, classical momentum cannot exist for massless photons and is denied by the wave equation. Finally, the revised Postulate 6 provides the physical justification as to why the Heisenberg uncertainty principle is not always true for physical properties that are observable in both quantum mechanics and classical physics as exposed above. It is apparent that the revised Postulate 6 makes use of the fact that EPR paradox is false [19].

Let us continue with the remaining two postulates related to spin. The seventh postulate stated in the Copenhagen interpretation section stays the same in the revised interpretation.

Copenhagen Postulate 7: The wavefunction of a system of electrons or particles with half-integral spins, must be antisymmetric with respect to

interchange of particles with half-integral spins, including electrons.

As for the final postulate on the intrinsic spin, we need to reconstruct it, and is given below.

Revised Postulate 8: The intrinsic spin ($s = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$) or the spin angular momentum is unique, specific and is immutable for each species or type of quantum particle. The said spin has no corresponding physical observable in classical mechanics and therefore, the Heisenberg uncertainty principle is always valid such that we cannot observe more than one (S_z or S_x or S_y) component of a quantum particle's spin simultaneously for all range of observation, $0 \leq r \leq \infty$.

The revised Postulate 8 establishes the fact that classically, the spin components (similar to orbital angular momentum components, L_x , L_y and L_z) cannot be defined because they are not observable physical properties in classical physics or the said observables are subject to Heisenberg uncertainty principle. In particular, $S_{x,y,z}$ and $L_{x,y,z}$ are associated to the wave-property of the quantum particle, which is not related to any particle-property in classical physics. Hence, $S_{x,y,z}$ and $L_{x,y,z}$ are subject to Heisenberg uncertainty principle for all range of observation, $0 \leq r \leq \infty$. Here, we have to be aware that $s = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \neq l$ where $l = 0, 1, 2, \dots$ as proved in Ref. [20].

Having said that, we can now state that the revised Postulates (1, 4, 5, 6 and 8) listed above do not require Bohr's philosophy that reads a particle takes a stand only upon measurement [5]. The above revised (1, 4, 5, 6 and 8) and Copenhagen (2, 3 and 7) postulates strictly imply that quantum mechanics is an ontic theory that deals with 'real' existing objects, but subject to the Heisenberg uncertainty principle or wave-particle duality. Here, wave-particle duality implies Heisenberg uncertainty principle. However, the wavefunction does not represent any sort of real physical wave. Consequently, we can rule out the idea that quantum mechanics is an epistemic theory (based on Bohr's philosophy), which defines that all quantum mechanical observables cannot exist (in whatever physical forms), until one measures them. This is simply because the wavefunction is not a real physical wave anyway.

Consequently, we can now deduce unequivocally that the orthodox position has got to be false. We are left with the realist and agnostic positions as the only possible and viable candidates. As a matter of fact, quantum mechanics is in favor of both realist and agnostic positions because even though the wavefunction is not a real physical wave (due to wave-particle duality, and from the definition of $\Psi(\mathbf{r}, t)$ given in the revised Postulate 1) but the wavefunction, together with the Hamilton operators, correctly represent the observables such that the observables are real and they do exist before the measurement.

2.1 New quantum-mechanical postulate

Let us now elaborate on the electron transition exposed above. The time-dependent state function (defined in the revised Postulate 1) evolution does not include the transition between energy levels because such transitions involve the existence of

‘stateless’ wavefunction, which is not captured by any other Copenhagen nor by the revised postulates of quantum mechanics exposed thus far. In particular, we have no other choice but to accept the following facts as truths, which are straightforward to be understood physically and technically.

F1: An electron does not disappear from one initial state and then reappear on another state by not occupying the forbidden energy gap.

F2: The time-evolution of $\Psi(\mathbf{r}, t)$ does not capture the transition between energy levels because the moment we define $\Psi(\mathbf{r}, t)$ to be so, then we do not need to guess the eigenfunctions for each energy level, where the time-evolution of $\Psi(\mathbf{r}, t)$ can be exploited to derive all the eigenfunctions for all the energy levels, including the ‘stateless’ eigenfunctions from the Schrödinger equation. We know for certain that this is never the case.

We know for certain that both **F1** and **F2** are absolutely true, and these facts lead us to the ninth Postulate, which is new.

New Postulate 9: When an electron makes a transition from one energy level (E_1) to another (E_2), then $\Psi(\mathbf{r}, t)$ evolves in such a way that this particular evolution is not captured by the Schrödinger equation. During this time-dependent evolution, the energy of the electron, E' is defined to exist between E_1 and E_2 such that $E' \neq E_1$, $E' \neq E_2$ and in addition, \mathbf{n}' , l' and \mathbf{m}'_l no longer represent the initial state wavefunction, $\Psi(\mathbf{r}, t)_{\mathbf{n}, l, \mathbf{m}_l}$. Therefore, the wavefunction during the transition is known as the stateless wavefunction, $\Psi(\mathbf{r}, t)_{\mathbf{n}', l', \mathbf{m}'_l}$ such that \mathbf{n}' , l' and \mathbf{m}'_l are undefined because $\Psi(\mathbf{r}, t)_{\mathbf{n}', l', \mathbf{m}'_l}$ is not subject to the Schrödinger equation where the electron is either in the process of emitting one (spontaneous) or two (stimulated) photons (if $E_1 > E_2$) or absorbing a photon if $E_1 < E_2$. Obviously, E' exists in the forbidden gap and this gap exists because the Schrödinger equation is blind-by-definition to the existence of E' , but not to the existence of E_1 and E_2 . Hence, we can always recover the quantum predictions for E_1 and E_2 as permitted by the Schrödinger equation.

In the revised Postulate 8 stated earlier, the technical reason why $\Psi(\mathbf{r}, t)$ cannot represent a stateless wavefunction is due to the fact that $\Psi(\mathbf{r}, t)$ is not a physical wave in the first place. Secondly, the changes to \mathbf{n}' , l' and \mathbf{m}'_l refer exclusively to wave-like properties of the electron that is making the transition by emitting or absorbing photons with appropriate energy. The changes to the position and momentum of an electron during the transition are observable if $r > \Delta\mathbf{r}$, and these changes to the position and momentum implies that one can indeed stop an electron from making a transition between energy levels as recently observed by Z K Minev *et al.* [21]. Recall here that $\Delta\mathbf{r}$ is the size of the electron or the radial distribution of the electronic wavefunction subject to the revised Postulate 5. The existence of this stateless wavefunction, $\Psi(\mathbf{r}, t)_{\mathbf{n}', l', \mathbf{m}'_l}$ also strictly implies the electron can never be a point-like particle because the excitation of an electron implies transformation of the wavefunction. Therefore, we can detect the disturbance to this wavefunction by switching on or off the disturbance during an electronic transition between two energy levels, which has been reported by Z K Minev *et al.* [21] recently. If an electron is a point-like particle, then stopping an electron from making the transition

is like flipping a coin—now you can, now you cannot.

2.2 New postulates from classical physics

We have to acknowledge that Bohr was correct in respecting classical physics such that quantum mechanics is not and cannot be independent of classical physics. For example, we have to find the link between quantum mechanics and classical physics, which we did when we discuss the quantum mechanical observables and under what circumstances that such observables are also observable in classical physics. For example, the classical position and momentum of an electron refers to the particle-property of an electron, while the momentum in quantum mechanics exclusively refers to the wave-property of an electron (see the correspondence rules in Copenhagen Postulate 2 and the discussion after that). Now, we shall introduce how special relativity comes into quantum physics.

We can construct the tenth postulate, which is new and is associated to time dilation effect.

New Postulate 10: Time dilation is always the effect that does not affect any physical processes, regardless of whether the processes are classical or quantum mechanical.

Here, the connection between time dilation and spacetime curvature can be captured by the fact that we cannot stretch the space without time dilation effect, nor permit time dilation without space stretching. In addition, time dilation effect implies non-universal ticking of time, in other words, consistent time dilation effect connects special relativity to gravity.

Postulate 10 also means that the relativistic effect solely implies correction to the non-relativistic value by adjusting its magnitude. Therefore, we can obtain the so-called ‘truthful’ value of a measurement by letting the observer and the observed to reside in the same frame of reference, and in this case, there is no need for relativistic correction. The value that does not require relativistic correction can be defined to be the truthful value. In contrast, if the observer and the observed reside in different frames of reference, then the measured value is nothing but the corrected value from the ‘truthful’ value due to relativistic effect. This leads us to the eleventh postulate that is new.

New Postulate 11: Truthful value either measured or observed means, a value without any relativistic correction.

A measured value with relativistic correction becomes the correct value from our frame of reference. Now to demonstrate that the physics stays the same even in the presence of time dilation effect is to show that the energy of the photons emitted by two hydrogen atoms (when the respective electrons make a jump from the triplet to singlet state) at different elevations (or gravitational potentials) or velocities is the same or identical. In particular, the famous 21-cm line due to hyperfine splitting in atomic hydrogen remains the same for the two atomic hydrogen, which should explain why the 21-cm radiation is one of the most ubiquitous forms of radiation in the universe. Of course, changes to the frequency of this emitted radiation can occur due to Doppler effect.

New Postulates 10 and 11 together give rise to the possibility that relativity and gravity do not introduce new physics. There is one more postulate that validates Galileo’s relativity postulate, which is given below.

New Postulate 12: If the relativity postulate is violated, then the speed of light cannot be a universal constant. This means that all physical laws cease to be invariant if the speed of light is not a constant.

We have proved the correctness of this new Postulate 12 in Ref. [10], which enforces the fact that Galileo’s relativity postulate demands the speed of light to be a constant.

Let us now derive the final postulate. It is worth noting that many researchers falsely and implicitly assumed the physical interaction to be reversible in the same way as time as recently reviewed in Ref. [22]. In particular, the following two false statements are implicitly assumed.

- (a) Time reversibility automatically implies reversible physical interaction.
 - (b) Each forward process is assumed to be reversible without any other changes.
- Therefore, we can construct the final new postulate (given below) to invalidate the above statements, (a) and (b), which are always assumed or activated *a priori* to prove the possibility of the second law of thermodynamics violation.

New Postulate 13: Time reversibility never implies reversible physical interaction and any forward physical process cannot be assumed to be reversible without any other changes.

The new Postulate 13 has been formally proved in Ref. [11] for any timescale and for any system size.

3 Conclusions

We have constructed a total of 13 postulates, 9 of which belong to quantum mechanics, while the last 4 are associated to classical physics. The primary conclusions based on these postulates are listed here. (1) The **realist** position is the only valid position in classical mechanics such that the **agnostic** position is meaningless, while the **orthodox** position is simply nonsensical. (2) In quantum mechanics, indeterminism has its origin in wave-particle duality and it has got nothing to do with indeterminism in classical physics, which is related to how accurate one measures the observables. (3) In quantum mechanics, we have to adopt the agnostic position for $r \leq \Delta \mathbf{r}$ because in this range of observation, both momentum and position of an electron are undefined. For $r > \Delta \mathbf{r}$ however, we can switch to realists position because one can measure both the particle-like momentum and position of an electron, as well as the wave-like momentum. (4) Indeterminism in quantum mechanics is not about how accurate one can measure the observables, but is determined by an intrinsic property known as the wave-particle duality, which then gives rise to the Heisenberg uncertainty principle. (5) The validity of this principle is shown to be limited—whether an eigenvalue is observable in classical physics, and whether the eigenvalue is confined (when $r > \Delta \mathbf{r}$) or spread beyond the range of our observation (when $r \leq \Delta \mathbf{r}$). Finally, (6) for both classical and quantum physics, the orthodox position turns out to be physically unacceptable.

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