

## Peer Review

# Review of: "The Complexity of Tullock Contests"

Alfonso De Miguel<sup>1</sup>

1. Zaragoza Logistics Center, Zaragoza, Spain

I would like to thank the authors and the Qeios platform for the opportunity to review the manuscript titled "The Complexity of Tullock Contests." This work examines the algorithmic complexity of computing pure Nash Equilibria (PNE) in Tullock Contests, offering significant insights into the interplay between elasticity parameters and equilibrium computability.

The study identifies three regimes of elasticity—small, medium, and large—each with distinct algorithmic properties. Notably:

1. When no contestant's elasticity parameter  $r_i$  lies within  $(1,2]$ , an efficient algorithm for computing PNE can be devised.
2. When many  $r_i$  values fall within  $(1,2]$ , determining the existence of a PNE becomes computationally intractable under the Exponential Time Hypothesis (ETH). In such cases, the authors design a Fully Polynomial-Time Approximation Scheme (FPTAS) to find an  $\epsilon$ -PNE when an exact PNE exists.

The manuscript is well-written, carefully structured, and clear in its exposition. I commend the authors for the quality of their work and their rigorous approach to a challenging problem. Below, I humbly offer some suggestions and comments for further refinement.

## ### Comments and Suggestions

### #### 1. \*\*Introduction\*\*

- While the discussion on Bitcoin and its relevance to contest theory is engaging, the manuscript could benefit from a broader perspective. Tullock Contests have applications in a wide range of domains, such as political economy, rent-seeking, innovation tournaments, and even war or conflicts (see the work of Sanjeev Goyal on this). Including these examples in the introduction would provide a more

comprehensive context and strengthen the relevance of the study.

- The term "network difficulty" is introduced but not explained. A brief definition or example would clarify its meaning for readers unfamiliar with the term.
- There is some redundancy between the first paragraph of the introduction and the motivation section. Merging or restructuring these parts could improve the flow and avoid repetition.

#### ### 2. \*\*Minor Typos and Errata\*\*

- "This paper develops algorithmic results for Tullock contests, an impactful and textbook-style contest model" — Correct Tullock to Tullock.
- "let  $\mathcal{S}$  denote the set of all (...)" — Capitalize let to Let for consistency with formal writing conventions.
- "The action shares of all active players falls into (...)" — Replace falls with fall for grammatical accuracy.
- "These properties are formally captured in Lemma 3, (...)" — I think it should be Lemma 4, not Lemma 3.
- In the Lemma 4 statement, index 2 is used for a bullet point that should be below the first one within the first property (indexed as 1). Subsequently, the second property, which now is labelled again as 1, should be read '2' instead.
- In Appendix D: Missing term in the fraction when defining  $a_i$  for  $i=N+1$ .

#### ### Final Thoughts

This manuscript presents an interesting contribution to the literature on algorithmic game theory, particularly in the context of Tullock Contests. The authors' identification of computational regimes and their development of both exact and approximate solution methodologies are notable achievements. The theoretical rigor and practical relevance of this work will make it valuable to researchers and practitioners alike.

I encourage the authors to address the above suggestions, which I believe would enhance the manuscript's accessibility and impact. Congratulations on an excellent piece of work.

## Declarations

**Potential competing interests:** No potential competing interests to declare.