The correlation of classic and experimental measurement results with quantum measurement theory

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Abstract: In classic measurement theory each physical measurement result has a precision which becomes infinitesimal as noise and distortion approach zero, i. e., exact repetitive measurement results (± a Planck) are theoretically possible. This classic measurement theory is not well correlated with experimental measurement results. When noise and distortion are minimized, repetitive experimental measurement results always display a Gaussian distribution. This paper first addresses experimental measurements by developing a formal measurement function and related definitions. The formal measurement function identifies that an exact repetitive classic measurement result is not possible in theory as well as in experiments. This new measurement function is then shown to explain the existing quantum measurement problems.

Keywords: *measurement theory, metrology, calibration, uncertainty, precision, quantization.*

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1. Introduction

1. In 1765 L. Euler stated: "Now, we cannot measure or determine any quantity, except by considering some other quantity of the same kind as known [i.e., a *reference*], and pointing out their mutual relation" [1]. Euler identified a *measurand*[1]–measurement–reference paradigm, which is a relative paradigm. A measurand (definition in Annex A) is represented by a local scale. In this paradigm a measurement result is relative to a reference as diagrammed in Fig. 1:

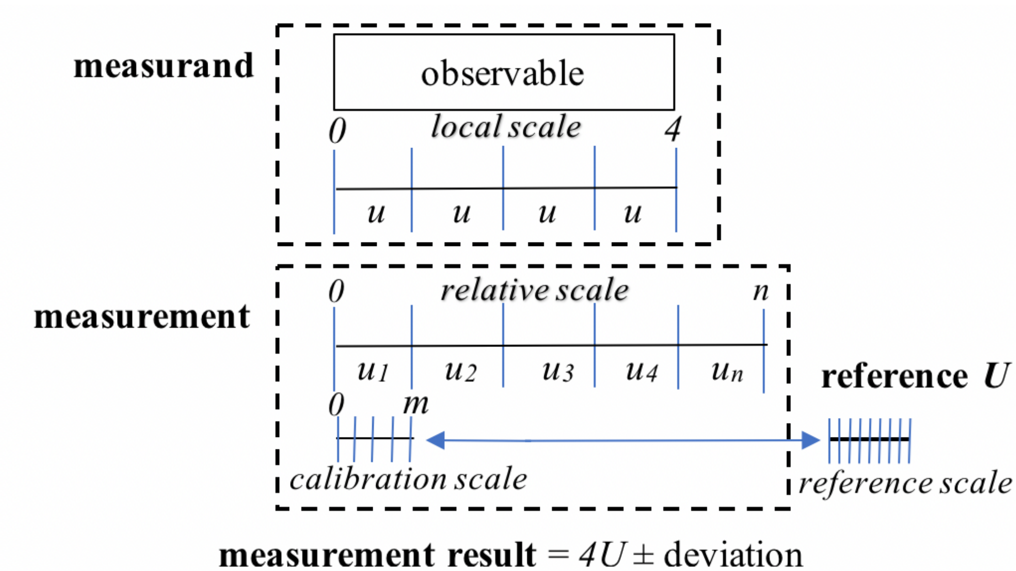

Figure 1. Relative measurement system in metrology

The paradigm applied in current measurement theory [2] in physics is based upon an observable–measurement–observation paradigm, which is a representational paradigm diagrammed in Fig. 2.

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[1] The first instance of a word, later defined (in Annex A or in the text), is italicized.

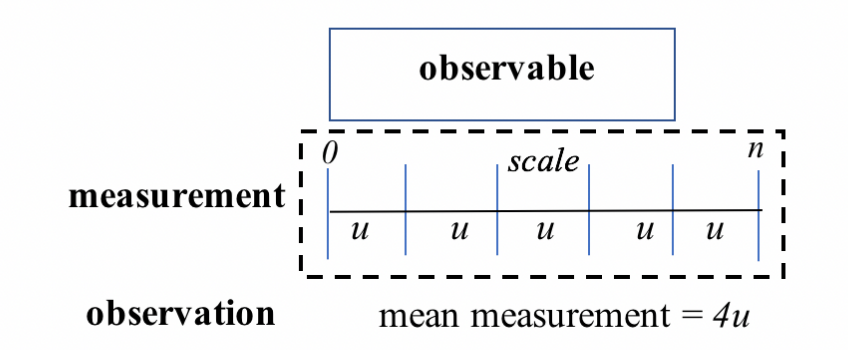


Figure 2. Representational measurement system in theory

In metrology (VIM [3]), all physical measurement processes follow Euler's paradigm, Fig. 1. However in physics theory, including quantum mechanics (QM), metrology is treated as empirical. In physics theory calibration to a reference is only for empirical reasons and does not appear in Fig. 2.

Relative Measurement Theory (RMT) [4] applies Euler's paradigm to all experimental and theoretical measurements. RMT recognized that all measurement systems, even in QM theory, are relative and must include calibration to a reference. The current paper formalizes the implementation of RMT and explains how RMT resolves the measurement problems [5] that appear in quantum mechanics.

2. Measurement result quantity

In physical measurements a *reference* is defined by a standard *quantity*, U. As example, a scale to measure mass (a physical measurement instrument) is calibrated to the standard kilogram, U). The scale is then is brought to a brick (observable), first the scale's *reference point* is set to zero, then the brick's mass quantity (measurand) is measured. That is, U, the kilogram reference, via the measurement instrument, defines one measurand of the brick's multiple local measurands (e.g., mass, length, width, height, volume, color, etc.).

A *quantity* is defined as the product of a numerical value (n) and a unit (u) (VIM 1.1), where the representational measurement assumption is that $u = U$ in theory. A standard unit, U may be treated as unity. However, as a unit of a measurement apparatus, u_n will deviate even in theory from U by ± a Planck at the minimum. That is, each u_n is not linear therefore a measurement function in theory must be, $\sum u_n$ not $n \cdot u$.

Developing Euler's relative measurement paradigm further, this paper proposes that a relative measurement system correlates an observable to U with four mutual relations. The first mutual relation is between an observable and a *local scale*, which establishes a measurand:

1. Establishes the measurability of an observable's measurand.

As all scales have a reference point, the second mutual relation is:

2. Common reference points when more than one scale is applied in a measurement process.

When an observable exhibits a local scale, the observable become a measurand. Then a count (local), but not a comparable measurement, can occur. As example, a wooden stick (observable) is determined to have a measurable length quantity by counting regular notches (local scale) on the wooden stick.

Calibration to U is accomplished by the next two mutual relations, which convert a measurand into a measurement result (see Fig. 3):

3. Quantization of calibration - numerical value of each u_n.

4. *Precision* of calibration - u_n relative to U.

When calibration to U is not included in a measurement theory (i.e., representational measurement theory), a comparable measurement cannot occur.

The measurement of an observable (e.g., determining the mass of a brick) requires calibration to a reference to:

- Define which measurand of an observable is being measured.
- Determine the numerical value of each interval of the relative scale.
- Determine each interval's precision relative to U.

Then the measurement result is comparable.

The correlation of each u_n on the local scale to the reference unit quantity, U, is the definition of *calibration* applied in this paper. This calibration defines the numerical value of the intervals of a *relative scale* and establishes a *relative quantity*.

A precise measurement result quantity (i.e., numerical value of u has a precision smaller than u) of a measurand, in theory or experiment, can only come from a measurand quantized into yet smaller states than one u. Therefore, the third mutual relation is required in any precise measurement theory as well as in any precise experimental measurement. The fourth mutual relation (precision) is required for comparable measurement results.

A local measurement process that produces a quantized local quantity requires at least the first mutual relation. A relative measurement process requires all four mutual relations. The measurement problems that occur when these four mutual relations are not recognized in the representational measurement paradigm are explained by first developing a new formal measurement function that recognizes these four mutual relations.

3. Measurement result Quantity

Calibration, which transforms a local scale to a relative scale, is considered solely empirical in representational measurement theory. When calibration is solely empirical, the precision (see definition in Annex A) of a quantity is also empirical. Then the precision of u of a measurand becomes perfect in a measurement system where noise and distortion are zero. The theoretical possibility of a perfect measurement result is a fallacy of the representational measurement paradigm.

RMT verified [6] that all u are not theoretically or empirically equal, especially at quantum scales (recognizing Heisenberg's uncertainty). When all u are not equal in theory, the quantization and precision of u must be included in a measurement theory which calculates a quantity.

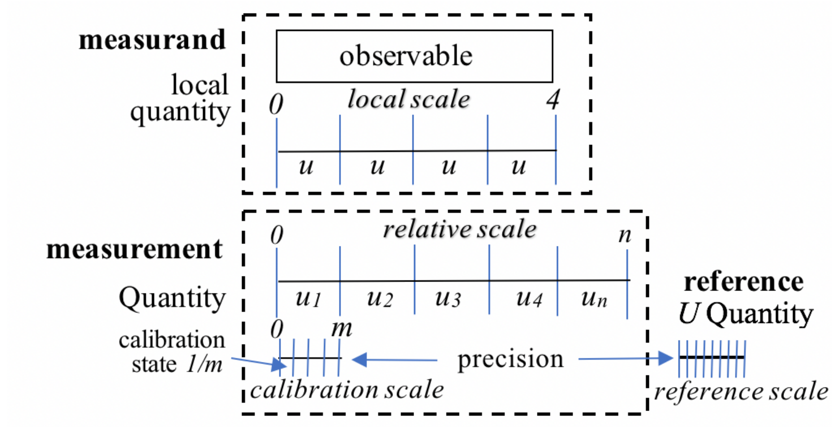
$$\text{quantized } \textit{Quantity} \;\; Q = \sum_{n=1}^{n} u_n \qquad (1)$$

In (1), u_n represents each of the smallest intervals of the relative scale. However, each u_n is not treated as equal in this paper, whereas each u on the local scale is assumed to be equal.

A quantity (usually $n \bullet$ mean u_n) is expedient for experimental measurements. A Quantity (capitalized), see (1), is a proper superset of a quantity, i.e., it includes the result $n \bullet$ mean u_n. Equation (1) is proposed as the first step towards a formal measurement function that applies to all measurements in theory and experiment.

4. A relative measurement system

Fig. 3 adds additional detail to Fig. 1. To support Euler's relative measurement paradigm the measurement apparatus (apparatus is formal or physical) determines a *relative quantity* which is correlated to both a *local scale* and a *reference scale*. This paper proposes that Fig. 3 applies to all relative measurements in theory or experiment. A physical measurement instrument may also include transducers (not shown) which convert a local measurand's quantity to a relative scale's quantity.



measurement result = (4 ± accuracy) • (u_n ± precision)

Figure 3. Relative measurement system in theory and metrology

In Fig. 3, measurement results of the measurand using the relative scale are a statistical sum: $u_1+u_2+u_3+u_4$. The four calibration states ($1/m$) quantify each of the u_n intervals on the relative scale. Since each u_n varies ± ($1/m$) relative to U, each statistical sum of the $4u_n$ creates a Gaussian distribution which identifies the deviation from 4(mean u_n) = mean measurement.

Any deviation of a measurement result quantity has three possible causes (operator errors are not considered): quantization ($1/m$ calibration states), noise (external to the measurement system), and distortion (internal to the measurement system). This paper focuses on quantization ($1/m$), as this has been previously overlooked in measurement theory.

Fig. 3 illustrates the successively smaller resolution (intervals between the small vertical lines), of each horizontal scale that are required for a functional measurement system. That is, the local scale resolution is u ($1/n$). u is equal (within a precision) to the relative scale resolution, the mean of u_n. u and u_n are larger than the calibration scale resolution (calibration states, $1/m$). The $1/m$ resolution is larger than the U reference scale resolution, which is the smallest identifiable change of U. Integers n and m represent counts when $1/n$ and $1/m$ represent the smallest resolution of their scales.

The first mutual relation in Fig. 3 establishes the measurand's measurability. The second mutual relation is the common 0 between the local and relative scales. The third mutual relation, quantization divides each u into m equal states, creating each u_n with an independent numerical value.

$$\text{quantized } u = u_n \tag{2}$$

The fourth mutual relation determines the precision of u_n relative to U. As example:

$$u \text{ when calibrated} = u_n = U \pm (1/m) \tag{3}$$

Eq. (3) identifies that the numerical value of each calibrated u_n is relative to U and is statistical (not fixed). Then a precise measurement function must be a statistical sum of each $\left(u_n \pm (1/m)\right)$. Including this in (1) produces:

$$\text{measurement result Quantity } = \sum_{n=1}^{n}\left(u_n \pm (1/m)\right) \qquad (4)$$

Over many repetitive measurement results, each statistical sum of (4) establishes a Gaussian measurement result distribution. In statistically rare cases, the distribution established by each ±(*1/m*) becomes increasingly dispersed (see the examples in Section 5). For a measurement function to represent the Gaussian distribution created by the statistical sums, a Quantity (summation) must be used. When a quantity (product) is used, the statistical sums are not treated and measurement problems appear.

5. Empirical measurement examples

Euler's paradigm as developed in the first three sections is a paradigm shift from the representational measurement paradigm applied in physics today. The following three examples are provided to support this new measurement paradigm.

5.1 Additive relative scale

An example of an additive relative scale is a thermometer which measures the quantity of thermodynamic temperature. This example demonstrates how additive imperfect intervals statistically increase the deviation of measurement results, producing a Gaussian measurement result distribution.

The measurement instrument consists of a hollow glass tube with a reservoir filled with mercury at one end, which fits inside another hollow glass tube that slides over the first. The two glass tubes are held together and placed in an adjustable temperature oven which has a resolution of 0.1^0 (degree). Then the outside glass tube is marked at the level of mercury which appears and each 1.0^0 u_n above this mark. $n + 1$ marks or *101* marks (a relative scale correlated to the resolution of the oven) are made to quantize the outside glass tube. Each of the *100* u_n is correlated using the chamber to $1/0.1 = 10 = u_n \pm 0.1^0$ precision.

After 101 marks are made, the instrument is removed from the oven and an ice water bath is applied to the tube with mercury. The outside glass tube is now slid over the inside glass tube until the top of the inside mercury column lines up with the first mark on the outside glass tube. Now one mark on the outside glass tube is referenced to the temperature of ice water (0^0C) which establishes a reference point on this relative scale (mutual relation 2).

Consider the temperature of a glass of water (measurand) in contact with the reservoir of this measurement instrument. If the temperature of the water is 80^0, the *81st* mark on the outside glass tube represents $80^0 \pm 0.1^0$ nominal precision or $\pm 8^0$ worst case precision. The $\pm 0.1^0$ nominal precision occurs when the $\pm 0.1^0$ precision of each *80* u_n is uniformly distributed and cancels. The $\pm 8^0$ precision occurs when each of the *80* u_n has the same $+0.1^0$ or -0.1^0 precision, which sums.

In the proper design of experimental measurement systems, the statistical sum of the quantization effects are reduced to less (usually) than the noise or distortion and is ignored. But in this thought experiment without noise or distortion, when each mark's precision is specified to be $\pm 0.1^0$, $\pm 8^0$ is very rarely possible. The statistical sum of the precision from $\pm 0.1^0$ to $\pm 8^0$ establishes a Gaussian distribution of measurement results (see Section 5.3 below). The effects on a measurement result of statistically summing this precision are ignored when equal u are assumed.

5.2 Length measurement instrument

A physical metre stick (a scale relative to a reference standard metre) is divided into *100* intervals (smallest u). Consider a measurand whose numerical value is $n= 70$. In the more rigorous measurement theory proposed here, the numerical value of each u_n is treated individually and then added to the next u_n (*70* times).

When first calibrated to U (e.g., the reference standard metre), each $u_n = (U/100) \pm (1/m)$ precision where, as example, each $1/m$ is $1 \cdot 10^{-6}$ metres (e.g., $m = 10^6$) and the *accuracy* of $n = n \pm (1/m)$ is small enough to be ignored. In the proposed theory, the Quantity deviation is established by the random application of $\pm (1/m)$ to each u_n producing a Gaussian distribution. In the statistically rarest two cases, when n of the u_n, all with a precision of $+(1/m)$, are summed and in another measurement of n of the u_n, all with a precision of $-(1/m)$, are summed, the maximum, and very rare, Quantity deviation appears $2(70)10^{-6} =1.4 \cdot 10^{-4}$ metres, which is sufficient precision ($\pm 0.7 \cdot 10^{-4}$) for a metre stick. When $m>>n$ the effect of quantization is often and realistically ignored. However, when n and m are both small, e.g., quantum measurements, it will be shown that the statistical effects of quantization become significant.

5.3 Gaussian measurement result distributions

Fig. 4 presents the characteristic Gaussian shape of a large distribution of repetitive experimental measurement results. This shape has been verified in many different forms of measurement results where noise and distortion have been minimized [7]. The ubiquitous nature of a Gaussian distribution of repetitive measurement results, caused by the summing of the $\pm(1/m)$ precision, strongly supports the measurement theory paradigm change proposed in this paper.

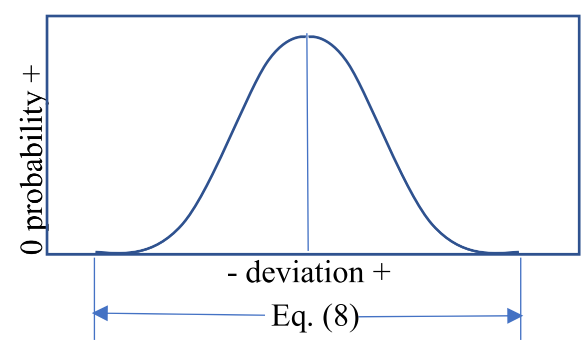


Figure 4. Gaussian distribution of measurement results.

6. The effects of quantization on accuracy, precision and deviation

From (4), applying the same calibration state to *n* (which determines the accuracy of *n*) produces the final form of the measurement function:

$$\text{measurement result Quantities } = \sum_{n=1}^{n\pm(1/m)} \left(u_n \pm (1/m)\right) \qquad (5)$$

Equation (5) is a measurement function that applies (without noise or distortion) to all formal and experimental measurement systems. From (5):

$$u_n \text{ precision } = \pm\left(1/m\right) \qquad (6)$$

This precision, ± *(1/m)* relative to *U* in each of the *n* u_n in (5), varies randomly (in theory) and statistically sums into:

$$\text{the worst case Quantity deviation of (5) } = \pm \sum_{n=1}^{n\pm(1/m)} \left(\pm(1/m)\right) \qquad (7)$$

Then from (7):

$$\text{the worst case range of (7) } = (2n/m) + (2/m) \qquad (8)$$

Equation (7) identifies that each calibrated measurement result Quantity has a deviation which is determined by both *n* and *m*. In (5) when *n* or *m* is large (a common experimental measurement, see Section 5.2), the ± (*1/m*) calibration states likely have a very small effect on deviation, due to the central limit theorem's effect on each statistical sum of (5) when *n* is large and/or due to the central limit theorem's effect on the sum of ± (*1/m*) calibration states in each u_n when *m* is large.

Conversely, when *n* and *m* are both small (a common quantum measurement), two repetitive measurement result Quantities of the same measurand will often be different because of the precision of each u_n. This quantity difference appears in QM as non-commuting numerical values (*n*), when all *u* are assumed to be equal.

7. Measurement problems in existing quantum measurement theory

The development presented in the first five sections follows from Euler’s paradigm. QM experiments and thought experiments which apply the representational measurement paradigm create the measurement problems. When the mutual relations which Euler’s paradigm requires are considered, the following four measurement problems are resolved.

7.1 Remote entanglement

There have been many attempts to understand remote entanglement. J.S. Bell's formalization (since verified experimentally [8]), which perhaps consolidates the earlier attempts, is addressed. J. S. Bell, in his paper [9] states: "...there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote." In the Stern-Gerlach experiments that mechanism is the second of the four mutual relation identified.

N. D. Mermin in 1981 [10] statistically analyzed the Stern-Gerlach experiments [11] that identify remote entanglement. Remote entanglement is described by Mermin without QM formalism, which indicates it may occur in all measurement results. Mermin identifies that the measurement result quantities of two entangled particles have a mutual relation that is not possible without an unknown interaction between the two measurement instruments. Not recognizing that a measurement process must establish mutual relations, Mermin identifies the unknown interaction - the correlation of two instruments' reference points - without realizing it.

In Mermin's device, the two measurement instruments each have a 3 position selector which selects the angle (one of three 120^0 intervals) of the spin quantity. Mermin's two measurement instruments require a common reference point among the three 120^0 intervals since the common quantity (spin) and common scale (three 120^0 intervals, each a u) are determined by experimental set-up.

There are 9 possible combinations of the two position selectors: Three (Mermin's case a) when each selector is in the same position and six (Mermin's case b) when the positions of the two selectors are different. When the two remote selectors are in different positions (the two measurement instruments' reference points are uncorrelated), each n (0^0 or 180^0) of the two particles spin vectors appears randomly over a large number of runs. Only in Mermin's case a is each n of the two particles correlated, because a common reference point (second mutual relation) between the two measurement instruments' scales is required. When this necessary reference point correlation appears in experiments, but does not appear in representational measurement theory, there is a problem.

7.2 Compton-Simon experiment

In the Compton-Simon experiment [12], the second measurement in the same experiment, which is not a repetitive measurement, is equal to the first. Similar to remote entanglement, it appears that the first measurement foretells the second measurement, before it is known. In actuality, when the third and fourth mutual relations remain the same, the two independent (taken at different times) measurements' numerical values remain equal.

7.3 Heisenberg's uncertainty

In Heisenberg's 1927 experiment [13], a single particle's two Fourier dual quantities (his notation: p, momentum and q, position) are compared multiple times identifying their precision p_1 and q_1. (Precision in his experiment is a range [where statistical sums occur but are not recognized] of the precision [defined in Annex A] over a set of repetitive measurements.) The Fourier dual quantities (having inverse time units) will vary inversely as time changes, which Heisenberg recognizes. However each quantity's precision is statistical and varies independently.

One form of Heisenberg's uncertainty principal (HUP) is $p_1 q_1 \sim h$. This is closely related to the minimum possible calibration state, $1/m$. Applying this paper's notation: $p_1 q_1 = \pm 2(1/m)$.

This identifies that HUP is consistent with the concept of a calibration state proposed in this paper. A formal analysis of the correlation between the different forms of uncertainty and precision is presented in the RMT paper.

Heisenberg's assumption that classic measurements in theory can be without uncertainty is shown to be invalid in Section 6, above.

7.4 Double slit experiments

Feynman's [14] explanation of the double slit experiments offers a good example of how a reference and the related relative scale selects which quantity of a measurand is measured. Feynman concludes, "...when we look at the electrons the distribution of them on the screen is different than when we do not look."

In these experiments the set of slits establish the local scale and the sensing screen is the relative scale which identifies two different measurands depending on how the operator looks. Observables are known to exhibit both wave and particle properties. The detectors at the local scale only identify one measurand (particle). However, an operator looking at the sensing screen, identifies a dot indicating a particle's quantity (momentum) or identifies a wavelength. The wavelength of the particle's quantity (frequency) is determined by the positions of the slits and the sensing screen.

The operators' selection of a dot pattern or wavelength on the sensing screen selects the reference applied. Consider the brick example: the selection of a mass scale (defined by a mass reference) selects the brick's mass measurand. If the measurement apparatus was a ruler (defined by a metre) the brick's length would be measured. When representational measurement theory does not recognize how the reference determines which measurand of an observable is measured, measurement problems appear.

8. Relating relative measurement theory to other theories

In 1891, J. C. Maxwell [15] proposed that a measurement result quantity is:

$$\text{measurement result quantity} \quad q = n \cdot u \tag{9}$$

In (9), n is a numerical value, and u is a unit ("taken as a standard of reference" [16]). Maxwell's proposal of a quantity equal (without $\pm$ precision) to a standard is representational. Then (9) suggests that perfect precision is possible in theory, which makes all units equal and the reference arbitrary in the same theory. This representational view of a measurement follows Fig. 2 and is not valid for small n or m. A reference is only arbitrary in first use and in 1927 Heisenberg identified that perfect precision is impossible.

In the 20th century, von Neumann's Process 1 QM measurement function [17] was developed. von Neumann's Process 1 formalizes a representational measurement as the sum of inner products of the stationary states (ϕ_n) of a observable and a scale of stationary states of the measurement apparatus. This is the diagonal of the observable/measurement apparatus matrix.

Process 1 includes a statistical projection operator, $P_{[\phi n]}$[18], which projects each statistical state [similar to ± (*1/m*)] on each inner product, which is then summed.

The Einstein, Podolsky, Rosen (EPR) paper titled: "Can quantum-mechanical description of physical reality be considered complete?" [19] can be answered in the affirmative when Euler's paradigm is applied. The EPR paper, based upon a representational measurement paradigm, considers a classic measurement can be as exact as is desired, which is shown to be incorrecct in Section 6, and does not recognize the requirement for a reference (identifying representational measurement theory as incomplete) for a relative measurement result to occur. Einstein's "spooky action at a distance"[20] is an illusion which appears between two measurement instruments caused by calibration between them.

Fig. 3 relates the local observable (representational) with the measurement apparatus and reference of a relative measurement system. The authors of the basic text on representational measurement theory [21] note that the theory does not recognize a quantity; assume measurement result comparisons can occur without a relative scale or reference; treat units as equal [22], which requires any calibration to be empirical [23]; and indicate that all measurement result deviation is due to noise and distortion in the measurement system [24]. RMT supports Euler's paradigm.

Since Euler's paradigm is not applied in current physics measurement theory, other measurement problems have been seen. RMT described the entropy change (*log m*) caused by a relative measurement which is seen as a collapse or decoherence in a representational measurement paradigm [25]. In Measurement Unification, 2021 [26], explanations are given of quantum teleportation experiments and Mach-Zehnder interferometer experiments. The Schrödinger's Cat thought experiment is explained in a short preprint [27]. These, together with the explanations in this paper, strongly support RMT which applies Euler's paradigm.

9. Conclusion

Representational measurement theory has been applied successfully to measurements made between Fourier duals. However, as the EPR paper identified, representational measurement theory is lacking when applied to measurements relative to a reference. Euler recognized that measurands can only be compared relative to a reference. Examined closely, all repetitive measurement result Quantities in theory and experiment have a deviation relative to a reference, *U*. Then a precise comparable measurement result *Quantity* is:

$$(n \pm \text{accuracy}) \bullet (u_n \pm \text{precision}) = (n \bullet \text{mean } u_n) \pm \text{deviation} \qquad (10)$$

The right side of (10) is how metrology is usually practiced.

Maxwell suggested in 1891 that $u = U$. Perhaps Maxwell assumed that a theoretical measurement result could be exact, whereby calibration is empirical and u or U is arbitrary. In any event, Maxwell's usage appears to have instigated what is now the representational measurement paradigm which has continued and was not reconsidered even after Heisenberg's uncertainty (1927).

When calibration is solely an empirical process, u and U may be unitary and are a factor of everything. Then u and U are without import and a ratio of two measurement results is relative to each other, rather than each relative to U. Then quantum uncertainty appears as relations between Fourier duals, rather than the precision of any quantized measurement result Quantity relative to U in a quantized state space. When Euler's paradigm is applied, quantum measurements no longer have problems.

Annex. Definitions

1.0 *Units* have multiple definitions

Currently accepted in physics and mathematics:

- In representational measurement theory (all physics except metrology) u is defined to be U.
- In metrology (currently assumed to be only empirical), u is commonly the mean u which is calibrated to U or a factor of U (relative measurement).
- In statistics, either a mean u (representational measurement) or U (relative measurement) may be the reference.
- In QM, bra-ket notation (representational), a state is a ket vector representing u or U, and is treated as unity [28].

In this paper (relative measurements):

- u identifies each of the smallest interval of a scale without calibration to U. Each uncalibrated u has a local numerical value, local quantity and undetermined precision.
- u_n has a numerical value (m/n). Each u_n of a relative scale is calibrated to U and has a numerical value, quantity and precision all relative to U or a factor thereof.
- *U standard unit* (capitalized), is a *reference quantity* with a defined numerical value in reference scale resolution states. U may be defined in theory as a true numerical value, even though a physical U will have a resolution in a quantized state space. U is represented by one of the seven different BIPM base units or their derivations [29].

2.0 Additional definitions used in this paper

The following definitions (different from VIM) codify the paradigm shift proposed in this paper. Other VIM definitions may also require changes.

Accuracy is the ± change of the numerical value (n) of a measurement result Quantity relative to its mean numerical value over repetitive measurement results.

Calibration state, $1/m$, is the smallest defined-equal state of a *calibration scale*. The calibration states quantify each u_n and are stationary states (time independent) within the set of u_n.

Deviation is the statistical sum of the ± precision and ± accuracy. The deviation of a measurement result distribution of repetitive measurement results may be standard, mean, worst case, etc.

Measurand, a quantity intended to be measured (VIM 2.3), meaning that it has order and additivity [30], which identifies that it can appear on a *local scale* with a reference point.

Observable, only a numerical value when $u = 1$ (representational), has order and additivity but does not reference U. Therefore an observable appears in experiments as a distribution relative to its mean and in current QM theory as a probability distribution.

Precision is the statistical sum of the ± change of each u_n of a Quantity relative to a U reference or factor thereof, as determined by calibration.

Relative scale is non-linear. A relative scale's quantity, u_n size and precision are correlated to U as determined by the third and fourth mutual relations. A relative scale establishes a *relative quantity*.

A *scale* establishes measurability by identifying order, additivity and a *reference point* (e.g. 0) of a set of n of u or u_n. A scale represents a measurand or a measurement apparatus.

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