

Concept of introverted space: is extroverted, multidimensional space an illusion?

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The quantum-phase-field concept of matter is revisited with special emphasis on the introverted view of space. Extroverted space surrounds physical objects, while introverted space lies in between physical objects. Space between objects leads to a network structure of matter: a network in which one-dimensional spaces connect massive elementary particles.

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I. INTRODUCTION

‘Space’ is one of the first perceptions a child makes after being born: space separates the child from its mother. It is reported that babies first see the world upside down because of the optic our eyes. The image, generated by the brain from individual signals of the visual nerves, is understood as field in a two-dimensional (2D) vector space. Since we have two eyes, our brain can add the third dimension. These empirical observations over thousands of years have been formulated in the mathematical language of physics as the concept of fields in a continuous multidimensional vector space. There is no question about the practical success of this approach, which culminates in Newton’s mechanics and Einstein’s general relativity.

But is ‘space,’ in which we are living, a multidimensional continuum on a fundamental level? Is it necessary to describe space as a vector space? What problems arise if we do so? What alternative descriptions are possible?

There are several approaches to the understanding general physics that are based on discrete spaces, string theory, loop gravity, or similar concepts. Typically, they embed discrete objects (objects with a reduced dimensionality) such as strings or branes into a high-dimensional continuous space-time. An infinite number of these discrete objects, so-called ‘quantum oscillators,’ are attached to each region in multidimensional space.

My concept of a discrete space is different: it develops a network of masses and spaces that are formed by quantum phase fields. Herein, I shall:

- illustrate the introverted and extroverted views of space;
- define the network structure of masses and spaces;
- revisit the quantum-phase-field theory with emphasis to introverted spaces.

II. INTROVERTED VERSUS EXTROVERTED VIEW OF SPACE

Let us start with the extroverted view of space; this is the common view in traditional physics, including quantum physics and general relativity. Extroverted space is something that exists with or without massive particles—leptons and quarks—embedded into it. Within the extroverted view, particles are placed into space. In this view, space is described mathematically as a real vector space of n dimensions, where $n \geq 3$. Empty space, i.e., space in which there are no particles, may not be seen as fully empty because there are quantum fluctuations. There are concepts that consider space to be related to some ‘substance,’ called ether, and this may be related to these quantum fluctuations.

Space is generally seen to be filled by ‘fields’: real or complex, scalar or vector valued, classical or quantum. These fields have a characteristic value at each point in space, and their best-known example is the electric field. Since extroverted space is formulated as a real and continuous vector space, classical fields have a measurable value ‘locally’ at each point in this vector space. In quantum mechanics, one has to also consider the gradients of the fields, which makes the description ‘non-local.’ In general relativity, one has to consider the invariance of the speed of light to connect three-dimensional (3D) real space with time, forming a 3D manifold in four-dimensional space-time. The fields transfer attractive or repulsive action between particles, and gravitation is not seen as an attractive force in general relativity, but as a consequence of the coupling between mass and the curvature of space-time.

I compare this to a billiard game (although in this analogy, action is only transferred when balls collide, but the indentation of the ball into the cloth may be of importance). The green of the billiard table represents space. The particles—billiard balls—are placed in this space (see Figure 1, left). They interact according to Newton’s laws of momentum and energy conservation, or their relativistic extensions. In a real billiard game, the player uses the cushions to mirror space. We may, hypothetically, push the cushions to infinity; then, our playground will be infinitely large. We may use periodic boundary conditions, or we might generalize flat space to Riemann

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geometries of different topology. The basic principle of the extroverted view of space stays the same: space exists with or without particles. Particles are placed into space, and space ‘surrounds’ these particles. I thus call this the extroverted view.

The introverted view is different; here, space, if you will, separates and connects particles (Figure 1, right). I call this view ‘introverted’ because it considers that space lies *between* particles. The space inside a building, which is mostly called ‘room’ in this context, is surrounded by walls and floors. Walls and floors are 2D objects in a 3D world; these objects do not exist as elementary entities in physics. Particles—leptons or quarks—are zero-dimensional, meaning they are so small that no extension in any direction can be attributed to them, they are point like. Therefore, introverted space lying in-between the particles has to be one-dimensional (1D): there is no other choice. Particles may then be represented as the vertices, or nodes, of a network structure. The connections—the edges of the network—can be seen as ‘spaces,’ and they are each defined by the distance between the two particles that they connect.

Here, we must take a moment to reflect: what does ‘distance’ mean? It is first of all a scalar value that is assigned to the relation between exactly two objects. The distance is small if the objects are close to each other, and it is large if they are not close. Considering ‘not close’ in contrast to ‘close,’ let us say that they have less to do with each other, that they interact less. We may measure the distance by a length in [m] or by a time to transfer action in [s]. We may also measure it according to the strength of interaction: by the binding energy between two particles in [J]. We will understand the interrelation of two particles being related to an energy![1]

The quantum-phase-field concept, as reviewed in this essay, describes mass and space as a network of energetic states. It is published in [2, 3], and a common version can be found in Chapter 8 of [4].

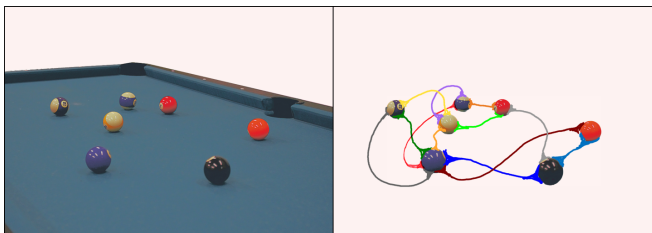


FIG. 1. Left: particles placed in space like billiard balls on a table. Right: particles and spaces as a network. Particles connect spaces (the line segments between the balls), and spaces connect particles. Note: The example on the left is 2D space (the table) and 3D billiard balls with finite volume; This is meant to be generalized to a network with point masses (nodes) and the coloured 1D spaces (edges).

III. THE NETWORK OF MASSES AND SPACES

Let us start with a formal definition of the network of particles and their connecting spaces.

The network is defined by nodes (particles) U_i , $i = 1 \dots n$, and edges (spaces) E_I , $I = 1 \dots N$. For a fully connected network, we have $N = \binom{n}{2}$, but we will also consider partially connected networks $N < \binom{n}{2}$. We postulate the following topology as the simplest form consistent with the above principles:

- One E -element $E_I \doteq E_{km}$ connects two U -elements U_k and U_m , $k, m \in 1 \dots n$.
- One U -element U_i connects a number of $2 \leq M \leq N$ E -elements E_K , $K \in 1 \dots N$.

Clearly, this leads to a network as depicted in Figure 1, right, where the U -elements are nodes and the E -elements are edges. Note that one needs a minimum of two elements of each type to form a primitive network (a ring of two edges connected by two nodes), which can be solved analytically. We will, however, mostly speak about *many* nodes and edges. Without loss of generality, we associate the U -elements with positive energy, the massive energy of particles, and the E -elements with negative energy, the energy of space. In the next chapter I will relate ‘edges’ and ‘nodes’ to ‘quantum phase fields’.

IV. FORMAL DEFINITION OF QUANTUM PHASE FIELDS

The basic object in the quantum-phase-field concept is a ‘phase.’ The phase distinguishes a piece of matter from other pieces in a different phase state; i.e., there is at least one attribute of the piece of matter in one particular phase state that distinguishes it from other pieces in a different phase state.

In quantum physics we call the phase a ‘quantum state’ and each state can be occupied only once. In condensed matter physics, the phase characterizes atomic order, or its absence: solid, liquid, gas, plasma, etc.; this also applies to magnetic order and so on. Each phase is distinguished from another phase by a so-called ‘order parameter,’ which is normalized to $0 \leq \phi \leq 1$.

We will allow *many* phases, ϕ_I , $I = 1 \dots N$, and each phase is connected to all other phases ϕ_K , $K \neq I$:

$$\sum_I \phi_I = 1. \quad (1)$$

This is a system without outer boundaries; the phases ϕ_I constitute a closed system, forming a ‘universe’. We do this in analogy to the multi-phase-field theory in condensed matter physics [5].

Here, ϕ_I is simply termed ‘phase’ because so far, no space has been defined. Each phase ϕ_I is an element of the system that is different from all other elements ϕ_K ,

but there are connections between the phases in situations where at least two phases have a value $\phi_I < 1$ due to the constraint (1). In fact, we will identify the pure phases $\phi_I = 1$ with the edges of the network E_I .

The task now is to define the nodes U_i . If the phase forms an edge, it must connect exactly two nodes. However, one node may connect many edges (if not *all* edges, as in a fully connected network). Later, when ‘space’ is introduced, we will simply denote all points in space where at least two phases are connected (with the condition $\phi_I < 1$, $\phi_k = 1 - \phi_I$) as ‘nodes.’

To give the phases a physical meaning, I associate them with the conserved quantity ‘energy’ $H = \langle \psi | \hat{H} | \psi \rangle$. Here, $|\psi\rangle$ is the quantum state of the system and \hat{H} is the energy operator. Furthermore, I postulate that $H = 0$, i.e., there is no net energy: all energetic states, positive and negative, have to sum to zero. The argument for this is simple: there is no evidence regarding where a finite energy of the universe should come from (see also the Wheeler–DeWitt theory [6]).

I allow changes $d\hat{H}$ such the zero-energy state—the state of ‘nothing’—separates into positive and negative energetic elements—the state of ‘something.’ This can be related to the Big Bang as the origin of our universe, if you will. We expand \hat{H} in the changes $d\hat{H}$ with respect to the phases ϕ_I . \hat{H} is thus itself a function of all phases $\{\phi_I\}$, $\hat{H} = \hat{H}(\{\phi_I\})$, and, as a reminder, all fields are connected by the sum constraint (1):

$$\hat{H} = \sum_I \int_0^1 d\phi_I \frac{\partial \hat{H}(\{\phi_I\})}{\partial \phi_I}. \quad (2)$$

The integral runs over the definition range of the phase as an order parameter from 0 to 1, meaning **yes** or **no**, **existing** or **not-existing**. We allow the fields to vary between these bounds; i.e., they are diffuse, as is usual in phase-field theory.

Introducing a length coordinate s_I (corresponding to an edge of the network), substituting $d\phi_I = \frac{\partial \phi_I}{\partial s_I} ds_I$, and

introducing the forces $\hat{h}_I = \frac{d\hat{H}}{ds_I}$ yields

$$\begin{aligned} \hat{H} &= \sum_{I=1}^N \int_{-\infty}^{\infty} ds_I \frac{\partial \phi_I}{\partial s_I} \frac{\partial \hat{H}(\{\phi_I\})}{\partial \phi_I} \\ &= \sum_{I=1}^N \int_{-\infty}^{\infty} ds_I \hat{h}_I(\{\phi_I\}). \end{aligned} \quad (3)$$

Space emerges; i.e., it is created by variations in the phases $d\phi_I$. We relate the line coordinate s_I to the distance Ω_I , which is defined by the negative inverse of the energy of an edge E_I (for the self-consistent proof see Section V) by the integral

$$\int_{-\infty}^{+\infty} ds_I \phi_I(s_I) = \Omega_I = -\tilde{\alpha} \frac{hc}{48E_I}, \quad (4)$$

where $\tilde{\alpha}$ is a dimensionless parameter to be defined, h is Planck’s quantum and c speed of light.

It is important to note that the space s_I is not ‘fundamental,’ but it is defined by the fundamental entity ‘energy’ as a real number. It is an auxiliary coordinate to link the concept to the view of physics rooted in wave mechanics. From now on, we will consider the phase ϕ_I as a field $\phi_I(s_I)$ in the line coordinate s_I that is intrinsic to this field.

The force operator \hat{h} is expanded in the phases and in their gradients:

$$\begin{aligned} \hat{h}_I &= u \left(\eta^2 \left[\left(\frac{\partial}{\partial s_I} \phi_I \right)^2 - \frac{1}{c^2} \left(\frac{\partial}{\partial t} \phi_I \right)^2 \right] \right. \\ &\quad \left. + P_I(\{\Phi_J\}) \right), \end{aligned} \quad (5)$$

$$P_I = \left(\gamma \sum_{\substack{J=1, \\ J \neq I}}^N \phi_I^2 \phi_J^2 + \tilde{\gamma} \sum_{\substack{J,K,L=1, \\ I \neq J \neq K \neq L}}^N \phi_I \phi_J \phi_K \phi_L \right). \quad (6)$$

Here, u is a positive constant with dimensions of energy per length, or force $[\frac{J}{m}]$, and η is a positive length to be determined at the end of this section [Eq. (13)]. The gradient contribution in time is included, which ensures relativistic invariance due to Lorentz contraction of the length η (see [2] for details). Furthermore, $P_I(\{\Phi_J\})$ is the Landau potential for phase I expanded in all connected phases J . In contrast to previous publications [2, 3], we employ a ϕ^4 potential in the current version of the theory, with force parameters γ and $\tilde{\gamma}$, for discussion see [7].

The second potential term connecting the four different phases in Eq. (6) has an intriguing consequence: this is the only surviving potential term in the limit $N \rightarrow \infty$. It states that there is a minimum of four phases to be connected, with $\phi_I \neq \phi_J \neq \phi_K \neq \phi_L$, which forms the minimum requirement for a space-filling body in three dimensions! All terms with lower numbers of connections vanish in the limit $N \rightarrow \infty$. To see this, we investigate the center of a node at which all connecting phases are equal $\phi_I = \phi_J = \frac{1}{N}$. In equilibrium, this center has the highest energy. The first term in the expansion of the Landau potential (6) is the sum over pairs $\phi_I^2 \phi_J^2 \approx \frac{1}{N^4}$. Since there are $\binom{N}{2}$ contributions, and $\binom{N}{2} \rightarrow \frac{1}{2} N^2$ for $N \rightarrow \infty$, this term vanishes for large networks. I retain this here for the analytical solvability of one edge between two nodes. The second term in the potential (6), $\phi_I \phi_J \phi_K \phi_L$, remains of order unity since $\binom{N}{4} \rightarrow \frac{1}{24} N^4$ for $N \rightarrow \infty$. Higher-order terms, ϕ^6 , ϕ^8 , and so on, may be added with the same argument, as long as all phases are independent and we sum over all combinations.[8]

The gradient contributions of the energy operator (5), $\frac{\partial}{\partial s}$ and $\frac{1}{c} \frac{\partial}{\partial t}$, shall be understood as operators acting either on the phase or on the quantum mechanical wave function $|\psi\rangle$. The kinetic equation for the evolution of phases is written down according to the Clausius–Duhem relation, with the relaxation constant $\tilde{\tau}$:

$$\tilde{\tau} \frac{\partial}{\partial t} \phi_I = -\frac{\delta}{\delta \phi_I} \int_0^{+\infty} dt \langle \psi | \hat{H} | \psi \rangle. \quad (7)$$

This equation has two parts: (i) a non-linear wave equation for the phase fields ϕ_I , and (ii) a linear Schrödinger-type equation for quantum-mechanical excitations within the phases. This procedure is not new; it can be traced back to the so-called de Broglie–Bohm double-solution program [9–12]. For further explanation, see [3].

We now separate the expectation value of the energy operator (3) into three different contributions. These are distinguished by whether the differential operators $\frac{\partial}{\partial s}$ and $\frac{\partial}{\partial t}$ are applied to the wave function $|\psi\rangle$ or the field ϕ_I .

Applying the differential operators to the phase components and using the normalization of the wave function $\langle \psi | \psi \rangle = 1$ yields the force $u_I \left[\frac{J}{m} \right]$ related to phase I :

$$u_I = u \left\{ \eta^2 \left[\left(\frac{\partial}{\partial s_I} \phi_I \right)^2 - \frac{1}{c^2} \left(\frac{\partial}{\partial t} \phi_I \right)^2 \right] + p_I \right\}. \quad (8)$$

The mixed contribution, in which one of the operators $\frac{\partial}{\partial s}$ and $\frac{\partial}{\partial t}$ is applied to the field ϕ and one is applied to the wave function $|\psi\rangle$, describes the correlation between the field and the wave function. This shall be set to 0 in the quasi-static limit. In this limit, we keep the field static for the evaluation of the quantum-mechanical force. Then, we take this force for the determination of the time evolution of the field. A coupled solution has not been worked out to date:

$$0 = \phi_I u \eta^2 \left[\frac{\partial \phi_I}{\partial s} \langle \psi | \frac{\partial}{\partial s} | \psi \rangle - \frac{1}{c^2} \frac{\partial \phi_I}{\partial t} \langle \psi | \frac{\partial}{\partial t} | \psi \rangle \right]. \quad (9)$$

It is shown in [2] that Eq. (9) is consistent with Newton’s second law of acceleration. Finally, we apply the momentum operators $\frac{\partial}{\partial s}$ and $\frac{1}{c} \frac{\partial}{\partial t}$ to the wave function $|\psi\rangle$, which yields the force $e_I \left[\frac{J}{m} \right]$:

$$e_I = u \eta^2 \phi_I^2 \langle \psi | \frac{\partial^2}{\partial s^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} | \psi \rangle. \quad (10)$$

This contribution applies to the bulk energy of the phase field $\phi_I = 1$. We will explicitly evaluate this after the structure of the solutions of the fields is discussed. One transforms the phase-field equation (7) into the moving frame traveling with the velocity v . Inserting

Eqs. (8)–(10) into (7), we find:

$$\begin{aligned} \tilde{\tau} \frac{\partial}{\partial t} \phi_I &= -\frac{\delta}{\delta \phi_I} \int_0^{+\infty} dt \langle \psi | \hat{H} | \psi \rangle \\ &= u \left[\eta^2 \frac{\partial^2 \phi_I}{\partial s^2} \left(1 - \frac{v^2}{c^2} \right) - p' \right] \\ &\quad + m_{\phi_I} \sum_{j=1}^N \Delta e_{IJ}, \end{aligned} \quad (11)$$

where: p' is the variation of the Landau potential with respect to ϕ_I ; $\Delta e_{IJ} = e_I - e_J$ is the difference in the volume force between two phases I and J according to Eq. (10); and m_{ϕ_I} is the appropriate coupling function.

The so-called doublon solution for two phases in a periodic setting is well known as the minimum solution of the classical part of the energy operator \hat{H} [Eq. (3)], as depicted in Figure 2:

$$\begin{aligned} \phi_I(s) &= \frac{1}{2} \left\{ \tanh \left(\frac{3(s - s_1 - vt)}{\eta_v} \right) \right. \\ &\quad \left. - \tanh \left(\frac{3(s - s_2 + vt)}{\eta_v} \right) \right\}, \end{aligned} \quad (12)$$

where the particles are located at s_1 and s_2 with distance $\Omega_{12} = s_1 - s_2$. The phases transform into each other with velocity $v \propto \Delta e$. $\eta_v = \eta \sqrt{1 - \frac{v^2}{c^2}}$ shows Lorentz contraction of the quantum length for accelerated particles (for details, see [2]). In this picture, u is the force of inertia. One phase is bounded by two solitonian waves, one right-moving and one left-moving. We call this object a doublon. Finally, we relate the size of the transition region η to the parameter u and γ by the minimum solution of Eq. (11) (the four-phase term proportional to $\tilde{\gamma}$ gives no contribution to the two-phase case):

$$\eta = \sqrt{\frac{36u}{\gamma}}. \quad (13)$$

V. VOLUME ENERGY OF ONE DOUBLON

From the doublon solution in Section IV, we can see that the field forms a 1D box with fixed walls and size Ω_I for phase I . We consider massless quantum fluctuations inside the box and find the explicit representation of the standing waves ψ_p in the length coordinate s ,

$$\psi_p = \sqrt{\frac{2}{\Omega}} \sin \left(\frac{\pi p}{\Omega} s \right). \quad (14)$$

The dispersion relation for massless particles is linear in momentum $\tilde{p} = p \frac{h}{\Omega}$, $p \in \mathbb{N}$, instead of quadratic in \tilde{p} , as for the case of massive particles in a box potential. According to Casimir [13], we have to compare quantum

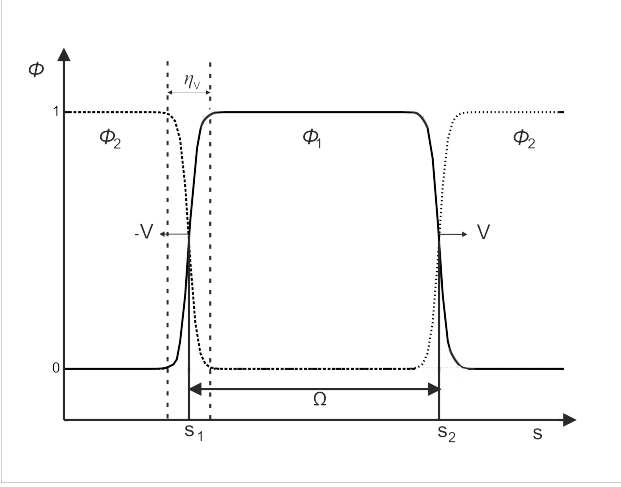


FIG. 2. Two doublons in a periodic setting. Each doublon is formed by a right-moving and a left-moving soliton. The velocity is proportional to the energy difference between adjacent doublons. The relative distance $\Omega_{12} = s_1 - s_2$ corresponds the length of the edges of the network.

fluctuations in the box with discrete spectrum p and frequency $\omega_p = \frac{\pi c p}{2\Omega_I}$ to a continuous spectrum. This yields the negative energy E_I of the space I :

$$E_I = \alpha \frac{hc}{4\Omega_I} \left[\sum_{p=1}^{\infty} p - \int_1^{\infty} p dp \right] = -\alpha \frac{hc}{48\Omega_I}, \quad (15)$$

where α is a positive, dimensionless coupling coefficient. I have used the Euler–Maclaurin formula in the limit $\epsilon \rightarrow 0$ after renormalization $p \rightarrow pe^{-\epsilon p}$. Since all parameters are positive, we see that ‘space’ is accounted for by negative energy, scaling in inverse proportion to the size of the doublon, which proves (4) for self-consistency. This energy scales like the energy of the gravitational field in Newtonian mechanics and general relativity. Therefore, the force—as the derivative of the energy (15) with respect to space—can be associated with a gravitational attraction between the nodes, which is transmitted by the spaces. The nodes can thus be interpreted as massive objects: elementary particles. They are associated with positive energy U , while the bulk energy of one doublon/space is negative. In contrast to Newtonian mechanics and in agreement with general relativity (see ‘gravitational waves’ [14]), the attraction is a wave phenomenon with time-dependent action. The coefficient α can be determined from the measured gravitational constant on Earth, as we will derive in the next section.

VI. CLOSED DOUBLON NETWORK

From the doublon solution (12), we see that the elements of the network—particles with energy U_i and spaces with energy $E_I \equiv E_{ij}$ —are both related to the

phases, or if we relate the bulk energy of a phase to a space coordinate, we say that they are related to phase fields. Particles relate to the gradient contributions, while spaces relate to the bulk in between particles. Values of $\phi \equiv 0$ have no physical meaning. Since each doublon decays to zero on either side, it has to connect to at least one other doublon on each side due to the sum constraint (1) within a finite transition region of size η_v . To get a handle on this size, as well as on the size of the bulk region Ω_{ij} , we need to evaluate the coupling constant α . We assume isotropy in the system and energy neutrality, with N being the number of particles connected to each single particle, dropping the subscript i . Since only $\frac{1}{2}$ of each doublon counts for one particle, we find:

$$0 = U + \sum_{I=1}^N E_I = U_i - \frac{1}{2} \alpha \frac{hc}{48} \sum_{I=1}^N \frac{1}{\Omega_I}. \quad (16)$$

With the mass of one particle $m = \frac{U}{c^2}$ and the average size of the network as the homogeneous mean $\bar{\Omega} = N \left[\sum_{I=1}^N \frac{1}{\Omega_I} \right]^{-1}$, we get:

$$\alpha = 96 \frac{mc\bar{\Omega}}{Nh}. \quad (17)$$

The force between two particles along one connecting doublon is evaluated:

$$\begin{aligned} f_I^{\text{micro}} &= - \left(\frac{\partial E_I}{\partial \Omega_I} + \frac{\partial E_I}{\partial \bar{\Omega}} \frac{\partial \bar{\Omega}}{\partial \Omega_I} \right) \\ &= \frac{mc^2 \bar{\Omega}}{N} \frac{1}{\Omega_I^2} \left[1 - \frac{\bar{\Omega}}{N\Omega_I} \right] \\ &= \frac{m^2 c^2 \bar{\Omega}}{M^u} \frac{1}{\Omega_I^2} \left[1 - \frac{\bar{\Omega}}{N\Omega_I} \right]. \end{aligned} \quad (18)$$

The number of particles N can be estimated from the mass in the visible universe M^u compared to the mass of one particle m . For distances $\Omega_I < \frac{\bar{\Omega}}{N}$, we note repulsive action with a force scaling with $\frac{1}{\Omega_I^3}$. Thus, the singularity of collapse of an agglomeration of point-like particles into one point is forbidden. For $\Omega_I \gg \frac{\bar{\Omega}}{N}$, Newton’s law of gravity is recovered. Identifying the prefactor in Eq. (18) $\frac{c^2 \bar{\Omega}}{M^u}$ with the coefficient of gravitation G , one obtains:

$$\bar{\Omega} = \frac{GM^u}{c^2} \approx 7.4 \times 10^{24} \text{ m} \approx 240 \text{ Mpc}, \quad (19)$$

$$\frac{\bar{\Omega}}{N} = \frac{Gm}{c^2} \approx 3 \times 10^{-53} \text{ m}. \quad (20)$$

I have used the numerical values $G \approx 6.67 \times 10^{-11} \left[\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \right]$, $c \approx 3 \times 10^8 \left[\frac{\text{m}}{\text{s}} \right]$, $M^u = 10^{52} [\text{kg}]$ [15], and $m = 4 \times 10^{-26} [\text{kg}]$ set to $\frac{1}{4}$, the mass of a hydrogen atom, or a neutron and its neutrino, each consisting of four fermions. Repulsive gravitational action at

the microscale is thus limited to distances $\frac{\bar{\Omega}}{N}$ below the Planck length. We have, however, to consider that this formal derivation considers the variation of a cosmological length $\bar{\Omega}$ with a very short length scale, e.g., between quarks inside a nucleon. From the formal definition of $\bar{\Omega}$ by the harmonic mean of all spaces $\Omega(I)$, which is dominated by short spaces, we see that $\bar{\Omega} \rightarrow 0$ for one single $\Omega(I) \rightarrow 0$, regardless of the large number of long and ultra-long spaces. The cutoff $\frac{\bar{\Omega}}{N}$ prevents this behavior, but on an unrealistically small scale, according to physical intuition. Therefore, the formal derivation of the micro force equation (18) has not been presented in previous publications [2, 3].

At ‘cosmological’ distances, we shall treat $\bar{\Omega}$ and Ω_I as independent. Variation of the energy of space (15) with respect to $\bar{\Omega}$ and Ω_I independently gives:

$$\begin{aligned} f_I^{\text{macro}} &= - \left(\frac{\partial E_I}{\partial \Omega_I} + \frac{\partial E_I}{\partial \bar{\Omega}} \right) \\ &= \frac{mc^2 \bar{\Omega}}{N \Omega_I^2} \left[1 - \frac{\Omega_I}{\bar{\Omega}} \right] \\ &= G \frac{m^2}{\Omega_I^2} \left[1 - \frac{\Omega_I}{\bar{\Omega}} \right]. \end{aligned} \quad (21)$$

Structures beyond the marginal distance $\bar{\Omega}$ repel each other, leading to an accelerating expansion. We further see from the generalized gravitational law (21) that in the limit $\Omega_I \gg \bar{\Omega}$, the force scales as $f_I \propto -\frac{1}{\Omega_I}$ instead of $f_I \propto \frac{1}{(\Omega_I)^2}$ for medium distances; i.e., repulsive gravitational action at ultra-long distances decays more slowly with distance than attractive gravitational action at medium distances. The consequences of this statement deserve further consideration in the future.

VII. CONCLUSION AND DISCUSSION

Phases ϕ_I and their variations $d\phi_I$ are the principal elements of the quantum-phase-field concept, forming a monistic theory. The model defines a network of particles and spaces that are both determined by the phases and their variation. Particles are not *placed in* space, as in traditional theories, but rather they *bound* spaces! Spaces connect particles, as particles connect spaces. Particles and spaces are two sides of the same coin: the doublon. One space element connects exactly two particles; one particle is the connection between many spaces. This defines the network structure of the physical world. Both elements of the network—nodes and edges—are defined by energy, which is the only fundamental substance in the concept. Space is introduced as a negative inverse of the energy of one edge. It is shown that this ansatz leads to a quantum problem on the edges, the solution of which reproduces the negative energy associated with

this space self-consistently. The quantum problem is defined as a linear Schrödinger-type equation. The equation of motion for the nodes is formulated as a classical non-linear wave equation derived from a Ginzburg–Landau-type Hamiltonian on the line coordinate of space. The non-linearity suppresses the spectrum of solutions, as is characteristic for linear Schrödinger-type wave solutions. This can also be compared with the theory of solitonian waves [16] and Goldstone modes in elementary particle physics [17, 18].

The doublons are the minimum solution of the classical part of the quantum-phase-field equation (11). The quantum part (10) defines gravitational attraction or repulsion; the mixed part defines the equation of motion of particles under gravitational action.

Energy is the only fundamental substance, and its net amount is zero, balancing the positive energy of particles with the negative energy of space. The problem then arises that a closed quantum system without energy has to be stationary according to the time-dependent Schrödinger equation (cf. the Wheeler DeWitt equation [6]):

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi = 0. \quad (22)$$

In the present concept, the problem is separated into a linear quantum problem on the individual doublons and a non-linear classical wave problem for the solitonian fronts. Equation (22) is linear in ψ , and time is seen as the observable conjugate to energy in the quantum-mechanical sense, and this part of the equation is reversible in time t .

\hat{H} itself is a non-linear function of the classical field variables $\phi = \phi(\tilde{t})$, and \tilde{t} shall be called ‘thermodynamic time’ as the observable conjugate to entropy. This distinction is introduced here to emphasize the different meaning of time in quantum mechanics (reversible in time) and thermodynamics, where time \tilde{t} is not reversible according to the second law of thermodynamics. The thermodynamic time governs the dynamic evolution of the system according to the non-linear classical wave equation (7) or (11). The evolution of the phases ϕ then changes the spectrum of quantum fluctuations within the doublons; i.e., it determines the time dependence of the wave function ψ . Although up until now only the quasi-static solution has been worked out, a coupled solution should exist. We may formally introduce a complex time variable, where \tilde{t} forms the real axis and $i\tilde{t}$ the imaginary axis (see also the considerations in [19]). I leave further discussion to future work.

Comparing the positive energy of mass and the negative energy of space leads to the prediction of a marginal distance $\bar{\Omega}$ beyond which gravitational action becomes repulsive. This distance compares well to the measured size of large voids in the universe [20]: massive objects at the rim of the voids repel each other such that they cannot enter the voids by gravitational forces.

Here, another fundamental problem arises: if energy is

conserved, according to Noether’s theorem, the universe must be stationary. We may recall the concept of self-similar distributions in materials science, e.g., coarsening in a multi-grain structure [21, 22]. Defining the relative length scale $\omega_r = \frac{\Omega}{\bar{\Omega}}$, one may reformulate the theory in this relative coordinate system. In the classical concepts cited above, the system reaches a self-similar distribution in the relative coordinates that is stationary in time: the system has to become self-similar if the system parameters are time-independent. We may assume that such a condition also holds for the universe, but a rigorous proof has not been given to date.

In the present treatment, space separates and connects particles. This defines the structure of the network of physical reality. Particles and spaces are two aspects of the doublons, the primitive object of the network; the doublon network of positive and negative states of energy defines the physical reality. It can be embedded into a 3D vector space or higher, but these spaces are not physical. In a multi-dimensional vector space, whether locally or globally defined, points are compact in any direction. In a network, points are compact only in one direction: the line coordinate s between nodes. Points in a multi-dimensional vector space that do not lie on edges or nodes of the network—i.e., which do not coincide with at least one doublon—are inaccessible: they have no physical reality.

A last comment shall be given regarding ‘general relativity.’ General relativity is based on the premise that

space is a multidimensional continuum which, in prevailing mathematical language, is formulated as a vector space. In this interpretation—the extroverted view of space—mass and space are treated as separate entities. Einstein’s equations present a closure of the non-convex problem of space and mass: they connect masses via the deformation of space. The closure is based on Riemann curvature, which is basically derived from a network structure of discrete spaces.

In the introverted view of space, where mass and space are part of the same entity (energy), the connection of mass and space emerges naturally. Energy is structured in the form of phases. Introducing space as a 1D line coordinate, one finds the doublon solution of phase expanded in this coordinate. The doublons are connected by the sum constraint (1) to form a ‘universe.’ Applying the gradient contribution of the Hamiltonian to the phases defines the positive energy of mass. Applying the gradient operators to the wave function and solving the quantum problem of acoustic excitation within the doublons defines the negative energy of the spaces. The theory defines gravitational attraction as a wave phenomenon. It is relativistic invariant and predicts repulsive gravitational action at ultra-long distances.

If we examine the arguments favoring the introverted view of space against the extroverted view, we are driven to conclude that multidimensional continuum space does not exist: it is a construct of our brains—an illusion.

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- [1] Particles are usually associated by a positive energy of their rest mass. Distance, space between particles, usually is not considered as an energetic state, although Newtons physics clearly draws this connection. I show that it has a negative energy, the energy of space. See section V.
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