

# Review of: "On the statistical arrow of time"

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The article addresses a long-standing issue: the origin of the arrow of time.

Namely, how can it be that fundamentally time-reversible dynamical laws lead to irreversible time evolutions?

The author argues against the typically held point of view that irreversibility is due to the impossibility of controlling all degrees of freedom of large systems, thus ultimately to the lack of knowledge on the part of the observer. The article aims at making explicit the danger of subjectivity accompanying such an argument; namely, the unwanted dependence of a seemingly objective law of nature, the one-way direction of time, on the amount of missing information.

The article clearly defines the context within which the conclusions are finally drawn. It starts by distinguishing the objective and subjective interpretation of probability.

*The objective interpretation regards the probability of an event as an objective property of that event and independent of the observer of the event. On the contrary, in the subjective interpretation probabilities arise due to the observer's lack of complete knowledge, or information, about the event. In other words, the appearance of probabilities arises due to the ignorance of the observer.*

Then, the article clearly assumes the subjective point of view, though remarking that this is not necessarily the author's point of view and states its goals.

*The intention of the article is rather to deduce the consequences, concerning the arrow of time, if the subjective viewpoint is taken. We will conclude that there are two possibilities:*

- *The arrow of time is not a fundamental property of Nature. It is subjective due to the ignorance of the observer. The apparent directionality to time thus exists only in the minds of ignorant observers.*
- *The arrow of time is a fundamental property of Nature. However, its origin has nothing to do with the concept of entropy and the second law of thermodynamics.*

The article then proceeds with a review of some tools from statistical mechanics: coarse-graining, ergodicity, tendency to equilibrium and entropy. Though the intention of this part of the article is didactical and thus much appreciated, it seems nevertheless a little redundant in relation to its overall purposes.

In particular, concerning ergodicity, the presence of an environment is discussed; however, it seems to me that its importance for the arrow of time is on one hand not fully developed. For instance, in relation to the presence of an

intrinsically irreversible dynamics as in diffusive, Brownian behaviors or measurement processes in quantum mechanics. Both possibilities indeed depend on the presence of something else, a noisy thermal environment or a measuring apparatus.

On the other hand, the section on ergodicity-breaking seems to convey the idea that ergodicity requires uniformity in the probability distribution, while an ergodic behaviour ultimately means that time-averages coincide with statistical mean values. In other words, the impression one gathers from this part of the article is that the micro-canonical ensemble (uniform probability distribution) is somehow to be preferred against canonical and gran-canonical ones. However, it is in fact when an environment is present and energy, particles and information are exchanged between system and environment that these statistical ensembles become important at stationarity, to which a system is led by an irreversible behaviour.

After introducing the concept of entropy as a measure of ignorance, that is lack of information or knowledge, the author makes explicit the fundamental question.

*Can the seemingly universal feature of the arrow of time, flowing towards the future for all observers, really owe its existence to the inability of the observer to completely specify the state of the system with infinite precision?*

The author argues that this can never be the case by means of the following argument:

*...presumably, a monkey is more ignorant, as compared to a human physicist, about the complete set of degrees of freedom characterizing e.g. the falling of a glass of wine. Yet, the human is more certain about what will happen to the glass of wine as it falls. The directionality of time does not seem less clear to the human despite having more precise information about the state of the glass of wine. This contradicts the logical consequence of the second law as stated above. There, the human is less limited than the monkey and therefore should loose information at a lesser rate and hence the monkey would have a greater sense of time's arrow. Thus, either the monkey does really have a much greater sense about the flow of time, or the directionality has nothing to do with the ignorance about the system possessed by the observer.*

Then, the author draws a conclusion.

*In this article, we have argued that time's arrow, as understood within the statistical interpretation of the second law of thermodynamics, is not compatible with the philosophical standpoint that its physical origin is an objective truth characterizing a fundamental property of Nature. Rather, the statistical arrow of time owes its existence to the ignorance of the observer. Whatever the true physical origin for time's arrow might be, it is our opinion that students and teachers of physics should be aware of this subtle subjectivity in the thermodynamic arrow of time.*

I personally appreciate the need felt by the author and the effort made to bring to the fore the subjectivity issue in connection with the arrow of time. However, my impression is that the subjective interpretation should not be as easily dismissed as I am now trying to explain.

I would reformulate the author concern as follows:

**If objective, the arrow of time cannot depend on the degree of knowledge on the part of any observer.**

However, from the argument presented by the author, it seems that one can only conclude that it is the qualitative perception of the arrow of time, and not its objective existence, that depends on the degree of knowledge of the observer.

My point is that this dependence can be gotten rid of. Concretely, the velocity at which information is lost because of an evolution certainly depends on the coarse-graining used to check and predict the development of the system trajectories in time. However, as shown by Kolmogorov and Sinai, the production of information in time (production because entropy is a measure of the lack of information before a look has been given, but also a measure of the information gathered after looking) leads to a coarse-graining independent characteristic invariant of the dynamics (the supremum taken over all possible coarse-graining) , which Pesin's theorem proves to coincide with the sum of its positive Lyapounov exponents.

Not only, but because of a theorem of Brudno, the Kolmogorov-Sinai entropy can be related to the algorithmic complexity production associated with the system trajectories; and the algorithmic complexity does not require any probability or any measure of lack of information.

Indeed, it relies on the reproducibility of coarse-grained based symbolic trajectories by means of a suitable universal Turing machine.

In other words, the objective exponential amplification of errors characteristic of chaotic classical dynamical systems, an objective feature of these systems, can indeed be accessed through the accumulation of subjective lack of knowledge; not only, but through an algorithmic, computer based notion of probability one can avoid referring to observers and their lack of knowledge.

Of course, there is a caveat, which I think makes the question of the origin of the arrow of time still an open issue: for a reversible dynamics, Liouville theorem, and thus conservation of probability, compensate with negative Lyapounov exponents the presence of the positive ones, the former emerging by going backward in time. What then ultimately decides to break the symmetry present in the fundamental laws of dynamics and by moving forward and not backward in time?

It is at this point that an intrinsically irreversible, time-directional dynamics, similar to those enforced by the presence of an environment, but without it as in the case of quantum dynamics with incorporated quantum measurements, may open new perspectives.

To be clearer, the question raised in this article has a quantum counterpart in that one needs reconcile the reversibility of the Schroedinger dynamics with the irreversibility of the collapse of the wave function due to measurement processes. In order to do so, a viable possibility is to assume that the dynamics is by itself irreversible, but that it reveals this feature only when a (macroscopic) measurement apparatus is present, and test the consequences against the experimental evidence.

