Qeios

Sfarti's Reply to Dingle's Clock Puzzle "Disproof"

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In a recent article,[1] Sfarti provided a simple disproof of Dingle's objection to relativity. The crux of Dingle's objection turns out to be based on Dingle's misunderstanding of synchronization, and perhaps in his inability to accept that his view of the circumstances might not be correct. This paper attempts to expand Sfarti's praiseworthy efforts to dispense with Dingle's misguided notion that relativity is incorrect. In this author's opinion, there are minor issues with Sfarti's response, but its major points, that Dingle did not properly consider intervals of time, and that his conclusion is wrong, are correct. This response discusses where Dingle went wrong, and addresses the minor issues in Sfarti's paper, after a separate refutation of Dingle's argument.

Some background on Herbert Dingle

Herbert Dingle is a controversial character in special relativity. An English Quaker, he earned a first degree from Imperial College, London at the age of 28 (poverty had kept him from attending college earlier). After graduation he worked on stellar spectroscopy under Alfred Fowler at Imperial and was elected as a Fellow to the Royal Astronomical Society four years later. Ten years on he was awarded a Rockefeller scholarship to study cosmology at Caltech, where he worked with R. C. Tolman.[2] For eight years, he was a professor of natural philosophy (as theoretical physics was called in the British universities until the mid-1950s) at Imperial, and then a professor of the history and philosophy of science at University College, London. Whatever else may be thought of Dingle, he was neither a fool nor ignorant of physics. His curious apostasy in later life defies easy explanation.

Though he had been involved in controversies before (with E. A. Milne and Eddington), these involved speculative theories, and his objections were based on mainstream physics (W. de Sitter was more on Dingle's side than Milne's or Eddington's). In the mid-1950s, Dingle somehow arrived at the conviction that relativity was wrong, and wrote a series of polemics against what he perceived as logical consistencies in Einstein's reasoning. Most of these arguments concerned the clock (or twin) "paradox" and related puzzles. He was answered over a decade or more by a large number of physicists (W. H. McCrae and Frank S. Crawford were indefatigable champions of relativity), but he could not be convinced.

In the fifty years since Dingle's last book[3] appeared, a very large body of experimental evidence has been built up by thousands of physicists. Einstein's special and the general theories of relativity have passed hundreds of rigorous tests, most recently in the celebrated success of LIGO. The evidence in support of relativity is beyond doubt.[4] Probably every physical theory has a better theory waiting to be found that will replace it. One day a broader version of general relativity (e.g., one at home with quantum mechanics, as Einstein's luminous theory is not) may be discovered. Whatever a replacement theory might look like, it must reproduce all the many successful predictions of relativity, and presumably will reduce to Einstein's theory as a special case. Perhaps Arzèlies' remark is too harsh.[5] But it is fair to say that within the physics community, to attack relativity as wrong, particularly on the basis of easily refuted arguments, marks one as belonging

[1] A. Sfarti, "Dingle's 'Clock Paradox' Short Disproof" v.2, *Qeios*, https://www.qeios.com/read/E5Q2JL.2,

[2] Dingle is thanked for his assistance with calculating Christoffel symbols on pp. 253-4 in Tolman's magisterial *Relativity, Thermodynamics and Cosmology*, (Oxford: Oxford U. Press, 1934).

[3] H. Dingle, *Science at the Crossroads.* (London: Martin Brian & O'Keeffe, 1972.) Much of this book is devoted to Dingle's years of fruitless struggle to get his unfounded criticisms of relativity taken seriously.

[4] A terrific survey suitable for lay folk is Clifford M. Will's *Was Einstein Right? Putting General Relativity to the Test*, 2nd ed. (New York: Basic Books, 1993.) There is a sequel, following dramatic recent events: Clifford M. Will and Nicholas Yunes, *Is Einstein Still Right?* (Oxford: Oxford U. Press, 2020.) Will has also written an "expert's edition", *Theory and Experiment in Relativity Physics*, 2nd ed. Cambridge: Cambridge U. Press, 2018.

[5] H. Arzèlies, *Relativité Generalisée Gravitation*, fascicle I. (Paris: Gauthier-Villars & Cie, 1961.) p. xxxii. He writes: "Mais tous simplement que les deux groupes possédent vraisemblablement des structures mentales quelque peu différentes; ce qui est

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to the fringe. That is the common view (which this author shares) of Dingle's unorthodox and incorrect beliefs.

Dingle's objection to relativity

Very probably the readers of this article are familiar with special relativity. There are two postulates:

- The laws of physics are invariant under uniform (non-accelerated) motion.
- The value of the speed of light is the same in all inertial (non-accelerated) reference frames.

A consequence of the first postulate is that it is impossible to discern which of two observers in uniform motion with respect to each other is at rest by any experiment whatsoever. That's what makes it a theory of *relativity.* Given a set of observers in uniform relative motion, each is equally entitled to regard herself at rest. And therein lies Dingle's quarrel with relativity.

According to relativity, a clock in uniform motion runs slowly with respect to a stationary clock. As relativity denies any difference between observers in relative motion, the usual effects (time dilation, length contraction) must affect each equally. One observer maintains that her clock runs at its usual time and the other person's clock runs slowly; the other observer believes *his* clock runs normally while *her* clock runs slowly. It is *logically* impossible, said Dingle, that two clocks can run slowly with respect to each other: one must be fast and the other slow; therefore, relativity is wrong. Call this **Dingle's objection**:

Two clocks cannot run slowly with respect to each other.

Contra Dingle, two clocks **can** run slowly with respect to each other, as will be shown below. This follows directly from the Lorentz transformations central to special relativity, in agreement with Sfarti. Frankly, the resolution of this apparent paradox—based on the tricky nature of simultaneity in relativity—is very well known,[6] and there's really no excuse for going through it again. It is worth another look however in terms of Galilean (or Newtonian) physics, with everyday speeds. This approach may not be new, but it is unfamiliar. The scenario then will be analysed relativistically, and Dingle's objection will be overcome. (It should be added that in the limit as $(v/c) \to 0$, the relativistic and Galilean results agree.) Finally the result will be found by a simple application of the Lorentz transformation. In the experience of this teacher, physical arguments convince skeptics more readily than arguments based entirely on the Lorentz transformations that Dingle's objection is baseless. Dingle himself refused to accept arguments analogous to that offered by Sfarti. As Dingle is no longer alive, there's no way to know if he would have been convinced by what follows; perhaps he would have had a harder time refuting it.

The relativity of synchronizing clocks

Dingle's objection fails to account for the subtleties of synchronizing clocks. Consider a very simple example at everyday speeds, using Galilean physics.

Suppose we have two brothers, B and L, standing on the ground, separated by a horizontal distance of $2R$. Place two identical tennis ball launchers on the ground halfway between the brothers, each one pointing at a target above each brother's head. Let their sister, A, approach B from some horizontal distance to the left of B, moving with constant velocity v parallel to a line connecting the brothers. (See Figure 1.) Let each launcher at the same moment fire a tennis ball with a speed v relative to the brothers at its corresponding target, and arrange the sister's distance such that she is immediately abreast of her brother B at the moment the left tennis ball hits B's target. The brothers are sure that each target is hit at the same time, because the targets are each a distance R from their respective launchers, and the tennis balls are shot at the same

évident, très clair, pour les uns, est obscur et absurde pour les autres. ... De toute façon, continuer à discuter entre physiciens, avec des arguments de physique ou de mathématique, est une perte de temps." (Quite simply, the two groups [physicists who do or do not believe in relativity] presumably possess somewhat different mental structures; what is obvious and very clear for some, is obscure and absurd for others. ... In any case, continuing a discussion between [the two groups of] physicists, using mathematical or physical arguments, is a waste of time.)

[6] See, e.g., T. M. Helliwell, *Special Relativity.* (Sausalito, CA: University Science Books, 2010.) §6.3, pp. 68-71.

moment. Each brother's tennis ball has to travel a distance of R at a speed of V. The time t at which the balls hit the targets is given by

$$t = \frac{R}{V} \tag{1}$$

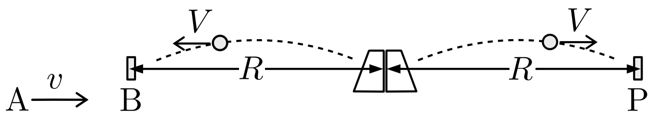

Figure 1: Galilean simultaneity, A moving

If their sister reaches B at the same time as the left ball reaches B, she must have been at a distance $\ell = vt$ to the left of her brother. That is,

$$t = \frac{\ell}{v} \tag{2}$$

If A reaches B when the left ball reaches B,

$$\frac{R}{V} = \frac{\ell}{v} \tag{3}$$

Their sister sees things differently. She regards herself as at rest, at a position she calls 0. The system of B, launchers, and L move towards her at constant velocity $-v$. (See Figure 2.) (Imagine putting the system of B, launchers, detectors, and P in a train car with glass walls, which moves towards a motionless A at a speed v.)

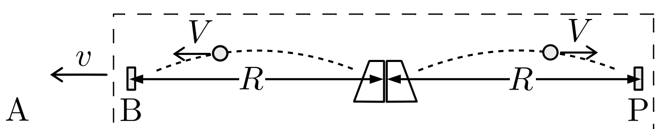

Figure 2: Galilean simultaneity, B and P moving

At the moment the balls are launched, A observes that B is the same distance ℓ away from A as B observed earlier. Denote her measurements of B's and P's position and time with a prime: x', t'. She describes the position of B and his target by

$$x'_{\mathrm{B}} = \ell - vt' \tag{4}$$

B reaches his sister when B's position is equal to A's, i.e., zero:

$$x'_{\mathrm{B}} = x'_{\mathrm{A}} = 0 = \ell - vt' \;\Rightarrow\; t' = \frac{\ell}{v} \tag{5}$$

This value of t' is the same as the value of t found earlier. The left tennis ball is supposed to hit the target at the same moment that B is at zero. The speed of the tennis ball towards A is given not by v, but by $V + v$; that's the way relative speeds add in Galilean physics. The direction of the velocity is to the left, negative. Then the position x_{left} is given by

$$x'_{\mathrm{left}} = \ell + R - (v + V)t' \tag{6}$$

The left tennis ball is supposed to reach the target when the target (and B) are alongside A; i.e. $x'_{\mathrm{left}} = 0$. That means

$$\ell + R - (v + V)t' = 0 \tag{7}$$

From the previous result, $vt' = \ell$, and so

$$R = Vt' \;\Rightarrow\; t'_{\mathrm{left}} = \frac{R}{V} \tag{8}$$

which is just what B and P say; the time t that both balls hit the targets equals R/V, so of course they say that the left ball hits at this time.

At what time t'_{right} does A observe the right tennis ball hit its target? When the balls are fired, P and his target are at a distance of $\ell + 2R$ away from A. The position of P is given by

$$x_{\text{P}} = \ell + 2R - vt' \tag{9}$$

and the position of the right ball is moving *toward* P, to the right:

$$x_{\text{right}} = \ell + R + Vt' - vt' \tag{10}$$

The right ball reaches its target when its position and P's position are equal:

$$\ell + 2R - vt' = \ell + R + Vt' - vt' \tag{11}$$

and canceling like terms,

$$R = Vt' \;\Rightarrow\; t'_{\text{right}} = \frac{R}{V} \tag{12}$$

which is the same time as t'_{left}. According to A, the balls hit their respective targets simultaneously, just as B is immediately abreast of her. The brothers observe the same. That's how things work in Galilean physics: simultaneity is the same for all observers in uniform motion.

Repeat the experiment, but this time using light instead of tennis balls, and Einstein's physics in place of Galileo's. Replace the tennis ball machines with a lamp midway between the brothers. (See Figure 3.)

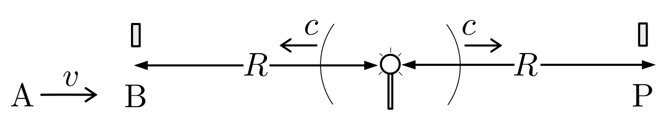

Figure 3: Relativistic simultaneity, A moving: the flash is equidistant from the detectors.

The lamp will emit a flash of light which propagates in a spherical wave front in all directions. Replace the two targets at B and P with two photoelectric cells, each linked to a stopwatch. The stopwatches start the moment the light flashes (call this $t = t' = 0$). A stopwatch ticks until its photocell is hit by the flash. Once again B and P are separated by a distance of $2R$, and A is a distance ℓ to the left of B, traveling towards B with a speed v. Exactly as before, the brothers believe (as the stopwatches will confirm) that each photocell is hit at the same moment, at a time

$$t = \frac{R}{c} \tag{13}$$

If A is to arrive alongside B as the left edge of the wave front hits B's photocell, the distance ℓ must be related to the time t as before;

$$\ell = vt \tag{14}$$

How does A observe the situation? As before, she regards herself as at rest, while the system of B, lamp, and P move towards her at a velocity $-v$. (See Figure 4.)

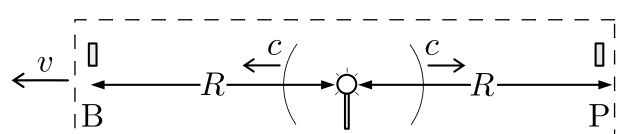

Figure 4: Relativistic simultaneity, B and P moving: the flash is closer to the right detector.

When the lamp flashes, it is at a position $\ell' + R'$ from A as she measures it. The position of B is given by

$$x'_{\text{B}} = \ell' - vt' \tag{15}$$

The speed of the light according to A is *not* $c+v$ but c. Unlike the Galilean relative speed of the tennis balls, the speed of light is the same for all inertial observers (as in the second postulate). Let x'_{left} be the position of the left edge of the flash's wave front. Then

$$x'_{\text{left}} = \ell' + R' - ct' \tag{16}$$

The flash hits the left photocell and its stopwatch stops when $x'_{\text{left}} = x'_{\text{B}}$. The time t'_B follows:

$$\ell' - Vt' = \ell' + R' - ct \;\Rightarrow\; t'_{\text{B}} = \frac{R'}{c-v} \tag{17}$$

In the same way, P's position is given by

$$x'_{\text{P}} = \ell' + 2R' - vt' \tag{18}$$

and the right edge of the flash's position by

$$x'_{\text{right}} = \ell' + R' + ct' \tag{19}$$

The flash hits the right photocell and its stopwatch stops when $x'_{\text{right}} = x'_{\text{P}}$, so time t'_P is found like this:

$$\ell' + 2R' - vt' = \ell' + R' + ct' \;\Rightarrow\; t'_{\text{P}} = \frac{R'}{c+v} \tag{20}$$

These two times are *not* the same; in particular,

$$t'_{\text{B}} > t'_{\text{P}} \tag{21}$$

This is very easy to understand physically. The speed of the wave front is the same, c, towards each photocell; but the one on the left is running away from the flash, while the one on the right is rushing towards it. So the one on the right is hit first. *What is simultaneous for the brothers is not simultaneous for their sister.*

It will be important to know the difference of these times:

$$\Delta t' = t'_{\text{B}} - t'_{\text{P}} = R'\Big(\frac{1}{c-v} - \frac{1}{c+v}\Big) = \frac{2vR'}{c^2 - v^2} = \frac{2vR'}{c^2}\cdot\gamma^2 \tag{22}$$

where γ is the ubiquitous factor in special relativity,

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \tag{23}$$

This quantity $\Delta t'$ is expressed in terms of primed quantities (t', R') which A measures. Her brothers measure these differently. First, from the consequences of relativity[7] come both length contraction ($\Delta x' = \Delta x/\gamma$) and time dilation ($\Delta t' = \gamma\Delta t$). That is,

$$\Delta t = \Big(\frac{1}{\gamma}\Big)\Delta t' = \Big(\frac{1}{\gamma}\Big)\Big(\frac{2v}{c^2}\cdot\frac{R}{\gamma}\cdot\gamma^2\Big) = \frac{2vR}{c^2} \tag{24}$$

This is a rule: If two clocks in the same frame, synchronized and separated by a horizontal distance L, are observed by someone moving horizontally with a speed v, she will observe that the leading clock is delayed by the quantity

$$\Delta t = vL/c^2 \tag{25}$$

One often hears this result expressed as a rule, "Leading clocks lag". The clocks will tick at the same rate (as they are in the same inertial frame of reference), but the clock in front (as determined by the velocity's

[7]Beautifully elegant and simple derivations of time dilation and length contraction may be found in Feynman's *Lectures*, freely available to read on line: `https://www.feynmanlectures.caltech.edu`, vol. 1, §15-4 (time dilation), §15-5 (length contraction). Helliwell (*op. cit.*) has similar derivations, as do many standard relativity texts.

direction) will register a lesser time, because it started later. This is the key to overcoming Dingle's objection, as will now be shown.

How two clocks can run slowly with respect to each other

Now imagine a third experiment. Swap the stopwatches for clocks, which start only when the flash hits them, and let all *three* siblings have a clock.

The clocks are synchronized in the brothers' reference frame by putting A, traveling at speed v towards B, at the appropriate distance ℓ away so that her photocell is hit by the same wave crest as B's just as she is alongside B. Call this Event 1. Let Event 2 correspond to when P and A are alongside each other. Finally, let the separation between the two brothers be L as measured in their coordinate system.

From the brothers' viewpoint, the times look like this:

	$t_{\rm B}$	$t_{\rm P}$	$t'_{\rm A}$
Event 1	0	0	0
Event 2	L/v	L/v	$L/(\gamma v)$

Table 1: The times according to B and P

($t'_{\rm A}$ is the time that the brothers observe on A's clock). The brothers' times for Events 1 and 2 should be self-evident; by design A's clock is arranged to start at the moment B's does, and B's is arranged to start at the same moment as P's. As far as the brothers are concerned, A's clock is synchronized with both their clocks.

For A's time at Event 2, the brothers work out how long it takes for A to move the distance L from B to P. This is easy; $L = vt$, so $t = L/v$. On the other hand, they observe that A's clock runs slowly with respect to theirs (time dilation):

$$\Delta t'_{\rm A} = \left(\frac{1}{\gamma}\right) \cdot \Delta t_{\rm B} = \left(\frac{1}{\gamma}\right) \cdot \Delta t_{\rm P} = \frac{L}{\gamma v} \tag{26}$$

The brothers can justifiably say to their sister, "Your clock runs slowly with respect to ours." But A is perfectly entitled to say that *she* is at rest, and the *brothers* are moving. So how can she say that *their* clocks are running slowly? Easily.

A's times on her own clock are straightforward. When she is at the same place as B, by arrangement both clocks start, reading 0. She believes the distance between the brothers is not L, but instead L/γ, because of Lorentz contraction. On the other hand, she believes that the B-lamp-P system is moving to her left at the very same speed, v, as the brothers think she moves past them. So the time required for the system to move from B to P past her is given by the distance over the time,

$$t'_{\rm A,\ Event\ 2} = \frac{L/\gamma}{v} = \frac{L}{\gamma v} \tag{27}$$

which is exactly what P reads on her clock, when they are face to face. And that is how things have to be. When they are at the same place at the same time, she must read the same time on his clock as he does on his own clock; he must read on her clock the same time as she does on her clock. But in general the two clocks display different times.

The interesting part is what A reads on the brothers' clocks at Event 1 (A and B coincide) and Event 2 (A and P coincide). First, consider P. From A's point of view, P's clock has a head start of vL/c^2, as above. On the other hand, when she is opposite P, they have to agree as to what is on each other's clock. P's time at Event 2 was worked out earlier to be L/v. When P and A are face to face, A must read the same on P's clock as P does: L/v.

What does A measure for B's times? When A and B coincide, they must agree as to what is on each other's clock. By arrangement, each reads 0 on their own clock and on their sibling's clock. When A and P coincide, A and P must both read L/v on P's clock, and $L(\gamma v)$ on A's clock. She also knows that B's clock is always behind P's by the quantity vL/c^2. Hence at Event 2, A must observe the time $(L/v) - (vL/c^2)$ on B's clock. Here are A's readings of the clocks.

	t'_{A}	t_{B}	t_{P}
Event 1	0	0	vL/c^2
Event 2	$L/(\gamma v)$	$(L/v) - (vL/c^2)$	L/v

Table 2: The times according to A

So how does A get to say that her brothers' clocks are running slowly with respect to hers?

The answer is that the clock reading at any moment is immaterial. What matters is the *interval* of time Δt or $\Delta t'$, the time between events; in this case, between Event 1 and Event 2. First, let's look at the brothers' point of view (the value of Δt_A following from time dilation):

$$\Delta t_{\mathrm{B}} = \Delta t_{\mathrm{P}} = \frac{L}{v} - 0 = \frac{L}{v}; \quad \Delta t'_{\mathrm{A}} = \Big(\frac{1}{\gamma}\Big)\Delta t_{\mathrm{B}} = \Big(\frac{1}{\gamma}\Big)\Delta t_P = \frac{L}{\gamma v} \tag{28}$$

As required, the brothers can say that their sister's clock runs slowly in the ratio of $1\colon\gamma$. What about the sister's times?

$$\Delta t'_{\mathrm{A}} = \frac{L}{\gamma v} - 0 = \frac{L}{\gamma v} \tag{29}$$

as expected. For B, A observes

$$\Delta t_{\mathrm{B}} = \left(\frac{L}{v} - \frac{vL}{c^2}\right) - 0 = \frac{L}{v}\left(1 - \frac{v^2}{c^2}\right) = \frac{L}{v}\cdot\left(\frac{1}{\gamma^2}\right) \tag{30}$$

In exactly the same way, for P, A observes

$$\Delta t_{\mathrm{P}} = \left(\frac{L}{v}\right) - \left(\frac{vL}{c^2}\right) = \frac{L}{v}\cdot\left(\frac{1}{\gamma^2}\right) \tag{31}$$

which is the same as she obtained for his brother. And that is how things must be, because they are in the same reference frame (they are at rest with respect to each other). More, we have

$$\Delta t_{\mathrm{B}} = \Delta t_{\mathrm{P}} = \left(\frac{1}{\gamma}\right)\cdot\frac{L}{\gamma v} = \left(\frac{1}{\gamma}\right)\Delta t'_A \tag{32}$$

Lo and behold, from A's point of view, her brothers' clocks run slowly compared to hers, in the very same ratio, $1\colon\gamma$ as the brothers observed her clock to run slowly with respect to theirs. Dingle called this *impossible*. It is not.

Unsurprisingly, this result can be obtained directly from the Lorentz transformations (which Dingle did not believe in). First, find A's coordinates in terms of the brothers' (Table 1). In what should be an obvious notation,

$$\begin{aligned}
t'_{\mathrm{A},1} &= \gamma\Big(t_{\mathrm{A},1} - \frac{vx_{\mathrm{A},1}}{c^2}\Big) &&= \gamma\Big([0] - \frac{v\cdot[0]}{c^2}\Big) &&= 0\\
x'_{\mathrm{A},1} &= \gamma(x_{\mathrm{A},1} - vt_{\mathrm{A},1}) &&= \gamma([0] - v\cdot[0]) &&= 0\\
t'_{\mathrm{A},2} &= \gamma\Big(t_{\mathrm{A},2} - \frac{vx_{\mathrm{A},2}}{c^2}\Big) &&= \gamma\Big(\Big[\frac{L}{v}\Big] - \frac{v\cdot[L]}{c^2}\Big) &&= \frac{L}{\gamma v}\\
x'_{\mathrm{A},2} &= \gamma(x_{\mathrm{A},2} - vt_{\mathrm{A},2}) &&= \gamma([L] - v\cdot[L/v]) &&= 0
\end{aligned} \tag{33}$$

Note that $x'_{\mathrm{A},2} = 0$, because A thinks she does not move. Note too that the time values agree with those in Table 1. Then as the brothers see it,

$$\begin{aligned} \Delta t &= t_{\mathrm{A},2} - t_{\mathrm{A},1} = \frac{L}{v} \\ \Delta t' &= t'_{\mathrm{A},2} - t'_{\mathrm{A},1} = \frac{L}{\gamma v} \\ \Delta t' &= \frac{\Delta t}{\gamma} \end{aligned} \tag{34}$$

Before working through the values in Table 2, some clarifications. The time $t'_{B,1}$ means: the time according to A when B is opposite A's position, namely 0. Similarly, $x'_{B,2}$ means: B's position according to A when P is opposite A's position. This is a distance of L/γ to the left of A's position, 0; $x'_{B,2} = -L/\gamma$. Here, then, are B's coordinates in terms of A's:

$$\begin{aligned} t_{\mathrm{B},1} &= \gamma\Big(t'_{\mathrm{B},1} + \frac{vx'_{\mathrm{B},1}}{c^2}\Big) &&= \gamma\Big([0] + \frac{v \cdot [0]}{c^2}\Big) &&= 0 \\ x_{\mathrm{B},1} &= \gamma(x'_{\mathrm{B},1} + vt'_{\mathrm{B},1}) &&= \gamma([0] + v \cdot [0]) &&= 0 \\ t_{\mathrm{B},2} &= \gamma\Big(t'_{\mathrm{B},2} + \frac{vx'_{\mathrm{B},2}}{c^2}\Big) &&= \gamma\Big(\Big[\frac{L}{\gamma v}\Big] + \frac{v \cdot [-L/\gamma]}{c^2}\Big) &&= \frac{L}{v} - \frac{vL}{c^2} \\ x_{\mathrm{B},2} &= \gamma(x'_{\mathrm{B},2} + vt'_{\mathrm{B},2}) &&= \gamma\Big(\Big[\frac{-L}{\gamma}\Big] + v \cdot \Big[\frac{L}{\gamma v}\Big]\Big) &&= 0 \end{aligned} \tag{35}$$

The time values agree with those in Table 2, found with physical arguments. Finally, here are P's coordinates in terms of A's:

$$\begin{aligned} t_{\mathrm{P},1} &= \gamma\Big(t'_{\mathrm{P},1} + \frac{vx'_{\mathrm{P},1}}{c^2}\Big) &&= \gamma\Big([0] + \frac{v \cdot [L/\gamma]}{c^2}\Big) &&= \frac{vL}{c^2} \\ x_{\mathrm{P},1} &= \gamma(x'_{\mathrm{P},1} + vt'_{\mathrm{P},1}) &&= \gamma([L/\gamma] + v \cdot [0]) &&= L \\ t_{\mathrm{P},2} &= \gamma\Big(t'_{\mathrm{P},2} + \frac{vx'_{\mathrm{P},2}}{c^2}\Big) &&= \gamma\Big(\Big[\frac{L}{\gamma v}\Big] + \frac{v \cdot [0]}{c^2}\Big) &&= L/v \\ x_{\mathrm{P},2} &= \gamma(x'_{\mathrm{P},2} + vt'_{\mathrm{P},2}) &&= \gamma([0] + v \cdot [L/\gamma v]) &&= L \end{aligned} \tag{36}$$

These time values also agree with those in Table 2. Note in particular that when A observes $t_{\mathrm{B},1} = 0$, she also observes $t_{\mathrm{P},1} = vL/c^2$, as was found earlier; the clocks of B and P, synchronized in their coordinate frame, are not synchronized in A's frame. A sees the intervals as

$$\begin{aligned} \Delta t' &= t'_{\mathrm{P},2} - t'_{\mathrm{P},1} = t'_{\mathrm{B},2} - t'_{\mathrm{B},1} = \frac{L}{\gamma v} \\ \Delta t &= t_{\mathrm{P},2} - t_{\mathrm{P},1} = t_{\mathrm{B},2} - t_{\mathrm{B},1} = \frac{L}{v}\Big(1 - \frac{v^2}{c^2}\Big) = \frac{L}{\gamma^2 v} \\ \Delta t &= \frac{\Delta t'}{\gamma} \end{aligned} \tag{37}$$

To recapitulate, what has been shown is that, according to both B and P, A is moving and A's clock runs slowly,

$$\Delta t' = \frac{1}{\gamma}\Delta t \tag{38}$$

while at the same time, according to A, B and P are moving, and their clocks run slowly:

$$\Delta t = \frac{1}{\gamma}\Delta t' \tag{39}$$

Despite Dingle's insistence that this is impossible, the clock of A runs slower than the clocks of B and P from their point of view, and the clocks of B and P run slower than the clock of A from her point of view. This is possible because of simultaneity is not absolute: it depends on the frame of reference.

Incidentally, just before Dingle's book came out, Hafele and Keating took two separate atomic clocks on round the world flights,[8] one eastward and one westward, all the while comparing by radio the times with an identical atomic clock on the ground ground. The eastbound clock was faster than the ground clock, and the westbound was slower, due to the earth's rotation. These experiments confirmed the relativistic slowing of the moving clocks, exactly as Einstein predicted.

What's wrong with Dingle's analysis?

There are two errors in Dingle's "proof" that relativity is inconsistent. The first is that Dingle only looks at the instant of a particular event. A particular moment is not a measurement; the clock can always be reset. Only an *interval*, unaffected by changes in the start time, is meaningful. Sfarti correctly addresses this mistake.

Sfarti correctly derives from the Lorentz transformations that those in frames moving relatively to each other will observe the other's clock running slowly with respect to their own. Dingle knew the argument, but unfortunately, refused to accept this because it seemed to him absurd on its face. His refusal is particularly peculiar because if one merely inverts the original set of Lorentz transformations (giving the pair (t', x') as functions of (t, x) to arrive at the set (t, x) in terms of (t', x'), one sees that for $x = 0$ and $x' = 0$, one obtains both $t = \gamma t'$ and $t' = \gamma t$. Yet these are not contradictory statements as the only place both x and x' are simultaneously zero is at $t = t' = 0$. (This conclusion does not tell you anything about the rates of the clocks; for that you need intervals, as has been argued.)

The second, fatal error is his misunderstanding to take into account how simultaneity works in relativity. In his "proof" he introduces four clocks (in addition to A, B, and P (called by him N), there is a fourth, H) but there's no reason to do so; it's easy enough to work out how things look to A's companion, and it doesn't add anything. He synchronizes N with B in a peculiar but legitimate way, and H with A in the same way, but only synchronizes A with B at a later moment when they coincide, and simply *assumes* that this also synchronizes A with N. As has been shown (twice), this is false. This author does not know if any of Dingle's adversaries replied with a calculation like this one, nor does he intend to spend any more time on Dingle or his erroneous conclusions, or anyone else arguing that relativity is bunk. Nature has answered in the affirmative.

Finally, addressing Sfarti's paper, the claim that $x = 0$ and $x' = 0$ are different conditions is not necessarily so. The standard way one synchronizes two reference frames K and K' moving relative to each other at constant velocity is to allow a light source fixed in one frame to emit a flash such that it reaches the origins of both frames ($x = x' = 0$) the moment they coincide, and to call this time $t = t' = 0$. It's only with such synchronized coordinate frames that the usual Lorentz transformations are applicable.

[8] J. C. Hafele and R. E. Keating, "Around-the-World Atomic Clocks: Observed Relativistic Time Gains", *Science* **177** (1974) 168-170; `https://en.wikipedia.org/wiki/Hafele-Keating_experiment`.