Review of: "Quantum mechanics and symplectic topology"

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Qeios review

Quantum mechanics and symplectic topology

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Clarification of concept of uncertainty is needed

This paper tries to use the uncertainty principle as a foundational principle for quantum mechanics in the sense that the Schrodinger equation can be derived from it. The inspiration comes the field of quantum information science (QIS). The basic idea is to define uncertainly of quantum states and define a measure able to distinguish two states so that uncertainty can be expressed quantitatively. The notion of fidelity in QIS is used for that. This approach fits in the modern data science philosophy and is actually very much in line with the old quantum theory of Einstein, Bohr, Sommerfeld and Ehrenfest.

The uncertainty principle refers to the variability in outcomes from measurements of conjugate variables, with position and momentum as the primary examples. The author proposes in the abstract and introduction to use the uncertainty relation as a fundamental principle. He proceeds to specify that a formulation of the principle on quantum states, i.e. wavefunctions, will be used. In quantum mechanics the wavefunction is not considered observable, i.e. measurable. Quantum tomography is a technique to use multiple measurements of identical systems to reconstruct a wavefunction from probabilities of measurements on variables that are the arguments of the wave function.

The “Squeezed coherent states” section then changes concepts again by introducing coherent states. While coherent states are a basis for arbitrary states, i.e. wavefunctions, they are not the full class of all possible states. The author then uses the parameters in the coherent states, positions and momenta, as the variables to be used in his presentation of the uncertainty principle and the further derivation of the Schrodinger equation. These variables are not the outcomes of measurements of position or momentum.

Hence the claim of the paper that the uncertainty principle is given a fundamental role becomes confusing, since it is not what is generally understood as the uncertainty relation.

Phase space and symplectic structure

In the “Finite distinguishability” section some corrections are needed. In classical mechanics, states are points in phase space and are therefore clearly distinguishable as either the same or different points, which is made possible by measurements with, in principle, infinite accuracy. The Liouville theorem is summarized correctly as stating that different points remain different. The next sentence is not precise in stating that trajectories cannot converge or diverge. The true statement is that they never intersect. They can diverge moderately or exponentially, as they do in chaotic systems.

It is stated in the paragraph starting with “In statistical mechanics” that all states become indistinguishable. The word “state” here means statistical state, i.e. the probability distribution on phase space, not the microscopic state that is a point in phase space. This needs to be written more carefully.

The paragraph starting “In quantum mechanics” about quantum states implies that states in quantum mechanics are not distinguishable, which is incorrect, or certainly misleading. The states are vectors in Hilbert space and they are either the same or different; the Hilbert space plays the role of phase space. It is true, because the state of a quantum system, i.e. the wavefunction, cannot be measured, that it is experimentally hard if not impossible to distinguish two quantum states that are close as measured in the Hilbert space geometry.

The ”Squeezed coherent states” section then introduces the phase space, that looks like classical phase space, of the coherent space position and momentum parameters and treats them as if they are the measured values obtained for quantum systems. This is confusing, as noted above.

The claim in the paragraph starting with “The appearance of complex-valuedness” is not clear and needs further clarification. It is not true that the position and momentum parameters in the coherent states take on complex values. So it is not obvious where this claim comes from.

The “Indeterminacy relation” section starts with “The coherent states … are the states which can be distinguished to the greatest resolutions.” This claim is not obvious and needs some substantiation.

Derivation of the Schrodinger equation

The definition of the symplectic capacity and the fidelity for comparing states and the subsequent derivation of the Schrodinger equation appear are, because of the previous remarks, not as general about quantum mechanics as claimed, but apply to the coherent state parameter space, which is not what is measured in experiments. The construction may need to be adjusted to connect it with the quantum mechanical wave functions and the uncertainty principle as it is defined in the textbooks of quantum mechanics.