Qeios

Peer Review

Review of: "Inverse Evolution Data Augmentation for Neural PDE Solvers"

Federica Ferrarese[1]

1. University of Ferrara, Italy

The manuscript presents a novel data augmentation strategy for training neural networks, leveraging inverse evolutionary strategies to bypass the need for numerically solving PDEs using explicit or implicit schemes. Indeed, explicit methods impose restrictions on the time step size, while implicit schemes are often computationally prohibitive. The proposed approach is validated through various numerical experiments on well-established test cases. Overall, I recommend the manuscript for publication, provided the authors address the following points to further refine and enhance its clarity.

Comments

1. When introducing the inverse PDE system on page 5 in the paragraph "Inverse Evolution", it is presented with an initial condition, which, in my opinion, should instead be a terminal condition. I suggest to check and correct it.

2. Paragraph "Data Generation Scheme": the overall idea is clear but in my opinion it is hard to be read. I suggest to revise the paragraph by considering of specifying the definition of input and output, which will be then used in the numerical experiments, and to the formula

L(U) = U \Delta t F (U)

as in [30].

3. Paragraph "Initialization of Inverse Evolution": from my understanding, the first time series introduced is obtained by directly solving the PDEs using forward methods. How are the other time series generated? For example, did you introduce uncertainty in the initial data? Additionally, I suggest modifying the notation for the n time series, as n is already used to denote the time steps.

doi.org/10.32388/B08AER

4. Eq. 7: how \lambda and C are defined?

5. Fig 1: I recommend providing a clearer explanation of what is shown in Fig. 1. Additionally, I suggest using different line styles or markers to distinguish the various lines more effectively.

6. Paragraph "Processing for data with Sharp Interfaces": how do you perform the scaling More in details, how did you define the constant a and C in the eq. on page 11?

7. Fig. 2: as for Fig.1, I suggest to use different line styles to mark the different lines.

8. Fig. 3-4-5: I suggest to specify what you are representing in each row. Additionally, please note that you haven't given a precise definition of input and output.

9. Sec 3.3-Sec 3.4: I suggest providing more details about the FNO abd UNet methods to make it accessible to readers who may not be familiar with it.

10. Tab 3: you state that the data augmentation strategy proposed in this paper offers advantages in terms of error reduction. However, when comparing the augmented and original datasets, this advantage is not clearly evident from the table. I suggest conducting additional numerical experiments where the performance of the proposed approach is more clearly demonstrated.

11. Paragraph "Results": I suggest adding a plot in Fig. 5 that represents the error between the real and computed solution. Although this is explained in the text and shown in Table 3, a visual representation could make it clearer.

12. Final questions: Have you considered using real data instead of generating them by directly solving the equations? Did you test your augmented strategy in terms of prediction of future steps?

Declarations

Potential competing interests: No potential competing interests to declare.