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Soliton Interpretation of Quantum Theory

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Abstract. This article proposes an interpretation of quantum physics based on the theory of solitons. According to this interpretation, the elementary particle (in particular, the electron) is a soliton solution of the system of nonlinear equations, while the linear equations of quantum mechanics for the wave functions represent the boundary conditions for soliton solutions. The nonlinear equations for the quantum electron are hypothesized to be the usual Maxwell equations in which the charge and current densities are expressed through quadratic combinations of the electromagnetic field strength. The complex wave function describing the motion of the electron in this case is the usual electromagnetic wave, where the real part is the electric field strength, and the imaginary part is the magnetic field strength. Soliton equations, Maxwell equations and quantum equations are easily written using 3+1 Pauli matrices, which indicates that the 3+1 system of coordinates of space and time is a natural realization of the particle-wave soliton world. The proposed interpretation allows combining both the Copenhagen interpretation and Bohm's theory of "hidden" variables.

Keywords: quantum theory, solitons, nonlinear Maxwell equations, Pauli matrices, wave functions, elementary particles

1. Introduction

Quantum theory has a developed and consistent mathematical apparatus describing all known facts and their consequences in a quite accurate way. Still, the basic, so called "Copenhagen" interpretation of quantum theory, presented in the works of Niels Bohr and Werner Heisenberg, is not as clear and understandable. The problem is that equations of quantum mechanics are written for wave function, while physical quantities characterizing a particle (coordinate, momentum, energy) are averaged over the square of wave function. It was this averaging that led the founders of quantum mechanics to a probabilistic interpretation of quantum phenomena. In his Nobel lecture Heisenberg said (Heisenberg, The development of quantum mechanics, 1933): «The statistical character of the laws of quantum mechanics, however, becomes apparent in that an accurate study of the energetic conditions renders it impossible to pursue at the same time a particular event in space and time». Polemicizing with him, Albert Einstein et al (Einstein, Podolsky, & Rosen, 1935) referred to the fact that physical reality must be described by predictable quantities. Since the wave function does not allow unambiguously determining the momentum and coordinate of a particle, it «does not provide a complete description of the physical reality», raising the need for another theory to provide this description.

Supporters of the statistical interpretation insisted that the physics of the microcosm, described by quantum equations, is quite different from the macrocosm that we live in and is governed by its own laws, different from those we are accustomed to. Bohr wrote (Bohr, 1963): «Notwithstanding the power of quantum mechanics as a means of ordering an immense amount of evidence regarding atomic phenomena, its departure from accustomed demands of

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causal explanation has naturally given rise to the question whether we are here concerned with an exhaustive description of experience». Heisenberg, in "Physics and Philosophy. The Part and the Whole" (Heisenberg, Physics and Philosophy. The Revolution in Modern Science, 1958) sets forth in sufficient detail the formulations of the opponents of the Copenhagen interpretation but ends his critical analysis with the following phrase: «The ontology of materialism rested upon the illusion that the kind of existence, the direct “actuality” of the world around us, can be extrapolated into the atomic range. This extrapolation is impossible, however».

Subsequently, many scientists tried to explain the statistical nature of quantum theory. The most popular interpretation of impossibility to simultaneously measure momentum and coordinates of the particle is usually based on impossibility to exclude the influence of measurement. Determining the exact coordinate of the particle introduces such a distortion that the value of momentum becomes unpredictable. Max Born in his philosophical work (Born, 1953) dedicated to explanation of the statistical nature of the quantum world, said: «The observation of atomic phenomena needs instruments of such sensitivity that their reaction in making measurements must be taken into account, and, as this reaction is subject to the same quantum laws as the particles observed, a degree of uncertainty is introduced, which prohibits deterministic prediction». However, the uncertainty relation is a consequence of the wave nature of the equations, not quantum theory: the smaller the region in which the wave is localized, the wider its frequency spectrum. Consequently, the problem is again in the interpretation of the wave function - whether it is a physical phenomenon or a function for calculating probability.

Besides explanation of a statistical nature of the quantum world through influence of measurements, it is worth mentioning several other interpretations. One of such interpretations, based on quantum ensembles, was proposed by Russian scientist Dmitry Blokhintsev (Heisenberg also mentioned this idea). According to this interpretation, it is necessary to consider not separate quantum particles, but their totality - ensembles, and then statistical behavior will relate to a large set of particles, which is quite observable and explainable behavior: "the wave function is not a quantity determining the statistics of any special dimension; it is a quantity determining the statistics of a quantum ensemble" (Blokhintsev, 1966). The ensemble approach was used by Peter Holland trying to "match" the Copenhagen interpretation with the interpretation of Louis de Broglie and Bohm (Holland, 1995). A similar, but more exotic interpretation was proposed by Hugh Everett (Everett, 1957), who proposed to consider a real system as a superposition of an infinite number of quantum wave systems, where "all elements of the superposition exist simultaneously, and the whole process is perfectly continuous", instead of the influence of the measurement process. Later this idea formed the basis of the so-called many-worlds interpretation (DeWitt, 1970).

Heisenberg divided (Heisenberg, Physics and Philosophy. The Revolution in Modern Science, 1958) all scientists who disagreed with the basic (Copenhagen) interpretation of quantum theory into three categories. The first one included those offering different interpretations which do not consider the physical essence of quantum theory but are only concerned about its philosophical understanding. The second group included those who do not argue with the experimental proofs of the Copenhagen school but try to find critical points in the quantum theory itself. Finally, to the third group Heisenberg referred those who, as he said, «expresses rather its general dissatis-faction with the results of the Copenhagen interpretation and especially with its philosophical conclusions, without making definite counterproposals». He also included Einstein to this third group, who opposing the statistical nature of quantum physics exclaimed in one of his letters that "God does not play dice with the universe" (Weisberger , 2019).

According to Heisenberg, none of these groups of opponents of the Copenhagen school has a chance of success, and one must simply accept the fact that nature on the micro-level is structured quite differently and cannot be reduced to the ideas of nature that one has on the macro-level. Consequently, we should not look for any interpretation other than the Copenhagen one. Heisenberg was right - after heated debates of the first decades of quantum theory development the number of active opponents of the Copenhagen school has greatly diminished. Recently either new methodologies and frameworks of statistical theory (Omnes, 1994), a kind of neo-Copenhagen interpretation, or various philosophical investigation of aspects of quantum theory interpretation have appeared (Adlam, 2022). Nevertheless, a new interpretation of quantum theory is possible, and we shall focus on it.

2. "Hidden" variables and the role of wave function

The most popular hypothesis, which is an alternative to the Copenhagen interpretation, is probably David Bohm's hypothesis of "hidden" variables. According to it, the motion of particles and fields at the microlevel is described by "hidden" variables in a way that allows to determine both the coordinate and its momentum, as in classical physics, at the same time. But due to the fact that at the macrolevel we interfere with the system and average some of "hidden variables", the description of the world becomes incomplete and results in uncertainty relations in particular. Although one cannot take into account all variables of the atomic structure, it is always possible to describe the structure of the microcosm in more detail. Bohm writes (Bohm, 1952): «We should never expect to obtain a complete theory of this structure, because there are almost certainly more elements in existence than we possibly can be aware of at any particular stage of scientific development. Any specified element, however, can in principle ultimately be discovered, but never all of them».

The hypothesis of "hidden" variables was supposed to solve the problem of violating local realism in quantum mechanics, which is known as the Einstein-Podolsky-Rosen paradox (Selleri, 1988). The interaction of two wave functions describing quantum particles (for example, in scattering) is non-local in nature, because the wave functions have no physical meaning and are not limited by the speed of propagation, as in the case of interaction of the particles themselves, including photons. For this reason, the supporters of the hypothesis of "hidden" variables believed that the problem with a lack of locality in the interaction of quantum particles is due only to the fact that we do not know additional variables that the wave function depends on.

The problem of local realism for quantum theory was failed to be solved experimentally for a long time, until in 1964 John Stuart Bell proposed a checking mechanism. He derived numerical conditions, later called Bell's inequalities, which must be fulfilled if the interacting wave functions have additional variables (Aspect, 1999). Experiments that were carried out on the basis of Bell's methodology demonstrated that local realism is not fulfilled in the interaction of quantum particles (in particular, photons). These experiments have been improved and tested until recently (experimenters Alain Aspect, John Clauser and Anton Zeilinger were even awarded with the Nobel Prize for this in 2022 (Pioneering quantum information science, 2022)), and all results were unambiguous - there are no "hidden" variables, the Copenhagen interpretation is the only true one.

However, the results of experiments on checking of Bell's inequality show only that there are no "hidden" variables in the wave function, and it is the linear equations of quantum mechanics that do not satisfy the principles of local realism. However, this does not mean that there is no other description of dynamics of elementary particles, without linear equation for wave functions. It is just that such description has not been found yet. One of the assumptions, alternative to "hidden" variables, is the assumption that there is another dimension of space-time, in which the wave function has a realistic nature. Alyssa Ney in her book (Ney, 2021) says: "...quantum theories tell us that seemingly spatially separated objects, connected by irreducible relations of quantum entanglement and capable of instantly affecting each other over spatial distances, are in fact manifestations of a deeper, multidimensional reality in which spatial individuation and nonlocal influence disappear. "

It is important to mention that physicists have been searching for ways to abandon the equations for wave functions violating local realism for a long time, albeit in vain. For example, Dmitri Blokhintsev described his search for such solutions as follows (Blokhintsev, 1966). The Schrödinger equation for a single electron is written through the wave functions ψ:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\vec{\nabla}^2\psi + V\psi,$$

which can be represented as the product of the square root of the charge density and the exponent of the real action function θ as follows $\psi = \sqrt{\rho}\cdot e^{i\frac{\theta}{\hbar}}$. This form automatically leads to the relation $\psi\cdot\psi^* = \rho$. Substituting this form of the wave function into the Schrödinger equation and separating the real and imaginary components results in two equations:

$$\frac{\partial\rho}{\partial t} + div\left(\rho\frac{1}{m}\vec{\nabla}\theta\right) = 0$$

$$\frac{\partial\theta}{\partial t} + \frac{1}{2m}(\vec{\nabla}\theta)^2 + V + \frac{\hbar^2}{m}\left[\frac{\vec{\nabla}^2\rho}{\rho} + \frac{(\vec{\nabla}\rho)^2}{\rho^2}\right] = 0$$

The first equation describes the law of conservation of charge, where the value $\vec{j} = \rho\frac{1}{m}\vec{\nabla}\theta$ is the current density, and the second equation for the action function θ is nonlinear. Blokhintsev believed that it is the nonlinearity of the second equation that prevents it from being used to describe the electron dynamics. Such formulation does not allow explaining the effect of wave superposition followed from the linearity of quantum equations. At the same time, we should note that the appearance of nonlinear equations in transition to real physical quantities is important in itself. Further, we will show that non-linearity is not only no a problem, but on the contrary, it can generate solutions in which wave functions and their superposition will be part of solving non-linear equations.

3. Wave dualism and soliton theory

Proposing his hypothesis of "hidden" variables, Bohm believed that the elementary particle is the very particle affected by the wave function. Another point of view was presented by Louis de Broglie, who back in the early last century believed that the particle must have a wave nature (Broglie, 2007). It is no coincidence that his ideas also formed the basis of the wave equations derived by Schrödinger. Louis de Broglie was looking for a wave interpretation of quantum theory. The additionality principle of coordinate and momentum is a consequence of the wave equation, and if the wave equations described the particle, there would be no problem with the interpretation of uncertainty relations. However, it was not possible to find solutions that would implement wave-particle dualism. One way of finding it was related to the description of the wave-pilot, affected by the particle, which was more in line with the Bohm's theory.

There is an interesting fact that the same Bell, who derived the conditions for testing local realism and implicitly helped the Copenhagen school to triumph, praised the work of Louis de Broglie. In his article (Bell, 1982) Bell exclaims: «Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?» One should pay tribute to perseverance of Louis de Broglie as at the end of his life he returned to the original definition of the particle as a wave with the properties of a particle. But he still could not find a solution.

At the same time, the wave-particle dualism is well known in the theory of solitons. The only difference from quantum mechanics is that wave equations in the theory of solitons must be nonlinear; the nonlinearity is a condition for forming a stable wave packets spreading like a particle. Generally speaking, nonlinearity in a distributed medium can often serve as a condition for the formation of stable waves or fronts (for example, in spreading of discharges in low-temperature plasma (Slavin & Sopin, 1992)), but soliton waves have not only the properties of rectilinear stable propagation, but also meet the laws of conservation of momentum, energy, etc., including the interaction with each other. Such unique properties are related to the peculiarity of solitons to scatter waves in such a way as to be "transparent" for them. It is this feature that will be used below for soliton interpretation of quantum theory.

The theory of solitons has a long history, dating back to the end of the 19th century (Allen, 1998). However, it was not until 1967 that Clifford Gardner and colleagues (Gardner, Greene, Kruskal, & Miura, 1967) found a method for obtaining analytical solutions of soliton solutions, which initiated a huge number of studies in both mathematics and applied fields (Manukure & Booker, 2021). Currently, the field of application of solitons is wide, using soliton solutions to model nonlinear processes both in hydrodynamics (where the soliton was actually first described by John Scott Russell) and in optics (Dudley, Genty, Mussot, Chabchoub, & Dias, 2019). Soliton solutions are used in medicine to model nerve impulses (Heimburg, 2022), in plasma physics (Bandyopadhyay & Sen, 2022) and even to describe nonlinear processes in electric transmission lines (Kengne, Liu, English, & Malomed, 2022).

The method used to obtain such a large number of analytical solutions is based on the solution of the inverse scattering problem. The assumed solution of the nonlinear equation is considered to be a potential barrier the waves are scattered on and the eigenvalues of bound states are calculated according to. The presence of bound states just testifies to the possibility of the existence of a soliton solution. The eigenvalues are used to reconstruct the solution of the nonlinear equation. Surprisingly, as in quantum theory, very few soliton researchers analyze physical meaning of the method of the Inverse Scattering Transform (IST). Ablowitz and Sigur probably advanced the most in understanding the meaning of IST method in their book "Solitons and the Inverse Problem Method" (Ablowitz & Segur, 1981), drawing parallels between the Fourier transform method for linear differential equations and the IST method for nonlinear equations.

Let us consider this similarity by the example of one-dimensional solitons spreading along the x-axis for time t. The direct Fourier transform method involves finding solutions of differential equations in the form of superpositions of eigenfunctions corresponding to the relations: $\frac{\partial}{\partial x}\tilde{\varphi} = i \cdot k \cdot \tilde{\varphi}$ and $\frac{\partial}{\partial t}\tilde{\varphi} = i \cdot \omega \cdot \tilde{\varphi}$,

where k is the wave number, ω is the frequency, and i is the imaginary unit. By substitution of eigenfunctions $\tilde{\varphi} = \mathrm{C}(\omega, k) \cdot e^{i\omega t + ikx}$into the linear differential equations, the multipliers depending on coordinate and time are reduced, and the relationship of wave numbers to frequency, called the dispersion relation, can be determined $\omega\ (k)$. In the general case (for example, for two- or three-dimensional equations), other special functions are used instead of trigonometric ones. The dispersion relation allows reconstructing the desired solution as an integral over the eigenfunctions, this operation is called the inverse Fourier transform.

It is easy to understand that in the case of nonlinear equations the Fourier transform method will not work, since eigenfunctions from solutions of linear equations in quadratic or other nonlinear terms in the general case will not be reduced at the substitution. However, it possible to try to find eigenfunctions that would satisfy, if not all, then at least a certain type of nonlinear equations. This is done by using similar equations for eigenfunctions, but instead of wave number and frequency, the quantities (K and Ω) are used, which are operators (for example, represented in matrix format) which commutator $[K, \Omega] = (K \cdot \Omega - \Omega \cdot K)$ is non-zero:

$$\frac{\partial}{\partial x}\tilde{\varphi} = i \cdot K(\zeta, x, t) \cdot \tilde{\varphi} \tag{2}$$

$$\frac{\partial}{\partial t}\tilde{\varphi} = i \cdot \Omega(\zeta, x, t) \cdot \tilde{\varphi}, \tag{3}$$

where ζ is the complex wave number. For these relations to be joint, the following equality must be satisfied:

$$\frac{\partial}{\partial t}K - \frac{\partial}{\partial x}\Omega + i \cdot [K, \Omega] = 0, \tag{4}$$

which is obtained by differentiating equation (2) by *t* and equation (3) by *x* and equating the right-hand sides.

The form of equation (4) shows why the values K and *Ω* should not commute with each other. If they do commute, i.e. $[K, \Omega] = 0$, then K and *Ω* must either be constant, which corresponds to the Fourier transform or have too simple dependence on *x* and *t*. But if the matrices K and *Ω* do not commute ($[K, \Omega] \neq 0$), in the general case the equation (4) is nonlinear and, depending on the type of matrices, describes a large class of equations, including soliton ones. It is for nonlinear equations (4) that we can create an analogue of the Fourier transform where equations (2) and (3) will serve as eigenfunctions.

Specifics of inverse scattering eigenfunctions is that they always represent vectors (columns) since K and *Ω* are operators (matrices), i.e. such eigenfunctions consist of at least two values, as presented below:

$$\tilde{\varphi} = \begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \end{pmatrix} \tag{5}$$

If the matrix $K(\zeta, x, t)$ is represented as:

$$K(\zeta, x, t) = \begin{pmatrix} -\zeta & -iq(x,t) \\ -ir(x,t) & \zeta \end{pmatrix}, \tag{6}$$

where $r(x,t)$ and $q(x,t)$ are functions constituting the nonlinear equation, then if $r(x,t) = -1$, the soliton Korteweg-de Fries equation can be obtained, and if $r(x,t) = -q^*(x,t)$ - the nonlinear Schrödinger equation (Ablowitz & Segur, 1981).

Equations (2), (3) are linear in $\tilde{\varphi}$ and well known (exactly due to the quantum theory). The value in the right-hand side is usually a potential, while solving such equations can be reduced to the problem of scattering of the wave function on the potential. According to the theory of such equations, scattering on a potential may produce bound states resulting in a wave passing through the potential without distorting the form (with a possible change in phase). On the one hand, this happens due to the nonlinearity, which "deforms" the waves, and, on the other hand, the fact that the eigenfunction is described not by one but by two waves $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$, the deformations of which compensate each other. This effect can definitely occur only at certain potentials.

It is this similarity that was used in the Inverse Scattering Transform method. If one can find the solutions for $r(x,t)$ and $q(x,t)$ which are both localized in space (rapidly decreasing at infinity) and "transparent" for the wave functions, then the superposition of these functions can produce a solution to describe a soliton solution. Thus, within the inverse scattering method, we should first solve the scattering problem (searching for functions for which the variable part of equations (2) is "transparent") and then solve the inverse problem to find the complete solution considering the dispersion relations. We can say that the IST method finds a soliton solution consisting of eigenfunctions "transparent" to the solution itself. This is the reason of such a unique stability of the soliton: it propagates with the same velocity without changing its shape (within the phase), and even when "colliding" with another soliton after interference, it restores its velocity and shape.

Let us highlight an important aspect of the soliton theory. Bound states appear in the presence of a discrete spectrum of wave numbers with an imaginary component. Thus, in the case of the nonlinear Schrödinger equation, this corresponds to the presence of the imaginary part of the wave number $\eta = Im(\zeta)$ (Ablowitz & Segur, 1981). In this case, the presence of a bound state does not depend on the position of the soliton, since the method of the inverse scattering problem describes the scattering of wave functions, the asymptotes of which are taken far from the soliton, while the soliton itself is localized in a small region. If equations (2), (3) will be integrated only for a continuous spectrum $\xi = Re(\zeta)$ as below:

$$\varphi_{1,2}(x;t) = \int e^{\pm i\xi x} \cdot \tilde{\varphi}_{1,2}(\xi;x;t)d\,\xi \qquad ,$$

leaving the values of the discrete spectrum as a parameter, then, for such integral wave functions $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ (which we shall write without the tilde at the top), we will most likely have a linear equation that depends only on the wave number of bound states of the soliton η:

$$\hat{P}\left(\frac{\partial}{\partial x}; \frac{\partial}{\partial t}\right) \cdot \varphi = F(\eta) \cdot \varphi, \tag{7}$$

where $\hat{P}$ is an operator, representing a function of derivatives and corresponding to the dispersion relation for a continuous wave number spectrum, while $F(\eta)$ is a function depending on the wave numbers of the bound states. This linear equation describes the wave functions for which the solitons will be "transparent," and the discrete spectrum is represented by the value of η. This very equation that can claim to be the analogue of the quantum theory equations.

A special property of solitons, besides describing soliton waves which are propagated at a constant speed, is that soliton equations have an infinite number of conservation laws (Xu, 1993): the law of conservation of density, energy, momentum, etc. That is, they fully possess the wave-particle duality that Louis de Broglie spoke of. However, despite the fact that the mathematical apparatus for solving soliton equations is now quite developed, unique analytical solutions have been found only for one-dimensional equations. Boris Malomed (Malomed, 2019) provides a comparative analysis of the two- and three-dimensional solutions investigated in the literature and shows that, in contrast to one-dimensional equations, no stable solutions have been found for them yet. In the case of three-dimensional solitons, the vortex component arises, which further increases the instability.

At the same time, the absence of solutions for three-dimensional solitons may just testify to the fact that, in contrast to the one-dimensional case, there are very few such solutions in three-dimensional space, maybe even the only one, which it has yet to be found. However, we can say now that the particles may well be described by nonlinear equations, with the classical quantum equations, which are linear, most likely a condition for the existence of a soliton solution, i.e. describing the eigenfunctions away from the soliton itself. The similarity between quantum wave functions and eigenfunctions of soliton solutions becomes even more obvious given the fact that the solution of soliton equations can be expressed through the squares of its eigenfunctions (Tian, Feng, & Liu, 2022).

4. Pauli matrices and 3+1 space-time

In soliton theory, complex 2x2 matrices, which can be expressed through Pauli matrices, occur naturally:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};\ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Any complex 2x2 matrix can be written as follows:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{a+d}{2} \cdot \sigma_0 + \frac{b+c}{2} \cdot \sigma_1 + i\,\frac{b-c}{2} \cdot \sigma_2 + \frac{a-d}{2} \cdot \sigma_3$$

In particular, soliton equation (2) can be written as follows:

$$\frac{\partial}{\partial x}\varphi = -i\zeta\sigma_3 \cdot \varphi + \left[\frac{q+r}{2} \cdot \sigma_1 + i\,\frac{q-r}{2} \cdot \sigma_2\right] \cdot \varphi \qquad (2a)$$

In this equation, it is no coincidence that the nonlinear terms depending on *q* and *r* are expressed through one Pauli matrix, while the term with the wave number ζ through another. Although the equation is one-dimensional, it mimics the propagation of a wave with three perpendicular polarizations to which the Pauli matrices correspond. Generally speaking, Pauli matrices describe 3+1 dimensional space, where $\sigma_1, \sigma_2, \sigma_3$ are spatial coordinates and σ_0 is the time coordinate. The four-dimensional vector $\hat{A} = (A_0; A_1; A_2; A_3)$ can be written as a two-dimensional matrix:

$$\hat{A} = A_0 \cdot \sigma_0 + A_1 \cdot \sigma_1 + A_2 \cdot \sigma_2 + A_3 \cdot \sigma_3 \qquad (8)$$

Since Pauli matrices are Hermitian conjugate $\sigma_\mu = \sigma_\mu{}^+$ (where μ = 0, 1, 2, 3), in the case of real values A_μ, the $\hat{A}$ matrix is also hermitian conjugate.

Apparently, using Pauli matrices will be more natural for the three-dimensional soliton equation rather than for the one-dimensional soliton. The waves reflected from the soliton will be scattered in three directions, but due to interference, the resulting wave function will be the one that the soliton is "transparent" for. Most likely, the vortex component will also play a significant role in the interference of wave functions.

However, Pauli matrices are important not only in the theory of solitons, but also in quantum mechanics, including relativistic mechanics. Quantum equations are just as naturally written through Pauli matrices. If we take the four-component wave function of an electron described by complex functions $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ and make two two-component vectors from it:

$$\psi^+ \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \varphi_1 + \varphi_3 \\ \varphi_2 + \varphi_4 \end{pmatrix} \text{ и } \psi^- \equiv \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \varphi_1 - \varphi_3 \\ \varphi_2 - \varphi_4 \end{pmatrix}, \qquad (9)$$

the quantum equations for a relativistic electron of mass *m* can be written in the following compact way:

$$\hat{D}_+ \psi^+ = -i\frac{mc}{\hbar}\psi^- \qquad (10)$$

$$\hat{D}_- \psi^- = -i\frac{mc}{\hbar}\psi^+ \qquad (11)$$

where *c* is the speed of light, ħ is Planck's constant and the following term is introduced:

$$\hat{D}_\pm = \frac{\partial}{\partial x_0} \pm \sum_{k=1}^{3} \sigma_k \frac{\partial}{\partial x_k} \qquad (12)$$

Here $x_0 = ct, x_1, x_2, x_3$ are time and space coordinates.

For description of the relativistic electron, 4x4 Dirac matrices are usually used, which allows combining (10) and (11) into one equation. However, this is unnecessary, and only reduces the universality, since Dirac matrices consist mostly of zeros. In order to make the relativistic equation compact, we should just make a 2x2 matrix from the wave functions, for example:

$$\Psi = \begin{pmatrix} \psi_1 & -\psi_4^* \\ \psi_2 & \psi_3^* \end{pmatrix}, \qquad (12)$$

where * means the complex conjugation. Then, the Dirac equation can also be written as a single equation using Pauli matrices only:

$$\widehat{D}_-\ \Psi = \frac{mc}{\hbar}\sigma_2 \Psi^* \sigma_1 \qquad (13)$$

Finally, the last surprising property of Pauli matrices. They are also a natural tool for describing Maxwell's equations for the electromagnetic field. For this, we should write the sum of the magnitudes of the electric E_k and magnetic H_k field strengths, as well as the magnitudes of the charge ρ and the current j_k using the Pauli matrices:

$$\widehat{M} = \sum_{k=1}^{3}(E_k - iH_k) \cdot \sigma_k;\ \hat{Q} = \rho + \frac{1}{c}\sum_{k=1}^{3} j_k \cdot \sigma_k \qquad (14)$$

Then Maxwell's equations will be written in the same compact form as the quantum equations:

$$\widehat{D}_-\ \widehat{M} = 4\pi\hat{Q} \qquad (15)$$

At this, the magnitudes of the electromagnetic field strengths are also easily expressed through the potential $\hat{A} = A_0 + \sum_{k=1}^{3} A_k \cdot \sigma_k$:

$$\widehat{M} = \widehat{D}_+\ \hat{A} \qquad (16)$$

Since the magnitude $\widehat{M}$ is expressed only through $\sigma_1,\ \sigma_2,\ \sigma_3$, the relation $Sp\widehat{M} = 0$ is fulfilled, which automatically leads to the gauge relation: $\frac{\partial}{\partial x_0}A_0 + \sum_{k=1}^{3}\frac{\partial}{\partial x_k}A_k = 0$. Similarly, of the condition $Sp\big(\widehat{D}_+\widehat{D}_-\ \widehat{M}\big) = 0$, should be the equality $\frac{\partial}{\partial x_0}\rho + \frac{1}{c}\sum_{k=1}^{3}\frac{\partial}{\partial x_k}j_k = 0$, which corresponds to the law of conservation of charge. We should also note that Maxwell's equations, in spite of the fact that they were discovered long before quantum theory, have not changed in quantum physics. Moreover, they fit seamlessly into quantum field equations by adding to the derivatives of the potential: $\frac{\partial}{\partial x_\mu} + eA_\mu$.

Thus, algebra based on Pauli matrices turns out to be natural both for soliton theory, quantum theory and electromagnetic field theory as well. Such coincidence may seem a surprising fact or, on the contrary, be considered as a consequence of something important. This "important" aspect is most likely the possibility of existence of stable solitons. We can suppose that the existence of stable and identical particles is a condition for the existence of our entire world. Therefore, the 3+1 dimensional space that we live in is due to the fact that it is this dimension that stable particles appear in. At this, electromagnetic field and quantum equations must describe dynamics of these stable particles - solitons.

5. Interpretation of quantum equations

Summing up the above mentioned, the following interpretation of quantum theory can be proposed allowing to combine soliton theory, quantum particle theory, electromagnetic field theory and fill them with physical and comprehensible meaning at the same time. In quantum theory, charge and current densities are a quadratic form of wave functions. Therefore, the role of the nonlinear equation generating solitons (in particular, electrons) similar to (4) is most likely performed by Maxwell equations (15), the right-hand side of which is equal to the square of the soliton solution eigenwave functions. These eigenwave functions are similar to functions (5). These wave functions have a physical meaning. Since they describe wave functions satisfying Maxwell's equation, they most likely represent electromagnetic waves, which being away from the particle (where charge and current are zero) are described by plane waves, in which the vectors of electric and magnetic field strengths are perpendicular, coincident in magnitude and shifted in phase. For example: $\tilde{\varphi} \sim E_1 + iH_2$, where $E_1 = B \cdot \cos(\omega t - k_3 x_3)$ and $H_2 = B \cdot \sin(\omega t - k_3 x_3)$.

It is easy to see that the product of the wave functions $\tilde{\varphi} \cdot \tilde{\varphi}^*$ is not only real, as it should be for the charge, but it also does not depend on the wave numbers. It can be opposed that the number of combinations from the vectors of electric and magnetic field strengths is 6 (the number of many pairs that can be made from the perpendicular vectors of the electromagnetic field), and the wave functions have four values. Apparently, the two combinations of electric and magnetic fields are associated with the vortex component, which is probably responsible for describing the electron spin, resulting in one of the directions becoming highlighted.

We should note that $\tilde{\varphi}$ wave functions are not the wave functions involved in the quantum equations. It is reasonable to interpret the quantum equations for the electron as equations for wave functions integrated over a continuous spectrum (earlier we agreed to denote them without the tilde). Such wave functions carry information about bound states or for "transparency," as in equation (7), rather than about specific soliton parameters related, for example, to its position. At this, the wave number responsible for the bound state, for the electron, is most likely represented by: $\eta = \frac{mc}{\hbar}$, which value can only be obtained by solving nonlinear equations. In this case, the integrals over the volume (V) of the squares of the wave functions of the nonlinear equations and the averaged wave functions should be equal:

$$\int dV\, \tilde{\varphi}\tilde{\varphi}^* = \int dV\, \varphi\varphi^*$$

This explains why quantum equations have statistical interpretation. Wave functions of quantum equations describe conditions of existence of particles - solitons, but not solitons themselves. If these wave functions are normalized to one (one particle), then their square will describe average probability of finding a particle in one point or another. But since quantum equations are linear, normalization does not change the solutions, that is why superposition of solutions of quantum equations is possible, which is not explained in any way from the statistical interpretation point of view. Figure 1 shows schematically where certain equations (Maxwell equations for the free charge, nonlinear soliton equations or quantum equations defining the conditions of solitons) are fulfilled.

In the shaded area, where no quantum particle (electron) is present at the moment, the electromagnetic field equations of the moving charged particle are fulfilled. In the region where the electron is present, Maxwell's equations become nonlinear and describe a soliton. Quantum equations for averaged wave functions are valid at the boundary of these areas, and determine the asymptotes of nonlinear equations indicating the presence of a particle in principle at any point in time. Quantum equations do not describe the dynamics of solitons, but only the conditions of their formation (the presence of bound states). Such interpretation explains why quantum equations can describe all quantum phenomena, including scattering of particles on each other, birth of electron-positron pairs, etc. All these phenomena are related to different bound states of wave functions, and are easiest to describe by linear equations, which allow calculating conditions of certain processes.

Let us note that the proposed interpretation is somewhat similar to the theory of "hidden" variables, but with an essential difference. Quantum equations describe wave functions averaged only by parameters not affecting scattering of real electromagnetic waves on a particle. They only describe the conditions required for the occurrence of soliton solutions. This is the reason why experiments to identify "hidden" variables by means of scattering cannot reveal limitations of the Copenhagen interpretation, it will be absolutely correct. Quantum equations describe the state of particles only at the boundary and therefore cannot describe the local interaction. Hence, the violation of local realism occurs.

Since electron dynamics cannot be described quantum equations, this results in the uncertainty relation between coordinate and momentum of a particle. It can be demonstrated by using the same Figure 1. If the unshaded region is reduced, the condition for the soliton solution is satisfied for the increasing spectrum of momentum (since quantum equations are wave equations). But this uncertainty relation concerns only the conditions for the existence of the soliton, not its dynamics. The real electron, described by nonlinear equations, has a definite form (a soliton wave) allowing to define both its position and its velocity (momentum).

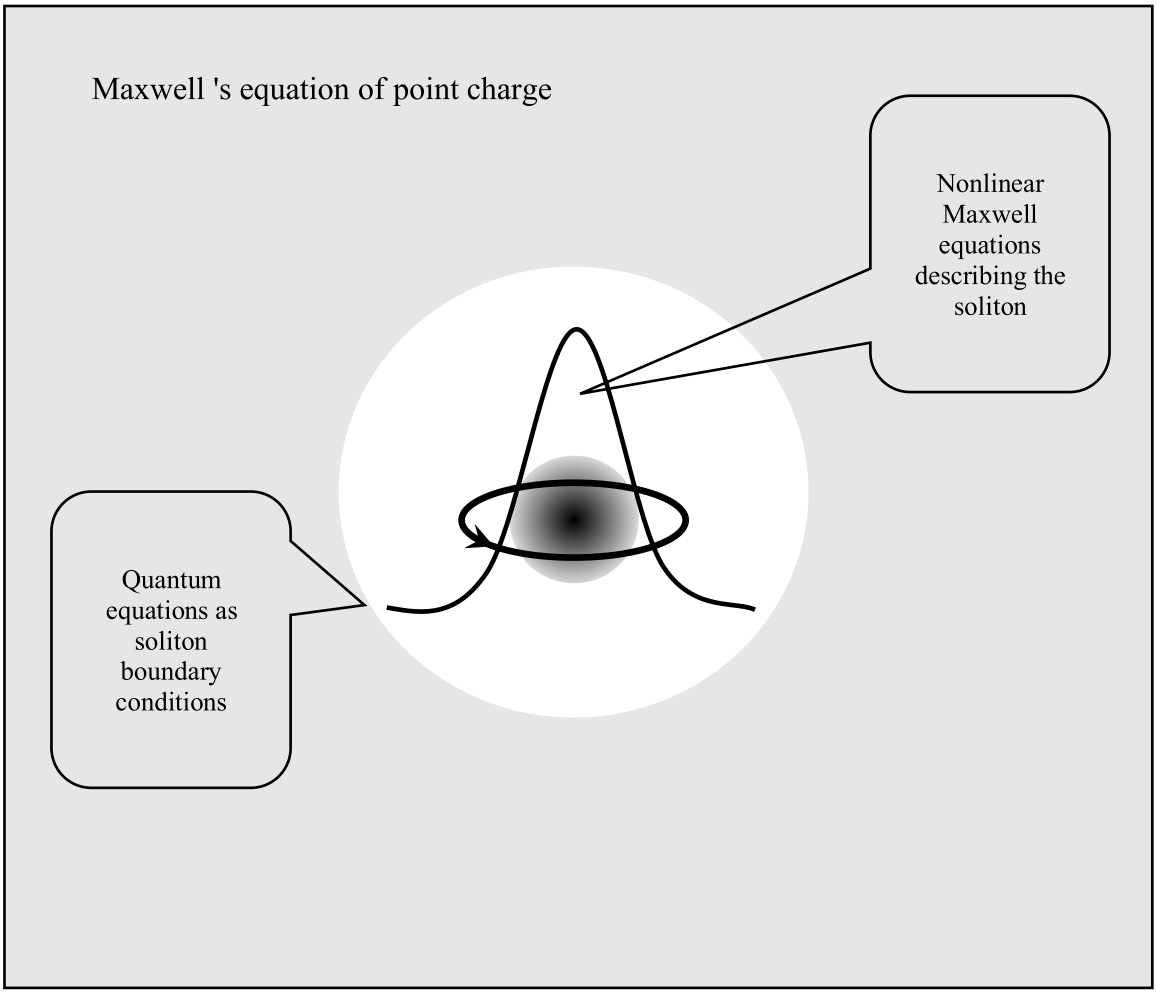

Figure 1. Areas of application of various equations

We should note that this interpretation does not eliminate the measurement problem. Any attempt to "look" at an electron (i.e., to illuminate it with photons or other particles) or to fix a coordinate while passing through a narrow hole, will certainly affect it resulting in changes in the wave functions. At the same time, if nonlinear equations are found, one could also find solutions under the influence of a measuring device, both of classical and quantum nature. This will allow to investigate particles in no less detail than we can do now with classical bodies. However, the description of scattering, birth and absorption of particles will still be more convenient with standard methods based on wave functions or operators. That is why the proposed interpretation does not cancel quantum theory, but only shows its location.

6. Conclusion

This interpretation is certainly a hypothesis until either 3+1 dimensional nonlinear equations for solitons are found or it is proved that such equations do not exist. However, if the solution of the above described nonlinear equations does exist, the proposed interpretation harmoniously explains various things considered a mystery today. First, it becomes clear why we live in the 3+1 dimensional world, as it is where stable formations (particles) naturally arise and the world becomes as we see it. Secondly, it becomes clear why Maxwell's equations, which seem to have nothing to do with quanta, fit so well into quantum field theory: they describe non-linear dynamics of those very quantum particles. Different elementary particles correspond to different wave numbers of the discrete spectrum.

Thirdly, it becomes clear why wave functions are written in a complex form, it is a natural representation of pairs of perpendicular to each other vectors of electric and magnetic field strengths of electromagnetic waves, which are not scattered on a soliton (elementary particle). That is why there is a wave superposition, this is a normal condition for electromagnetic waves in free space. And last, but not least, the role of quantum equations becomes clear. They represent boundary conditions for the existence of quantum particles, but do not describe their dynamics. If we normalize the squares of functions of the wave equation by one, we will certainly have a probability distribution, but this does not mean that we cannot say anything else about a quantum particle. We can fully describe its motion using the nonlinear equation, although measurement problems remain.

The proposed interpretation does not halt the statistical approach, but suggests that this approach refers not to the elementary particles themselves, but to the conditions of their existence and the conditions related to scattering, birth and absorption. This is quite different from Bohm's interpretation of "hidden" variables, because the linear equations of quantum theory are not approximate, but precise, containing no hidden variables within. Still, they describe boundary conditions for particle dynamics only, while real particles are real soliton waves of an electromagnetic field.

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Data availability statements

No Data associated in the manuscript

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