Entangled simultaneity: Testing Lorentz and light speed invariance with quantum and classical entanglement

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Abstract

In a rod of length AB $= L$, rotating uniformly, any two spatially separated points along the rod are connected in a way that shows analogies with the quantum entanglement of the spin of particles. This "classical entanglement" reflects the simultaneity preset in the system, which can be used for syncing two distant clocks, one at A and the other at B. Since it differs from Einstein synchronization, this procedure can be adopted for testing the one-way light speed and Lorentz invariance. Applications to optical Sagnac effects confirm that a consistent interpretation requires the adoption of absolute versus relative simultaneity.

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1 Introduction

The concept of simultaneity is often related to the stimulating theme of measuring the one-way speed of light, which has been widely discussed in

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literature. In quantum physics, simultaneity is linked to the quantum entanglement of the spin of two particles when spatially separated. For two entangled particles, one at A and the other at B, with AB $= L$, when a spin component of one of the particles is measured (e.g., at point A), then, the statistical distribution of another component of the spin of the particle at point B is instantaneously determined, because the measurement involves the quantum system as a whole with a "spooky action at a distance" [1], [2].

Since there is experimental evidence of quantum entanglement [3]-[6], we expect the transmission of the information about the measurement, to occur at faster-than-light speeds. However, according to Siegel [7], we do not yet know of an explicit way to exploit quantum entanglement for information transmission or for determining simultaneity between two events occurring at A and B, respectively.

In line with standard special relativity (SR), the simultaneity of the two events occurring at locations A and B can be checked using two clocks synchronized employing Einstein synchronization procedure. Einstein considers the average round-trip light speed $c = 2L/T$, where the time interval T is measured by clock A for the light round-trip from A to B and back to A. With his procedure, Einstein assumes that the one-way light speed coincides with the average speed c and then, the synchronization consists of setting the clock at B at $t = L/c$ when light reaches it.

Einstein synchronization procedure was soon met with criticism by epistemologists [8]-[12] and physicists, who pointed out that, in principle, the one-way speed from A to B can be different from the return speed from B to A. Hence, by relying on the observable round-trip average speed c, Einstein "two-way" synchronization leaves undetermined and arbitrary (conventional) the one-way speed. As the interpretation of special relativity has been evolving, in 1977 the physicists Mansouri and Sexl [13] introduced a set of coordinate transformations from frame S to S$'$ in relative motion with velocity v in agreement with Einstein synchronization, but with the speed from A to B that may differ from the speed from B to A, depending on the arbitrary synchronization parameter ε:

$$
\begin{aligned}
t' &= \frac{t}{\gamma} - \frac{\varepsilon x'}{c^2} = \gamma[t(1+\frac{\varepsilon v}{c^2} - \frac{v^2}{c^2}) - \frac{\varepsilon x}{c^2}] \\
t' &= \gamma(t - \frac{vx}{c^2}) \ \ \text{LT} \qquad\qquad t' = t/\gamma \quad \text{LTA} \\
x' &= \gamma(x - vt) \qquad\qquad y' = y \qquad\quad z' = z \\
c' &= c'(\varepsilon) = \frac{dx'}{dt'} = \frac{c}{1 + v/c - \varepsilon/c}
\end{aligned} \tag{1}
$$

In the generalized transformations (1) the factor $\gamma = (1 - v^2/c^2)^{-1/2}$, while the parameter ε can assume any arbitrary value from $\varepsilon = 0$ to $\varepsilon = v$. With $\varepsilon = v$ we have the standard Lorentz transformations (LT), while with $\varepsilon = 0$ we obtain the Lorentz transformations based on absolute simultaneity (LTA) [14], adopted, even if with different names, by many physicists adhering to conservation of simultaneity [15]-[29]. The time transformations of the LT and LTA differ by the value of ε only. The one-way speed of light is assumed to be the same as the average two-way speed c in frame S, while in frame S$'$ the local (differential) light speed is $c' = c'(\varepsilon) = dx'/dt'$. The time interval taken by light to traverse the distance L' is $t' = L'/c$ for the LT, and $t'_{\varepsilon=0} = L'/c' = (L'/c)(1 \pm v/c)$ for the LTA. The difference $\delta t' = t'_{\varepsilon=0} - t' = \pm(v/c)(L'/c)$ corresponds to the time gap (de-synchronization) of the LT in (1) when $L' = L/\gamma = x$. Light speed invariance, $c' = c$, holds for the LT only ($\varepsilon = v$).

According to Mansouri and Sexl [13] and the physicists adhering to the "conventionalist" view [30]-[33], internal clock synchronization procedures, such as clock transport from A to B, turn out to be equivalent to Einstein's, and thus the transformations (1) are considered as physically equivalent and interchangeable. In this scenario, the transformations (1) are supposed to foresee the same relativistic effects, even when adopting different ε values, while instantaneous action at distance and simultaneity are considered to be conventional and indeterminable. Thus, although for quantum entangled systems the concept of simultaneity is naturally present and reflected by the nonlocality of the quantum wave function, in standard SR we are met with the potentially different view that simultaneity and action at a distance cannot be verified. Unless we find a synchronization procedure different from Einstein's, within the classical relativistic scenario, it seems impossible to discriminate relative from absolute simultaneity.

The purpose of this article is to introduce a procedure for the synchronization of distant clocks that, differing from Einstein synchronization, can be interpreted as a sort of "classical entanglement" preset in the physical system. By comparing and discussing quantum and classical entanglement and the related action at a distance, we consider the similarity between the intrinsic simultaneity of the spin quantum entanglement and the "natural" simultaneity of the classical entanglement. If viable, the classical entanglement can be used, at least in principle, for syncing two spatially separated clocks that can be exploited to test the Lorentz and one-way light speed invariance. As applications of the natural sync, we consider first a simple test of light propagation along the distance AB $= L$, and then also along the moving closed contour of the linear Sagnac effect [34]-[36]. In both cases, it is possible to discriminate relative from absolute simultaneity and, for the Sagnac effect, we show that the consistency in the interpretation favors the LTA *versus* the LT.

2 Entanglement-based quantum clock synchronization

The possibility of synchronizing distant clocks using the approach of entanglement-based quantum clock synchronization has been discussed by several authors [37]-[39] and, according to them, the clock synchronization method based on quantum entanglement can improve the accuracy and precision of measurement. In some of these works, discussions are presented about the simulation experiment and simulation results. The entanglement-based clock synchronization may be useful in several scenarios, such as, for example, the context of space-based quantum network. The methods of clock synchronization are mainly based on two classical protocols proposed by Einstein [40] (Einstein synchronization) and Eddington [41] (clock transport) respectively.

To achieve quantum clock synchronization, in general, a quantum entangled state is sent from a node to another node. For example, Charlie prepares a singlet state that is sent to Alice at A and Bob at B. There is an unknown relative phase φ in the singlet of Alice and Bob, reflecting the two different basis conventions. Moreover, in sending the qubit from Alice to Bob, the different phase conventions and time offset have to be related to the probability p of an error. Quantum clock synchronization is achieved in subsequent steps by means of state purification to a degree of fidelity that, however, requires having clocks conveniently synchronized to some extent. Information related to time synchronization can be transmitted through a common, usually classical, channel. Hence, the time difference between Alice and Bob depends also on the global clock synchronization convention adopted, which could be, for example, Einstein (or equivalently, clock transport) or GPS synchronization procedures, which differ by the synchronization parameter ε in the time expression (1). Then, the time difference (usually denoted by δt or Δ) between Alice and Bob's first quantum operation, the related probability p, and the phase φ, may all depend on ε.

We do not know whether the approaches used in Refs. [37]-[39] or other works, are suitable to determine the value of ε because the authors do not consider the case of the arbitrariness of clock synchronization. Since, a priori, the time difference between Alice and Bob may depend on ε, i.e., $\delta t = \delta t(\varepsilon)$, any effective synchronization procedure is expected to be able to determine ε, unless we accept, in line with conventionalism, that the one-way light speed and simultaneity are not observable and, thus, ε is not determinable in principle. Of course, since quantum entanglement implies a sort of instantaneous action at a distance or a preset simultaneity between A and B, we believe $\varepsilon = 0$ to be the expected value valid for the quantum entanglement scenario.

In the next section we show that ε is not arbitrary and can be determined experimentally. In fact, as discussed in detail in Refs. [15]-[25] and also recognized by conventionalists [13], [30], [31], [33], Einstein synchronization ($\varepsilon = v$) fails when applied to light propagation along the closed contours of the optical Sagnac effects, where only the absolute sync ($\varepsilon = 0$) is viable. Actually, in order to achieve clock synchronization in the Global Positioning System (GPS) on Earth, the GPS engineers have to take into account the Sagnac effect and adopt absolute sync ($\varepsilon = 0$) [42], [43].

Although it is not clear if the entanglement-based protocol can discriminate Einstein synchronization ($\varepsilon = v$) from that of the GPS ($\varepsilon = 0$), the above-mentioned theoretical and experimental evidence, related to the Sagnac effects and the GPS, requires $\varepsilon = 0$ and thus the GPS synchronization should be the one to be used to establish the time difference δt between Alice and Bob. In any case, once the parameter ε has been determined one way or the other, it appears that the procedure of entanglement-based quantum clock synchronization represents significant progress in achieving greater precision in distant clock synchronization [37]-[39].

3 A physical system with classical entangled simultaneity

The rod AB of Fig. 1 has rest length L and radius $r << L$ and is stationary in the inertial reference frame S$'$ extended along the x' axis. The rod possesses two slots, or channel grooves, which are parallel to the y' axis when, initially, the rod is not rotating. In the circular cross-section at A, the pointer s_A mimics the spin of the particle at A, and the pointer s_B mimics the spin in the opposite direction of the particle at B. The pointer s_B can be moved from A to B at the distance L, simulating the separation of the two quantum particles while the system keeps its total spin equal zero. After applying an external rotational impulse to the rod, which is free to rotate about its longitudinal symmetry axis x', it reaches a steady-state uniform rotational motion. Then, supposing that there are no torsional stresses along its length L, every point of the rod possesses the same uniform angular velocity ω_0' and is bound to be rotating in phase with any other point. We may say that the two pointers s_A and s_B are "classically entangled" because the slots act as constraints that force them to keep opposite directions even when they oscillate up and down while the rod is rotating at the angular velocity ω_0'. Consequently, in the absence of permanent distortions, all the points along any line parallel to the direction A$-$B, initially in phase when the rod is not rotating, will be in phase when the rod is in steady-state rotational motion.

For this classical physical system, the tip A* of the pointer s_A intersects the y' axis of frame S$'$ at A when, simultaneously, the tip B* of the pointer s_B intersects the y_B' axis of frame S$'$ at B. Hence, the system possesses a natural built-in simultaneity with reference to the two events corresponding to the intersection of the two rotating points A* and B* with the y' direction. This natural, "classically entangled", simultaneity can be used to synchronize two spatially separated clocks, one at A and the other at B, by having both clocks set at $t' = 0$ when, simultaneously, the points A* and B* cross the corresponding axis in the y' direction.

Analogy and difference between quantum and classical entanglement. For the quantum system of two entangled particles with opposite spins, the term "entangled" reflects the condition of interwoven or intertwined spins that evolve in time while keeping in phase as the system maintains zero total spin. When the spin of one of the particles is measured, e.g., at point A, then (statistically) the opposite spin of the particle at point B is instan-

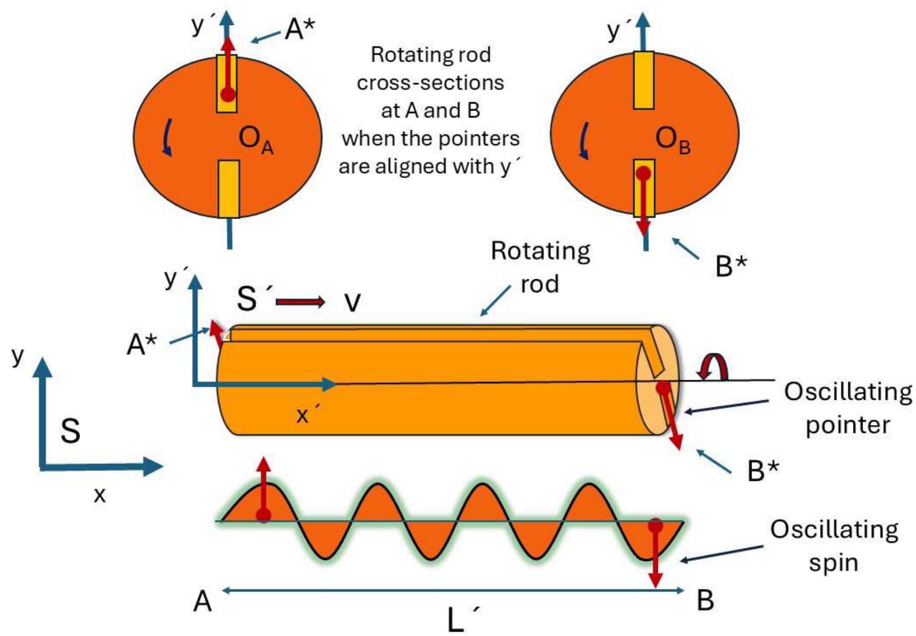

Figure 1: The classically entangled pointers at A and B are fixed inside the slots and are forced to turn around in phase when the rod is rotating with uniform angular velocity. Since the tips A* and B* of the pointers intersect simultaneously the y' direction, the corresponding events can be used to synchronize two spatially separated clocks at A and B. By oscillating up and down, the two pointers mimic the oscillations of the particle quantum entangled spins connected by the wave function, as shown at the bottom.

taneously determined.

Analogously, for the classical system of the rotating rod, the vector OA* keeps in phase with the vector OB* pointing in the opposite direction while rotating. Since s_A and s_B always point in opposite directions, the "classically entangled" tips of the pointers A* and B*, keep rotating in phase at the same angular velocity, while imitating the oscillating up and down spins of the corresponding two quantum entangled particles shown at the bottom of Fig. 1. However, although we may say that the two vectors s_A and s_B are classically entangled, there are still differences with the quantum entangled spins, as the term "classical" indicates that we are dealing now with a system that is different from the "quantum" one.

Yet, the classical and quantum entanglements have in common the important concept of built-in simultaneity because both entanglements share a preset simultaneity that is not tied to any finite-speed transport of information from A to B. Instead, the standard concept of simultaneity linked to Einstein synchronization requires the transport of information at the light speed c for establishing the synchrony of the two spatially separated clocks at A and B. Similarly, for the case of the synchronization procedures equivalent to Einstein's, if we consider that of clock transport from A to B, or from B to A, we find that the information about synchronization, carried in this case by the transporting clock, is affected by the clock's motion because of the intrinsic effect of time dilation [13], [33]. Hence, with clock transport and other analogous procedures involving finite-speed transport of information, the resulting sync is equivalent to Einstein synchronization. However, our rotating rod synchronization procedure does not even require the transport of information from A to B, or from B to A. In fact, for the rod in uniform rotational motion, the simultaneity of the two intersection events with the y' axes (y'_A and y'_B) is naturally preset in the system. Therefore, we are met with the important result that the corresponding "natural sync" related to the rotating rod is not necessarily equivalent to Einstein's.

In any case, although the described analogy between quantum and classical entanglement may be of help to clarify the notion of simultaneity and thus possess heuristic validity, the analogy is mainly of form, because, from the physical perspective, quantum and classical physics keep differing in many aspects. In fact, for example, the direction of the classical pointers is always observable, even when one of the pointers is transported from A to B. Instead, according to quantum mechanics, the spins become observable only after the measurements are performed.

Clock synchronization and faster than light signals. Let us consider the reference frame S where space is isotropic and we wish to synchronize the clocks distributed along the x-axis. In this "preferred" frame, the one-way light speed coincides with the average two-way speed c. Using Einstein synchronization, or our natural sync with the rotating rod, distant clocks in frame S can be equally and perfectly synchronized, and thus they will show the same readings simultaneously. If, hypothetically, we send a faster than light signal ($c \to \infty$) from one clock at the origin to another located at the distance x, in its motion the signal will find all the clocks displaying the same readings. Then, simultaneity and faster than light signals are compatible within frame S.

If we now consider the inertial frame S$'$ moving with velocity v relative to S, we can have all the clocks along the x' axis equally internally synchronized using either the natural sync or the hypothetical faster than light signal ($c' \to \infty$). In this case, there is conservation of simultaneity between frame S and S$'$ because the simultaneity of events corresponding to clocks along the x-axis displaying the same reading $t = 0$, is compatible with the simultaneity of events corresponding to clocks along the relatively moving x' axis displaying the same reading $t' = 0$. This result is consistent with, and reflected by, the transformations from S to S$'$ with $\varepsilon = 0$ in (1), i.e., the LTA based on absolute sync.

If we adopt Einstein synchronization in frame S$'$, we find once more that simultaneity and faster than light signals are compatible, as long as they are considered within frame S$'$. However, for the resulting LT, simultaneity between S and S$'$ is not conserved. Indeed, for the LT with $\varepsilon = v$ in (1), the readings of the clocks in S$'$ along the x'axis are given by $t' = -\gamma vx/c^2$ when $t = 0$ in frame S. Yet, if the hypothetical faster than light signal ($c' \to \infty$) is sent along the x' axis of S$'$, it may be expected to meet all clocks displaying the reading $t' = 0$, a result that seems in contradiction with $t' = -\gamma vx/c^2$. The point is that, in standard SR, simultaneity is meaningful within each inertial frame only, and the hypothetical faster than light signal ($c' \to \infty$) in S$'$ cannot be considered as a faster than light signal ($c \to \infty$) when observed from frame S. Hence, faster than light signals are not operationally meaningful within standard SR, where simultaneity is relative and a necessary consequence of Lorentz and light speed invariance.

In any case, the rod's natural sync is not necessarily equivalent to Einstein synchronization and in the next Section and in the Appendix we indicate several physical situations, discussed in literature, where the LT and LTA are not physically equivalent. For the convenience of the reader and as applications of the natural sync, we delve below into two of these situations where the conventionality of the one-way light speed may not be valid.

4 Measuring the one-way speed and describing light propagation using the natural sync.

Assuming that frame S is a "preferred" frame where space is isotropic, and the one-way light speed is c, we calculate the time interval taken by light to traverse from A to B the rod co-moving with S′, as in Fig. 1. Here, we suppose that clocks on S′ are internally synchronized using the natural sync. Using the LT and the LTA, elementary kinematics provides the following results:

Traveling at speed c along the rod in frame S′, moving at speed v relative to S, light reaches point B when $ct = L'/\gamma + vt$ and the time interval measured from frame S is,

$$t_B = \frac{L'}{\gamma(c - v)}, \tag{2}$$

where L'/γ represents the Lorentz contracted length of the moving rod.

a) LT. According to standard SR based on the LT with $\varepsilon = v$ in (1), light from A reaches B after the interval,

$$t'_B = \frac{L'}{c}, \tag{3}$$

as measured by clock B in frame S′.

b) LTA. According to the LTA with $\varepsilon = 0$ in (1) and with $\gamma^{-2} = (1 - v^2/c^2) = (1 + v/c)(1 - v/c)$,

$$t'_B = \frac{t_B}{\gamma} = \frac{L'}{\gamma^2(c - v)} = \frac{L'(1 + v/c)}{c} = \frac{L'}{c'}. \tag{4}$$

Hence, the LTAs foresee an observable result different from that of the LT, indicating that different transformations imply different physical realities, if the natural sync is not equivalent to Einstein synchronization.

Changing the orientation of the rod. We now place the rotating rod AB along the y' axis and the light signal is sent from A to B in the y' direction.

As seen from frame S, with $c_x = v$, $c_y = (c^2 - v^2)^{1/2} = c/\gamma$, light from A reaches B when,

$$t_B = \frac{L'}{c_y} = \frac{\gamma L'}{c}. \tag{5}$$

a) LT. The LTs foresee,

$$t'_B = \frac{t_B}{\gamma} = \frac{L'}{c}. \tag{6}$$

If confirmed experimentally, the results (3) and (6) indicate that, for the LT, light speed is isotropic in frame S$'$ and we may conclude that the natural sync is equivalent to Einstein synchronization and the LTs are equivalent to the LTA.

b) LTA. The LTAs foresee,

$$t'_B = \frac{t_B}{\gamma} = \frac{L'}{c}. \tag{7}$$

The results (4) and (7) indicate that for the LTA the light speed observed from S$'$ depends on v and its relative orientation. Hence, unless disproved experimentally, with the natural sync we should be able to detect the velocity $\mathbf{v}$ of S$'$ relative to the chosen "preferred" frame S. We show below that this seems to be the case for the Sagnac effects, where the lab frame is assumed to represent the "preferred" frame S and the clock on the frame S$'$ is co-moving with the optical fiber at the speed v relative to S.

As mentioned above, according to the physicists adhering to the conventionalist view, all the transformations (1) are physically equivalent and foresee the same observable results because they differ only by the arbitrary synchronization parameter ε. In their test theory of SR, Mansouri and Sexl [13] show that any of the transformations (1) can interpret all the known experiments supporting SR. Hence, the set of experiments supporting SR with the LT, equally support SR with the LTA or any of the transformations (1). However, with the different results (3) and (4), the possibility that the natural sync is not equivalent to Einstein synchronization challenges the conventionalist thesis that clock synchronization is arbitrary and that the LTs are equivalent to the LTA. The first important consequence of the natural sync and the related results (4) and (7), is that conventionalism is, at least in principle, disproved: clock synchronization and the one-way light speed are not necessarily arbitrary. As shown in other works about the interpretation of the optical effects of the Sagnac type [34]-[36] given in literature [44]-[47] and mentioned in the Appendix, the resulting set of experiments supporting SR with the LTA is wider than that of the LT.

To show in detail what are the consequences of a synchronization different from Einstein's, in the next section we test relative *versus* absolute simultaneity by interpreting the linear Sagnac effect, experimentally verified by Wang et al. [35], [36].

4.1 Testing the natural sync *versus* Einstein synchronization with the linear Sagnac effect.

In Sagnac effects, spacetime continuity requires the local light speed along the moving optical fiber to be $\simeq c \pm v$.

For the Sagnac effects, we find the well-established result that, if the one-way light speed is assumed to be c in the "preferred" lab frame where the effects are interpreted, the one-way light speed along the moving contour where light propagates, is $\simeq c \pm v$, as Sagnac [34], Selleri [15]-[17] and many other physicists [18]-[25] have been claiming through more than a century. Although the Sagnac effects have been widely discussed, unfortunately, the relativistic interpretation of the linear Sagnac effect [18]-[25], considered below, is barely mentioned in the literature.

As well known [28], [18]-[31], the LT and Einstein synchronization fail to describe light propagation along a closed contour. To show where the LTs fail, it is convenient to focus on the linear Sagnac effect [35] of Fig. 2, where an optical fiber slides at speed v on the two pulleys A and B with AB $= 2L$. The device C* (a clock) is fixed to the fiber and moves from the lower to the upper section while a counter-propagating photon performs the round trip in the time interval T. Let S$''$ be the inertial rest frame of C* when located on the fiber lower section and by S$'$ the rest frame when on the upper section. For the counter-moving photon, the round-trip interval measured by C* co-moving with the fiber (with refractive index $n = 1$), is [48], [18]-[25],

$$T = \frac{2L}{\gamma(c+v)} = \frac{2\gamma L}{\gamma^2(c+v)} = \frac{2\gamma L(1-v/c)}{c}. \tag{8}$$

We wish to check the effective ground distance covered by the photon traveling at the ground local speed c in the time interval T given by (8). This task requires measuring separately the intervals T''_{out} and T'_{ret} taken by the photon to traverse the lower and upper sections respectively. We choose to measure T''_{out} and T'_{ret} by means of two clocks always in uniform motion. For the out trip on the lower section, the first clock is C* co-moving with S$''$ and, for the photon return trip on the upper section, we use, as shown in Fig. 2-b, the second clock C$'$ co-moving with S$'$, where C$'$ is set at $t' = t'' = 0$ at point A when facing C*. This way, we may neglect the dimension of the pulleys and the effect is completely linearized (as drawn in Fig. 2-b) and described from the inertial frames S" and S$'$.

Since, as seen from C* co-moving with frame S$''$, the speed v of A and the ground local light speed $c'' = c$ are known, the initial position of C* relative to A (Fig. 1-b), can be chosen in such a way (AC* $= (v/c)L/\gamma$) that the counter-propagating photon leaving C* reaches B when, simultaneously in S", C* reaches A, as shown in Fig. 2-b. Then, the photon reaches B in the interval,

$$T''_{out} = T_{out} = \frac{L''}{c''} = \frac{L}{\gamma c}. \tag{9}$$

Hence, the fiber ground length covered at speed $c'' = c$ by the photon in the out trip T_{out} from C* to B, is $L'' = \gamma^{-1}L$.

With $w = 2v/(1 + v^2/c^2) \simeq 2v$ the relative velocity between S$''$ and S$'$, the corresponding LT are [18], [19],

$$x' = \gamma_w(x'' - wt) \qquad t' = \gamma_w(t'' - \frac{wx''}{c^2}) \tag{10}$$

with the relations,

$$\begin{aligned} \gamma_w &= \gamma^2(1 + v^2/c^2) = (1 + w^2/c^2)^{-1/2} \\ \gamma_w(1 + w/c) &= \gamma^2(1 + v/c)^2 = \frac{1 + v/c}{1 - v/c}. \end{aligned} \tag{11}$$

The photon return trip time interval from B to clock C$'$, can be calculated from S$''$ from the equation $wt'' = L/\gamma - ct''$, and,

$$\begin{aligned} T''_{ret} &= \frac{L}{\gamma(c + w)} = \frac{\gamma_w L(1 - v/c)}{\gamma c(1 + v/c)} \\ T_{ret} &= T'_{ret} = \frac{T''_{ret}}{\gamma_w} = \frac{\gamma L(1 - v/c)^2}{c}, \end{aligned} \tag{12}$$

where the proper interval $T'_{ret} = \gamma_w^{-1}T''_{ret}$ is the same for the LTA and LT.

The same result $T_{ret} = T - T_{out}$ is obtained by calculating it from S$'$, after determining the initial position of the photon on S$'$ at $t' = 0$. On account of relative simultaneity, we have from (10) $t'' = wx''/c^2 > 0$. Hence, the photon, from $x''_B = L/\gamma$ at $t'' = 0$, has moved to $x'' = L/\gamma - ct'' = L/\gamma - wx''/c$, corresponding to $x'' = L/\gamma(1 + w/c)$. Since at $t' = 0$ we have $x' = x''/\gamma_w$, using relations (11), after simple algebra we find that the initial position of the photon on S$'$ is $x'(t' = 0) = \gamma L(1 - v/c)^2 < L$, in agreement with (12). Hence, due to the nonconservation of simultaneity of the LT, for S$'$, at $t' = 0$, the photon is already at point K$'$ as shown in Fig. 2-b.

Spatial ground distance covered by the photon according to the LT. In the out trip, the ground distance covered is $L'' = \gamma^{-1}L$, as given by (9). The return trip interval T'_{ret} is given by (12). According to the LT, the return ground light speed is c on S$'$ and $T'_{ret} = L'/c$. For the observer on S$'$,

$$L' = cT'_{ret} = \gamma L(1 - v/c)^2 \simeq L - 2(v/c)L < L, \tag{13}$$

indicating that the photon does not cover the whole path $\simeq L$ on the return trip. The total ground path covered at speed c by the photon, L'' on S$''$ and L' on S$'$, is exactly,

$$L'' + L' = \gamma^{-1}L + \gamma L(1 - v/c)^2 = 2\gamma L - c\delta t' < 2L, \tag{14}$$

where the term $\delta t' = \gamma_w \gamma^{-1} wL/c^2 = 2\gamma vL/c^2$ represents the "time gap" from S$'$ to S$''$ due to the relative simultaneity foreseen by the time transformation of the LT in (10).

Hence, at the ground local speed c, the photon cannot cover the whole fiber length $2\gamma L$ in the round-trip interval T. This result implies a breach in spacetime continuity by $c\delta t'$ because, at the ground local speed c, there is a "missing path" $2\gamma vL/c = c\delta t'$ that has not been covered in the interval $T = T_{out} + T_{ret}$. The result (14) shows that Einstein synchronization fails because it does not provide a consistent interpretation of Sagnac effects. In fact, at the invariant photon local speed c, to cover the whole contour $\simeq 2L$, the resulting round-trip interval measured by C* and C$'$, should be $(T)_E \simeq 2L/c$, and not the observed T as in (8). As remarked by Klauber [28], with Einstein synchronization applied to a moving closed contour, a clock turns out to be out of synchrony with itself.

Imposing spacetime continuity with the LTA in deriving T. Assuming with the LTA that the one-way light speed is c in the "preferred"

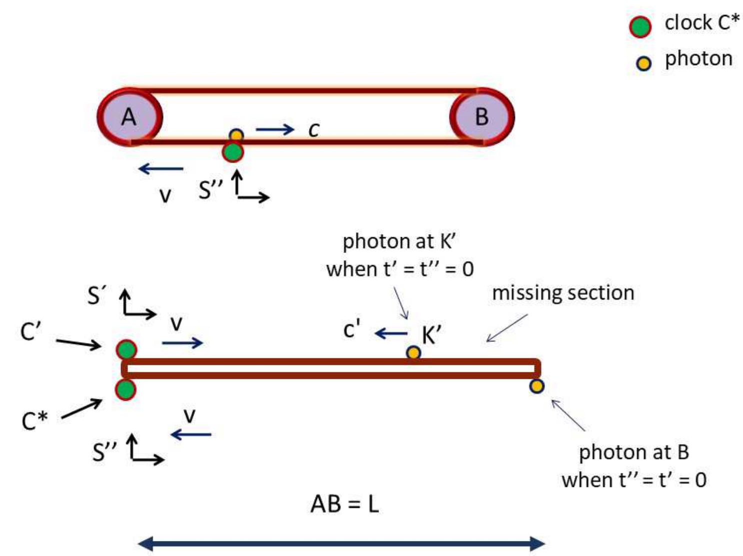

Figure 2: In the linear Sagnac effect, light propagates along an optical fiber sliding at speed v on the two pulleys A and B. The frames S$'$ and S$''$ are co-moving with the upper and lower fiber sections, respectively, possessing opposite velocities v relative to the lab frame AB. Their origins coincide at point A at $t' = t'' = 0$. *a*) After being emitted by the clock C*, initially co-moving with the fiber lower section at $t'' < 0$, the photon reaches B when A reaches C* at $t' = t'' = 0$, as shown in *b*). As observed from C* in frame S$''$, the photon has covered the distance L/γ in the interval $T_{out} = L/(\gamma c)$. According to the LT, in the return trip on the upper section, the photon is at K$'$ at $t' = 0$ and covers the shorter distance $\gamma L(1 - v/c)^2$ in the interval $T_{ret} = \gamma L(1 - v/c)^2/c$ measured by clock C$'$. The missing section K$'$B $= c\delta t' = 2\gamma(v/c)L$ has not been covered for $t' > 0$ in the observed interval T'_{ret}.

lab frame S where the frame AB is at rest, according to the result (8), the average ground speed along the contour of length $2\gamma L$, is,

$$c_g = c' = c'' = \gamma^2(c+v) = \frac{c}{1-v/c}. \tag{15}$$

Then, as seen from the frame S$''$ of the fiber lower section, $T_{out} = \gamma L/c'' = \gamma L/\gamma^2(c+v)$ and, for the fiber upper section, $T_{ret} = \gamma L/c' = \gamma L/\gamma^2(c+v)$. Hence, at the ground speed c_g, the photon covers the whole contour $2\gamma L$ in the observed interval $T = T_{out} + T_{ret} = 2\gamma L/\gamma^2(c+v)$ in agreement with (8), while spacetime continuity is conserved as expected.

Even physicists adhering to the conventionalist view agree [28], [30] that the LTs based on Einstein synchronization fail in interpreting these optical and other physical effects. Still, claiming that synchronization and the one-way light speed are conventional, the predominant view of conventionalists [13], [30]-[31] has been that, although these experiments are clearly and correctly interpreted using the LTA, the LT are not disproved because of the equivalence of all transformations (1). Nevertheless, since the results (3) and (4) plus the breach in spacetime continuity (14), confirm that the LT and LTA foresee different observable results and are not equivalent, the conventionalist strategy used to solve paradoxes by interchanging the LT with the LTA becomes conceptually untenable.

It is convenient to seek more experimental and theoretical evidence to verify whether relative and absolute simultaneity are equivalent or not. Then, besides the well-known paradoxes of SR, it is opportune to mention other problems arising with the use of standard SR, such as the less-known electrodynamics equilibrium paradoxes [49]-[51] and the interpretation of the electromagnetic interaction in the effects of the Aharonov-Bohm (AB) type (denoted recently as the "AB paradox"), for which a modification of the standard Lorentz has been derived from the *electric charge-magnetic dipole* interaction Lagrangian [52]. We are considering discussing and, if feasible, even proposing a test for our synchronization method in a future contribution. For the moment, we can mention that the approaches discussed in Refs. [17]-[27] and the performed Sagnac experiments [34]-[36] validate our claim that the one-way speed of light and Lorentz invariance can be tested. Furthermore, in the Appendix we mention some other experiments (such as the "reciprocal linear Sagnac effect"), or physical situations, either favoring the LTA versus the LT, or corroborating the nonequivalence of the two transformations.

Although potentially very interesting, the implications of adopting absolute simultaneity for existing established theories are certainly incipient and at an embryonic stage for the moment. However, what appears to be quite promising is that the approach based on conservation of simultaneity seems to be able to solve many of the paradoxes of standard special relativity based on relative simultaneity, as recognized even by conventionalist physicists [30], [31].

5 Conclusions

We have shown that the kinematical system, consisting of the rod AB in uniform rotational motion, possesses a natural preset simultaneity connecting the spatially separated points A* and B*. This inherent "classical entangled" simultaneity can be exploited to "internally" synchronize two distant clocks, one at A and the other at B. Since, in principle, this natural sync differs from Einstein synchronization, it can be adopted to remove the indeterminacy of the synchronization parameter ε in the transformations (1) and to test Lorentz invariance by measuring the one-way speed of light.

Within some limits, the entangled simultaneity suggests that we may establish an analogy between the classical behavior of the pointers in the rod of Fig. 1 with the fluctuating spin of two quantum entangled particles. In both cases, there is no need for transmission of information at a finite speed, since simultaneity is preset in the system. Once the parameter ε has been determined, the procedure of entanglement-based quantum clock synchronization appears to be relevant for achieving greater precision in distant clock synchronization.

By means of the natural sync, we show that the LT and LTA foresee different results for various physical situations. For the linear Sagnac effect, application of the natural sync confirms that, in the interpretation using the LT, the propagating photon cannot cover the whole closed contour traveling at the local speed c in the observed interval T. Instead, consistency is achieved in the interpretation adopting the LTAs, which conserve simultaneity and spacetime continuity.

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Data availability statement
No new data were created or analyzed in this study.

7 Appendix

Examples of physical situations where the LT and the LTA foresee different observable results.

1- Sagnac effects. There are in literature different interpretations of the Sagnac effects, circular and linear [34]-[36], as discussed by Selleri [15]-[17] and other physicists [23]-[29]. In these effects, the average one-way light speed along a moving closed contour can be measured with a single clock. If the one-way light speed is c in the laboratory frame, the local light speed in the clock frame must be $\simeq c \pm v$. As recognized even by conventionalists (Klauber [28], Lee [30], and Mamone Capria [31]), Einstein synchronization fails when applied along closed moving contours.

2- GPS (Global Positioning System). As considered by Gift [42], Ashby [43], and other authors, according to GPS engineers, to achieve clock synchronization while using Einstein synchronization in the GPS and maintaining accuracy, the GPS must apply to the light signals a velocity correction that corresponds to the Sagnac effect. If the local speed of light is c in the Earth Centered Inertial (ECI) frame, it must be $c \pm v$ on the rotating Earth surface [34], [15]-[17], [23]-[29].

3- The reciprocal linear Sagnac effect. The analysis by Spavieri and Haug [21], [22] of the reciprocal linear Sagnac effect, shows that the LTA and LT foresee different resulting values for the interval T. Hence, the two transformations are not equivalent and represent different physical realities. In agreement with the relativity principle, this effect is reciprocal to the standard linear effect for the LTA, while it is not reciprocal for the LT.

4- The Thomas precession. In many textbooks, the Thomas precession [53] is derived using standard SR with the LT. Spavieri and Haug [21], [22] follow the same procedure adopted by Jackson [54] but using the LTA. The result shows that, unlike the LT, the LTAs do not foresee the Thomas precession. In fact, the two transformations have different inherent symmetries [15]-[17]. For the LTA, the spin-orbit interaction is explained with the Dirac equation and its nonrelativistic limit [55], [56] without the need to introduce the classical Thomas precession.

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