Review of: "On bundles of varieties V_2^3 in PG(4, q) and their codes"

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Report

on the paper "On bundles of varieties V_2^3 in PG(4,q) and their codes" by Rita Vincenti

In this paper two related classes of algebraic geometric linear codes C_X and C_X' over a finite field GF(q) having low rank and high minimum distance are introduced and discussed.

The construction is based on the spatial representation of the projective plane Pi = PG(2,q^2) in the projective space PG(4,q). Fix a hyperplane Sigma' of Sigma containing a regular spread S of lines. Now consider a bundle X of varieties V_2^3 of Sigma having in common q+1 points of a conic C^2 of a plane pi_0, where pi_0 intersects Sigma' in a line l_0 in S (thus representing an affine line of Pi) and a further affine point O not in pi_0.

So X represents a bundle of non-affine Baer subplanes of Pi, each of them having one point at infinity (corresponding to a line of S), having in common a subline of affine points of Pi and a further affine point. C_X is by definition the linear code obtained by taking X as a projective system of Sigma.

The main result is Theorem 3.4 where it is shown that C_X is an [n,k,d]_q - code with code length n = q^3 + 2q^2 + q +1, rank k = 5 and minimum weight d = q^3 + q^2 -q. Deleting the coordinates at infinity one gets the related "affine" code C_X' = C_X_aff as an [n',k,d']_q -code with length n' = q°3 + q^2 -q, rank k = 5 and minimum distance d' = q^3 -q. (In the proof of Theorem 3.4 there should be given a more explicit argument for rank k = 5, only k<= 5 is trivial.)

Since the rank of the codes is k = 5 is rather small for reasonably large q these codes are not very interesting for substantial error correction applications, but the results appear of high interest for understanding the geometry of those bundles X of Baer subplanes under consideration. It is reasonable to expect that these codes display many interesting features of such a "bundle geometry". It may be easily possible to classify the minimum weight vectors of C_X resp. C_X' using the counting methods of the paper. It also would be interesting to determine the code automorphism groups of C_X resp. C_X'. I guess that the code automorphism group of C_X acts transitively on the varieties corresponding to the full set of q+1 non-affine Baer subplanes having in common the subline c_2 and the point O in Lemma 3.1 . I also conjecture that the automorphism group in each case acts transitively on the set of minimum weight vectors of the codes.

Reading the paper requires, of course, a basic understanding of finite geometry, in particular of translation planes, and some basics of algebraic geometry. Also understanding many subtle counting arguments, sometimes quite technical, is necessary. The paper represents a very interesting topic and uses finite geometry for the construction of linear codes in a fine and very nice way.