Is charge-energy conversion more energetic than mass-energy conversion?

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Abstract

In this article, we derive the relation between energy and charge in the form of charge-energy equivalence using gauge-gravity duality and special relativity. In comparison with Einstein's energy-mass equivalence, our result shows that the conversion of charge into energy leads to more energy gain than the conversion of mass into energy. The result also shows that the energy increases with the increase of charge. For our result, the energy also depends on the Newton's gravitational constant.

Keywords: gauge-gravity duality, double copy, mass-energy equivalence

1 Introduction

The double copy theory shows that there is a surprising relation between quantum gravity to gauge theories[1, 2]. Most importantly, the double copy theory enables calculations in both theories to utilize foundational elements derived from gauge theory, thus yielding outcomes within the realm of gravitational theory. This renders the double copy a potent instrument for producing gravitational amplitudes, especially pertinent for the scattering of massive external lines essential for black hole scattering applications [3, 4]. The double copy is important for high-precision predictions [5] which are required by current and future LIGO and VIRGO experiments. The success of the double copy theory hinges on the relation between the gauge theory coupling, and the gravitational coupling. Most interestingly, this relation also involves charge and mass. On the other hand, Einstein showed that there is a relation between mass and energy. And the combination of these two concepts leads to an interesting result. This article is organized as follows: In Sec. 2 we provide an overview of the derivation

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for the relation between the electromagnetic coupling constant g and the Newton's gravitational constant G_N, also we provide an overview of the derivation of energy-mass equivalence, and then deduce the relation between energy and electromagnetic coupling constant g. Finally, in Sec. 3 we summarise the article.

2 Gauge-gravity duality and Special Relativity

2.1 The relation between g and G_N

First, we provide an overview of the derivation of the relation between g and G_N from classical double copy of Wilson lines [6]. The gravitational Wilson line on spacetime manifold M can be defined as the phase of the action S for a test particle of mass m in a gravitational field. For any loop $\mathcal{C}$ on M the gravitational Wilson line can be expressed as

$$\begin{aligned} W_{grav}(\mathcal{C}) &= e^{iS}(\mathcal{C}) \\ &= \exp\left[im \int_{\mathcal{C}} ds \left(g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} \right)^{\frac{1}{2}} \right] \end{aligned} \tag{1}$$

Let us choose the metric of the form $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ and then the expansion of $W_{grav}(\mathcal{C})$ to the first order in κ leads to

$$W_{grav}(\mathcal{C}) = \exp\left[i\frac{m\kappa}{2} \int_{\mathcal{C}} ds \left(h_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} \right) \right] \tag{2}$$

where κ is a constant that is related to the Newton's gravitational constant, and s is a length(path) parameter. Now, the holonomy of one gauge field along the same path $\mathcal{C}$ leads to the following expression

$$W_{grav}(\mathcal{C}) = \mathcal{P} \exp\left[ig \int_{\mathcal{C}} ds \left(A^a_\mu \frac{dx^\mu}{ds} T_a \right) \right] \tag{3}$$

Thus we realize that the double copy rules for a Wilson line should be in the following way

$$T_a \to \frac{dx^\nu}{ds}, \quad g \to \frac{m\kappa}{2}, \quad \kappa^2 = 16\pi G_N \tag{4}$$

where G_N the Newton's gravitational constant, T_a is the colour generator of the gauge group, and g is the gauge theory coupling. In quantum electrodynamics, g is directly proportional to electric charge. Note that they exactly replicate the methodology of the double copy for scattering amplitudes, exchanging color data with kinematic data and the gauge coupling constant with its gravitational counterpart. From Eq. (4) the double copy rules, $g \to m\kappa/2$, indicate the connection between charge g and mass m. Surprisingly, the same connection arises from the bozonization of the massless $(1+1)$ Schwinger model, as a result, the Schwinger boson mass M_s [7] is expressed in terms of charge g as, $M_s = g/\sqrt{\pi}$.

2.2 The relation between e and m

Next, we provide an overview of the relation between mass m and energy e in the sense of Einstein [8]. Let e_0 be the energy of a test particle in the coordinate system (x, y, z). Let $(\tilde{x}, \tilde{y}, \tilde{z})$ be a new coordinate system which is uniformly moving with respect to the system (x, y, z), and whose origin is moving along the x-axis with velocity v. Let h_0 be the energy of the test particle in the coordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$. The test particle emits a light of energy $L/2$ as measured in the system (x, y, z) and the emitted light from the test particle forms an angle θ with the x-axis. If e_1 and h_1 are the energies of the test particle after the emission of light, as measured with respect to (x, y, z) and $(\tilde{x}, \tilde{y}, \tilde{z})$ respectively, then we arrive at the following formulas

$$\begin{aligned} e_0 =& e_1 + L/2 + L/2 \\ h_0 =& h_1 + \left[(L/2) \frac{1-(v/c)\cos(\theta)}{\sqrt{1-(v/c)^2}} + (L/2) \frac{1+(v/c)\cos(\theta)}{\sqrt{1-(v/c)^2}} \right] \\ =& h_1 + \frac{L}{\sqrt{1-(v/c)^2}} \end{aligned} \tag{5}$$

where c is the speed of light. The combination of equations for e_0 and h_0 leads to the following expression

$$(h_0 - e_0) - (h_1 - e_1) = L \left[\frac{1}{\sqrt{(1-(v/c)^2) - 1)}} \right], \tag{6}$$

Since the test particle is at rest in the coordinate system (x, y, z), the energy difference differs from the particle's kinetic energy k with respect to the coordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$ by a constant C. Therefore, we can write

$$\begin{aligned} h_0 - e_0 = k + C \\ h_1 - e_1 = \tilde{k} + C \end{aligned} \tag{7}$$

Since C is a constant, we obtain

$$k - \tilde{k} = L \left[\frac{1}{\sqrt{1-(v/c)^2}} - 1 \right] \tag{8}$$

Let us denote $k - \tilde{k}$ by Q. After the expansion we obtain

$$Q = (L/c^2)(v^2/2) \tag{9}$$

The above formula is for the kinetic energy, so we deduce that $m = L/c^2$, then we get

$$L = mc^2 \tag{10}$$

This result of Einstein shows that the energy L is related to mass m, and a constant c. If $c = 1$, then the amount of mass m is equal to the amount of energy L

$$L = m \tag{11}$$

2.3 The relation between g and L

Now, we are going to establish the relation between g and L. As already mentioned, g is directly proportional to electric charge in quantum electrodynamics. The expression for g is given by the double copy theory, and the expression for L is given by the special relativity, more precisely, energy-mass equivalence. To achieve this we just use Eq. (4) and Eq. (11). Since from Eq. (4), g=$m\kappa/2$, and from Eq. (11), $L = m$, we get

$$L = 2g/\kappa = \frac{g}{2\sqrt{\pi}} G_N^{-1/2} \tag{12}$$

In comparison with $L = m$, we realize that the conversion of g into energy is more energetic than the conversion of mass m into energy because of the factor $G_N^{-1/2}$ in Eq. (12). It is worth emphasizing that the presence of $G_N^{-1/2}$ indicates a connection to gravity in this case. In addition, since a small amount of mass can contain an enormous amount of charges, so the conversion of charge into energy would have an advantage over the conversion of mass into energy.

3 Summary

In this article, we derived the relation between energy L and charge g using the double copy theory and special relativity. The result shows that energy increases with the increase of charge. We made a comparison between Einstein's energy-mass equivalence, $L = m$, from Eq. (11), and Eq. (12) and we found that the conversion of charge into energy is more energetic than the conversion of mass into energy. This is due to the presence of the factor $G_N^{-1/2}$ which indicates a connection to gravity.

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