Review of: "Possible connections between relativity theory and a version of quantum theory based upon theoretical variables"

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Potential competing interests: No potential competing interests to declare.

Review of paper, by Harish Parthasarathy. The author gives a nice description of accessible and inaccessible variables and the notion of gauge groups acting on fields leaving the Lagrangian density invariant in field theory. I think that in the author's view, an inaccessible variable is like a position-momentum pair $\xi = (q, p)$ (or more generally a finite set of non-commuting observables that in classical mechanics/field theory determine the state of the system at any given time) that do not commute with each other and hence cannot be measured simultaneously while on the other hand, an accessible variable is like a single observable that is expressible as a function $X = f(\xi) = f(q, p)$ of the inaccessible variable. Such a single observable can be measured. Indeed, events concerning X are expressed in terms of the spectral measure EX(.) of X. The event that $X \in B$ for B a Borel subset of R is represented in the logic of quantum mechanics by the orthogonal projection operator EX(B). Moreover, given a state ρ, we compute the probability of this event using Gleason's theorem as $P(X \in B) = T\,r(\rho.EX(B))$ Here $X = Z\,\lambda.EX(d\lambda)$ is the spectral representation of X. More generally, we can think of an accessible variable as a set of observables given by functions of an inaccessible variable (that describes the state in classical mechanics) such that all these observables mutually commute. My feeling is that a maximal accessible variable should then be expressible as commuting functions of an inaccessible variable such that this set of functions forms a complete set of commuting observables in the quantum theory which means that any pure state/wave function for the resulting quantum system can be represented as a function of the joint values assumed by this set of observables. In the author's work, two maximal accessible variables $f1(\xi)$, $f2(\xi)$ can be related by $f2(\xi) = f1(k.\xi)$ for some $k \in K$ where K is a group of transformations acting on the inaccessible variables. In classical mechanics with (q, p) as the quantum inaccessible variable, K would consist of symplectic transformations acting on the phase space. The author also makes a statement that if G is a group acting on the accessible variables, then for any $g \in G$ and an accessible variable $f(\xi)$ where ξ is inaccessible, there should be a $k \in K$ such that $g.f(\xi) = f(k.\xi)$. This equation should be cast in the language of group representation theory, ie, writing $U(k)f(\xi) = f(k\ {-1}\ \xi)$, we observe that the group G is simply U(K) where U is a representation of K acting in an appropriate function space on the set of inaccessible variables. In classical mechanics, if the allowed functions f(q, p) are for example assumed to be square integrable then U would be a unitary representation of the symplectic group on the Hilbert space of square integrable functions. It should be noted that in classical mechanics, two canonical set of position and momenta (ie, satisfying the canonical Poisson bracket relations) are related by a symplectic transformation which is generally a nonlinear transformation of phase space 2 while in the quantum theory, the analogue of this is the Stone-Von-Neumann theorem which states that if q, p satisfy the canonical commutation relations as operators in $L^2(\mathbb{R})$, then there is a unitary operator U acting in L (R) such that $q = Uq_0U^{-1}$, $p = Up_0U^{-1}$ where q_0 is multiplication by x and $p_0 = -i\partial/\partial x$. So in the quantum context, perhaps the group K acting on the phase variables (q, p) should be the adjoint representation of the unitary group, ie, $k.(q, p) = U(q, p)U^{-1}$ for some unitary U. The author further introduces the notion of permissible maximal accessible variables, by stating that if $f(\xi_1)$ and $f(\xi_2)$ are two such, then $f(\xi_1) = f(\xi_2)$ for some inaccessible variables ξ_1, ξ_2 implies that $f(k.\xi_1) = f(k.\xi_2)$ for all $k \in K$ in the quantum theory. Some analogy with the classical situation is really required here. Since ξ_k like (q, p) cannot be measured, it cannot assume definite classical values and hence giving a classical interpretation of $f(\xi_1) = f(\xi_2)$ is difficult. In the classical context, if (q_1, p_1) and (q_2, p_2) are two canonical position-momentum pairs then they are related by a symplectic transformation $(q_2, p_2) = M(q_1, p_1)$, ie, a transformation M on the phase space that preserves the Poisson bracket which amounts to saying that if $J_M(q, p) = \frac{\partial M(q,p)}{\partial(q,p)}$ is the Jacobian matrix of M, then $J_M(q, p)J_0J_M(q, p)^T = J_0$ where $J_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ The equation $f(q_1, p_1) = f(q_2, p_2) = f(M(q_1, p_1))$ for all (q_1, p_1) then can be expressed as $f = foM$ from which it follows that for any other symplectic transformation M_1, we have $foM_1^{-1} = foMoM_1^{-1}$, or equivalently in terms of the unitary operator U_1 induced by M_1 on $L^2(\mathbb{R}^2)$, we can write $U_1((f) = U_1(foM)$ (Note that $U_1(f) = foM_1^{-1}$) The author directly translates this classical equation into the quantum scenario. The author should mention that the fact that an inaccessible variable ξ cannot be measured is reflected in the no-hidden variables theorem for quantum mechanics as is known to be a consequence of the inequalities of John Bell taking the example of non-commuting spin observables that assume only two values ±1. Bells inequality holds for three such Bernoulli random variables but fails for the Pauli spin triplet $(\sigma_x, \sigma_y, \sigma_z)$ and this means that we cannot construct a large probability space into which non-commuting observables in quantum theory can be embedded because otherwise Bell's inequality proved using properties of classical probability distributions would be satisfied which is manifestly false for the Pauli spin triplet. The author then proceeds to give a nice contradiction within general relativity from the standpoint of quantum mechanics and gauge groups. He states that the equivalence principle on which general relativity is based amounts to saying that locally, gravity and acceleration are equivalent and hence to measure the gravitational field locally, it is sufficient to measure the acceleration of particles locally. However, to relate gravity at two different points using the tensor transformation laws is not possible by virtue of non-commutativity of observables. I 3 think that in this context, some additional material to the following effect can be added: Two observers whose space-time coordinates are related by $\bar{x}^\mu = \bar{x}^\mu(x)$ choose to measure a four vector field A_μ like for example space-time differential coordinates, or energy-momentum values of a particle. Let A_μ be the canonical position field and $P_\mu(x)$ the canonical momentum field of a Lagrangian vector field theory, as for example, electromagnetism, Yang-Mills gauge field theory etc. On canonical quantization, assuming no constrains, these fields at equal time satisy the canonical commutation relations (CCR): $[A^\mu(x), P_\nu(y)] = \delta^\mu_\nu \delta^3(x-y)$, $x^0 = y^0$ In general relativity, we also have the diffeomorphism relating the contravariant and covariant four vector observables as seen by the two observers: $\bar{A}^\mu(\bar{x}) = A^\nu(x)\frac{\partial \bar{x}^\mu}{\partial x^\nu}$ $\bar{P}_\mu(\bar{x}) = P_\nu(x)\frac{\partial x^\nu}{\partial \bar{x}^\mu}$ As a consequence of these commutation relations, each observer can measure either only A_μ or only P_μ at his location, not both. Moreover, $x^0 = y^0$ does not imply $\bar{x}^0 = \bar{y}^0$. Thus, equal time commutation relations for the first observer do not transform generally into equal time commutation relations for the second observer. Thus, diffeomorphism covariance of general relativity is lost when we try to quantize the theory based on conventional Hamiltonian dynamics. This can be remedied by using Ashtekar's variables and loop quantum gravity wherein canonical position and momentum fields are defined in such a way that after taking into account constraints, the

Hamiltonian for gravity appears as some sort of a non-Abelian gauge Hamiltonian that contains only upto fourth degree terms in the position fields and only second degree terms in the momentum fields, so that the problem of renormalization encountered in a straightforward quantization is circumvented and further diffeomorphisms of space-time appear in the Hamiltonian as constraints. Divergence of path integrals is also overcome in loop quantum gravity owing to the fact that the connection fields that appear as SU(2) gauge fields are replaced by their holonomies ie, by elements of SU(2) obtained by parallely displacing the SU(2)-Lie algebra valued connection field around a closed loop. Since the elements of SU(2) (a compact group) are bounded in norm unlike the corresponding Lie algebra elements, by using the Haar measure on the compact group SU(2), we obtain finite path integrals. By resorting to the Feynman path integral approach to quantization rather than the Hamiltonian approach, we are able to obtain a diffeomoprhism invariant quantum gravity theory and this theory of loop quantum gravity can be extended to include non-Abelian gauge fields, Dirac fields and the Higgs field all within the loop framework in interaction with gravity. In short, loop quantum gravity is background independent since the metric field is also quantized and hence lowering and raising of indices of tensor field using a background metric 4 is of no consequence in such a theory, it ensures a quantum theory of gravity that respects diffeomorphism invariance. The author then introduces the notion of a coherent quantum state in terms of a unitary operator $T(u)$ parametrized by a vector u acting on the vacuum: $|u>= T(u)|0>$ and then considers the action of $T(u)$ induced on observables A: $A \to A(u) = T(u)AT(u)^*$ so that $A(u)|u>= T(u)A|0>$. In this context, I think that some mention of coherent states on the Bosonic Fock space should be provided as an example: If $a(n), a(n)^*, n \geq 1$ are canonical Bosonic annihilation and creation operators satisfying the CCR $[a(n), a(m)] = 0, [a(n), a(m)^*] = \delta(n, m)$, then for $u = (u(n)) \in l^2(Z)$, the coherent state $|u>$ is defined by $|u>= W(u)|0>$ where $W(u) = \exp(-u^* a + u^T a^*)$, $u^* a = X n \bar{u}(n)a(n)$ is the Weyl translation operator, or equivalently, $|u>= \exp(-|u|^2/2)\exp(u^T a^*)|0>$ The importance of coherent states in quantum field theory should be emphasized namely that they represent eigenstates of the positive frequency part of the electric and magnetic fields (since $a(n)|u>= u(n)|u>, n = 1, 2, ...$ and the positive frequency components of the electric and magnetic fields are expressible as linear combinations of $a(n), n \geq 1$) and secondly that the position-momentum uncertainty represented by the product of their variances has a minimum value in a coherent state. Thus if an operator A on the Boson Fock space is invariant under Weyl translations, then $[A, W(u)] = 0 \forall u$ in which case it follows that $A|u>= AW(u)|0>= W(u)A|0>$, so it follows that if A leaves the vacuum invariant, then all coherent states are eigenstates of A with the same eigenvalue, ie, A assumes a definite value in each of the coherent states. The author proceeds to given an analogy between his concept of inaccessible and accessible variables with the conventional operator formalism of quantum theory. He states that if $\theta = f(\xi)$ is a an accessible variable with ξ inaccessible, then we associate an operator $A\theta$ with θ. Suppose that K is a group acting on the inaccessible variables, then the operator associated with the accessible variable $\theta(u.\xi)$ should be given by $A\theta(u) = T(u)A\theta T(u)^*$ where $u \to T(u)$ is a unitary representation of K. Here, the operators $T(u), u \in K$ generate the coherent states by their action on the vacuum, ie, $|u>= T(u)|0>$ is the coherent state with parameter u. This because, $A\theta(u)|u>= T(u)(A\theta|0>)$. Since coherent states span a dense manifold of the Boson Fock space, we expect to be able to generate all states by considering the closure of the linear span of the $T(u)$ 0 s acting on the vacuum. Indeed, if $u \to T(u)$ is an irreducible representation of K, then the closure of the span of the $T(u)|0>, u \in K$ is a T-invariant subspace of the Boson Fock space and by irreduciblity of T, it equals the entire Fock space. Since we expect two maximal accessible variables $\theta 1 = f1(\xi)$, $\theta 2 = f2(\xi)$ to be K related, ie 5 $f2(\xi) =$

$f_1(u.\xi)$ for some $u \in K$, the operator associated with θ_2, is given by $A\theta_2 = T(u)A\theta_1 T(u)^* = A\theta_1(u)$. This is natural to expect because then $A\theta_2|u\rangle = A\theta_1(u)|u\rangle = T(u)A\theta_1 T(u)^* T(u)|0\rangle = T(u)(A\theta_1|0\rangle)$ in agreement with the correspondence $\theta_1 = f_1(\xi)$, $\theta_2 = f_2(\xi) = f_1(u.\xi)$ In this way, knowledge of the quantum operator associated with any one maximal accessible variable, enables us the generate the quantum operators associated with all the other maximal accessible variables In this context, a general observation can be made: Let G be a group acting on a manifold X equipped with a G-invariant measure μ. Let $g \to U(g)$ denote the unitary representation of G in $L(X, \mu)$ defined by $U(g)f(x) = f(g^{-1}.x) = f_g(x)$. Then $U(g)$ acts on the space of Hilbert Schmidt operators B in $L^2(X, \mu)$ by $B \to B_g = U(g)BU(g)^* = Ad(U(g))(B)$ so that $B_g f_g = U(g)(Bf)$. The space of Hilbert-Schmidt operator in $L^2(X, \mu)$ becomes a Hilbert space with inner product $\langle B_1, B_2\rangle = Tr(B_1^* B_2)$ and $Ad(U(g))$ leaves invariant this inner product and hence $g \to Ad(U(g))$ defines a unitary representation in this Hilbert space of operators. In particular, if a self-adjoint operator A commutes with the representation U, then the representation U leaves each of the eigen-spaces of A invariant if f_0 is an eigenvector of A with multiplicity one with eigenvalue c, then $U(g).A|f_0\rangle = A.U(g)|f_0\rangle$ gives $AU(g)|f_0\rangle = cU(g)|f_0\rangle$, $\forall g \in G$, ie, A assumes the value c in each of the state $U(g)|f_0\rangle$, $g \in G$. More generally, if $W = N(A-c)$ is the eigenspace of A corresponding to the eigenvalue c, then W is left invariant under the operators $U(g)$, $g \in G$, ie, $U(g)(W) \subset W$, $g \in G$ and hence, A assumes the same value c in all the states $U(g)|f\rangle$, $g \in G$, $f \in W$. The author the proceeds to gives a nice interpretation of symmetries of a Lagrangian density of a field with respect to a local gauge group. Specifically, I think that he considers a Lagrangian $L(\varphi(x), \partial_\mu\varphi(x))$, $x \in X = R^d$ that is invariant under a local gauge group $g : X \to G$ where G is a Lie a group acting according to a unitary representation $D(g)$ of G on the finite dimensional Hilbert space X as in the conventional treatment of the Yang-Mills matter and gauge field theory, ie $L(g(x)\varphi(x), \partial_\mu(g(x)\varphi(x)) = L(\varphi(x), \partial_\mu\varphi(x))$. Such a locally gauge invariant Lagrangian density can easily be constructed from a function $L(\varphi, \chi_\mu)$ where $\varphi, \chi_\mu \in X$ and L is globally G-invariant, ie, $L(D(g)\varphi, D(g)\chi_\mu) = L((\varphi, \chi_\mu)\forall g \in G$ by assuming that the fields consist of matter field components ψ and gauge field components A_μ on X with the latter assuming values in the Lie algebra of dD(}) where } is the Lie algebra of G and setting $\varphi(x) = \psi(x)$, $\chi_\mu = \nabla_\mu\psi(x) = (\partial_\mu + eA_\mu(x))\psi(x)$ and assuming that under local gauge transformations, the matter field $\psi(x)$ transforms as $\psi(x) \to D(g(x))\psi(x)$ while the gauge field $A_\mu(x)$ transforms as $A_\mu(x) \to D(g(x))A_\mu(x)D(g(x))^{-1} + e^{-1}D(g(x))(\partial_\mu D(g(x))^{-1})$ ensuring thereby that $\nabla_\mu\psi(x)$ transforms to $D(g(x))\nabla_\mu\psi(x)$ thereby implying 6 from global G-invariance of $L(\varphi, \partial_\mu\chi)$, the local invariance of the Lagrangian density $L(\varphi(x), \nabla_\mu\psi(x))$. The author then proceeds to assume that G acts transitively on the range space of the fieldX thereby ensuring from the standard theory of group actions on a manifold, that X is isomorphic to G/H where $H = \{g \in G : g\varphi_0 = \varphi_0\}$ for some fixed non-zero $\varphi_0 \in X$. H is called the isotropy subgroup of G at φ_0. I think that at this point, the author should include a discussion of spontaneously broken symmetry and unbroken subgroups along the following lines and should relate his discussion of the gauge group of symmetries of the Lagrangian to this concept. Specifically, if φ_0 is chosen as the vacuum expectation of the field and if $\varphi = \varphi_0 + \varphi_1$, then the spontaneously broken Lagrangian $L_1(\varphi_1) = L(\varphi_0 + \varphi_1)$ would not be invariant under the whole of G but only under the broken subgroup H: $L_1(h.\varphi_1) = L_1(\varphi_1)$, $h \in H$ because L is invariant under G and φ_0 is invariant under H and $H \subset G$. The author may then proceed to explain how massless particles are generated by such spontaneous symmwetry breaking along the following lines: Let t_A be the Hermitian generators of the Lie algebra of G (More precisely, t_A are the Hermitian generators of the Lie algebra $dD(g)$ of the unitary group $D(G)$). Then, from G-invariance of L, follows the G invariance of the quantum effective potential $V(\varphi)$: $V(\varphi + .t_A\varphi) = L = V(\varphi) + o()$, $\to 0$, $\forall\varphi$ or equivalently, X n ∂V (φ) ∂φn (tAφ)n = 0, forallφ which gives on differentiating w.r.t φm, setting φ = φ0 the vacuum expectation of φ and noting that ∂V (φ0)/∂φn = 0 since V attains minimum value the the vacuum expectation that X m,n ∂V (φ0) ∂φm∂φn (tAφ0)n = 0 implying thereby that tAφ0, if non-zero is an eigenvector of the mass matrix ((∂V (φ0) ∂φm∂φn)). Now, tA is a generator of the broken symmetry group iff, then it does not annihilate the vacuum, ie tAφ0 6= 0 (by definition, the unbroken symmetry group H is the set of all h ∈ G which leave the vacuum state φ0 invariant, so that its Lie algebra consists of all t ∈ gfor which t.φ0 = 0). Thus, corresponding to each broken symmetry generator t, t.φ0 is a non-zero eigenvector of the mass matrix. This means that corresponding to each broken symmetry there is a massless particle which is called a massless Goldstone Boson. In short, there 7 is a massless Goldstone Boson corresponding to each broken degree of freedom. Finally, I would like to point out the following basic facts about quantum blackhole physics that the author must address using his formalism of inaccessible and accessible variables. In classical blackhole physics, one observer is located within the critical Schwarzchild radius of the blackhole and another outside and it is known that the two cannot communicate by means of light signals or by means of any other particle, massive or massless. However, quantum mechanically, the two can communicate by means of sharing an entangled state. Entangled states between the two observers can be created as follows: At time t = 0, the joint state of the observer within and the observer outside is pure and separable: |ψ >= |ψ1 > ⊗|ψ2 >. This pure state evolves according to a unitary operator U that can be represented as U = P k Vk ⊗ Wk, thereby generating an entangled state U|ψ >= P k Vk|ψ1 > ⊗Wk|ψ2 > and hence when the observer within makes a measurement of his state, the overall state collapses and consequently, observer outside by measuring his state gets to know what measurement the observer within has made. In this way, the observer within can transmit classical information to the observer outside by choosing what to measure about his state. Likewise, the observer outside can also transmit classical information to the observer within by choosing what state of his part to measure. If the within states Vk|ψ1 >, k = 1, 2, ... are orthonormal and complete and so are the outside states Wk|ψ2 >, with the same norm, then we get a maximally entangled state and we know that using such a maximally entangled state in which each observer has d qubits, he can even teleport any d qubit state |φ > to the observer outside by tensoring |phi > to his shared qubits and then making a clever measurement of his 2d qubits and reporting the outcome of his measurement to the observer outside by means of 2d classical bits of information. However, in the blackhole context, there is no way by which the observer inside can transmit classical information outside without using entaglement, so quantum teleportation is ruled out. However, by using a maximally entangled state of d qubits shared by the two observers, the observer within as noted above can transmit d-bits of classical information by making an appropriate measurement on his part of the d shared qubits thereby causing the overall state to collapse which the observer outside can detect from a measurement. Specifically, if P i |ei > ⊗|fi > is a maximally entangled state, and the observer uses the PVM |ei >< ei |, i = 1, 2, ..., 2 d and chooses to measure |ej >< ej |, then the total state collapses to |ej > ⊗|fj >, and the observer measures his state as |fi > based on his PVM |fi >< fi |, i = 1, 2, ...2 d and thus gets to know j, the number in {1, 2, ..., 2 d} that the observer within intended to transmit. The other aspect of quantum blackhole physics is based on the fact that if |ψ > is a pure state shared by the two observers, one inside and the other outside the critical radius of the blackhole, then the overall entropy is zero but the two mixed states T rout(|ψ >< ψ|) and T r1(|ψ >< ψ|) of the two observers have the same non-zero entropy. Finally, quantum mechanically, particles can 8 tunnel through the critical radius of the blackhole as follows by writing down the Klein-Gordon or the Dirac equation in the blackhole metric and observing that if probability mass is initially concentrated within the blackhole, then under the quantum evolution, after a finite time, some of its mass will also be present outside the the blackhole. This fact can also be seen by an easier approach, namely noting that the proper time interval for radial motion in the Schwarzchild metric is $d\tau^2 = (1 - 2m/r)dt^2 - (1 - 2m/r)^{-1}dr^2$. For a photon, $d\tau = 0$ giving $dt = (1-2m/r)^{-1}dr$. Integrating from $2m-0$ to $2m+0$ w.r.t the radial variable gives $\delta t = 2m.\ln(r - 2m)|_{2m-0}^{2m+0} = 2mi\pi$, namely imaginary time to cross the critical radius. This has the physical interpretation of changing the amplitude $\exp(i\omega t)$ of a sinusoid by $\exp(i\omega.\delta t) = \exp(-2m\pi\omega)$ and hence gives a probability of $\exp(-4m\pi\omega)$ of a photon of frequency ω to tunnel through. Equating this to the Gibbs probability $\exp(-h\omega/2\pi kT)$ gives the Hawking temperature at which the blackhole starts radiating a $T = h/8\pi.mk$ where $m = GM/c^2$, M being the blackhole mass and k Boltzmann constant. The question is whether this formula of Hawking can be derived using the inaccessible-accessible variable approach of the author ? Other more rigorous methods to compute the tunneling probability of a Klein-Gordon or a Dirac particle include solving the radial wave equation in the Schwarzchild metric by assuming the wave field amplitude to be $\psi(t, r) = \exp(2\pi iS(t, r)/h)$ and noting that the solution would give the action function $S(t, r)$ to acquire an imaginary part after tunneling. Can the author's approach be extended to these results ? I would rate the author's work as 3.5/5 and would increase it to 4/5 if he can provide partial explanations to my queries