Short Communication

On the Three-Components-in-One Volatile Capacitor

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The first author has realized a volatile capacitor as two partially-contacting resisters with differential current which can take both positive and negative capacitance values. We refer to this as a two-resistor structure capacitor (TRSC). Experimental results on the real device appeared in[1] and will soon appear in[2] and in papers in preparation.
This note provides some plausible elucidations and additions to the underlying philosophy and reasoning of these and forthcoming papers.
In order to make this surprising device more intelligible to a wider audience, we shall explain its use as a three-components-in-one device among many other practical uses.
We shall also give a speculative explanation of the virtual differential current in TRSC from the point of view of the governing role of the tunneling current.

1. Introduction

First we recall the basics of conventional capacitors.

A **capacitor** (which is an idealized form of **condenser**) is a device for storing charge, consisting of two conducting plates (or surfaces) of any shape called **electrodes**, with one carrying positive charge $+Q$, say, and the other negative charge $-Q$. The two electrodes are kept in electrolyte. Charges assembled at pole plates yield capacitance. If the voltage difference between two plates is V and the positive charge is Q, then $C = \frac{Q}{V}$ is called the **capacitance**, whence it follows that

$$Q = CV, \quad I = \frac{dQ}{dt}, \tag{1.1}$$

the second being because the (instantaneous) rate of change of the charge is the current.

A **parallel plate capacitor** is a special type of capacitor with two conducting plates of the same shape and of area A, being at a distance d apart. The capacitance C is given by $C = \frac{A}{4\pi d}$, whence the shorter the distance, the bigger the capacitance, and thus the bigger the amount of charge. The working principle depends on the following EDL.

An **electric double layer** EDL generally means an aggregate of electric dipoles on two thin surfaces, close to each other, on one of which positive charges (holes) are uniformly distributed and on the other of which negative charges (electrons) are found, as discovered by Helmholz in 1879.

The capacitance C in (1.1) of a parallel plate dielectric condenser may be stated as

$$C = \frac{\Delta Q}{\Delta V},$$

which implies that if the plate charge Q increases by ΔQ according to the increase ΔV of voltage V, then C takes positive values and is known as a positive capacitance condenser—an ordinary capacitor. But if the **plate charge** Q **decreases** by ΔQ according to the increase of voltage, then C takes negative values—a negative capacitance condenser. There exists material, rather rare, which has negative capacitance. The strong dielectric HfO_2 is one of rare substances with negative capacitance.

In **ferroelectric substance**s, however, there exists a proper electric field in themselves without an impressed electric field. Hence under an impressed electric field, the plate charges of a ferroelectric substance tend to decrease until it cancels out the inner electric field, presenting the property of negative capacitance. If charges by conductive currents I_c (which is the sum of conductive current I_E and polarized current I_P but we denote it just by I_c) and tunneling current I_T are assembled at both electrodes of a conductive substance, they give rise to negative capacitance. This is not necessarily a dielectric substance. Hence by setting nano electrodes and metal ones at a suitable distance (with nearly vacuum medium), we may construct a condenser with negative capacitance in the wide sense. The conductive currents reach the other pole plate under the resistance of the nano material while tunneling currents reach the other pole plate without resistance through vacuum. *The velocity of I_T is bigger than that of I_c, and I_T lasts up to higher frequency of the impressed DC current (than I_c).*

This has given rise to the new invention[1] of TRSC as a *tunneling current dominating capacitor with capacitance of both signs*. Or we may describe it as **a volatile capacitor made from conductive substance with metal pole plates and a nano pole plate (tunnel pole plate)**. We sometimes call this a NC condenser or a device.

This new capacitor with (positive and) negative capacitance is volatile, meaning that the device works only under exertion of the electro-motive force. This is really surprising since we are mostly familiar with positive capacitance condensers.

2. Basics of circuit and some plausible evidence

We assemble basic knowledge of circuits for easy reference of readers.

Definition 2.1. *The principal components (also called elements) of an electric circuit are capacitor, inductor, resister and transformer. Since they may spend, store electric energy, but do not produce and energy, they are called* ***passive components****. The capacitor, inductor, and resister have only two terminals, so that they are called a* 2 *terminal component or a* 1 *port component.*

We confine to the three main ingredients in (electrical) circuits—capacitor (an idealized condenser) (C), inductor (an idealized coil) (L), and resister (R). The inverse electro-motive force generated by these components is given respectively by

$$V_C = -\frac{1}{C}\int_0^t I\,dt, \tag{2.1}$$

$$V_L = -L\frac{dI}{dt}, \tag{2.2}$$

and Ohm’s law

$$V_R = -RI, \tag{2.3}$$

where $I = I(t)$ and $V = V(t)$ with subscript indicate the current and the voltage of the prescribed component, respectively. We refer to the difference of potential as the voltage V and sometimes write e.

(2.1) is to mean that when the current $I(t)$ is impressed,

$$V_C = V_C(t) = \frac{1}{C}\int_0^t I_C(t)\,dt \tag{2.4}$$

while (2.2) is to mean (in its equivalent form) that when the voltage $V(t)$ is impressed,

$$I_L = I_L(t) = \frac{1}{L}\int_0^t V_L(t)\,dt. \tag{2.5}$$

We state the table on p. 24[3] slightly modified.

EF	Capacitor	Inductor	Resister
Current	(C-1) $V_C(t) = \frac{1}{C}\int_0^t I_C(t)\,dt$	(L-1) $V_L(t) = L\frac{dI_L(t)}{dt}$	$V = RI$
Voltage	(C-2) $I_C(t) = C\frac{dV_C(t)}{dt} = \frac{dQ(t)}{dt}$	(L-2) $I_L(t) = \frac{1}{L}\int_0^t V_L(t)\,dt$	$I = \frac{1}{R}V$

Table 1.1. Characteristics of three components according to the electro-motive force

Here EF=Electro-motive force. (C-1)=(2.4), (L-1)=(2.2), (L-2)=(2.5).

We immediately observe that for a capacitor, (C-1) and (C-2) are connected by integration and differentiation and foe an inductor, (L-1) and (L-2) are related by differentiation and integration. Further we observe

Proposition 2.1. *A capacitor and an inductor are dual components in the sense that when the impressed electro-motive force is changed from voltage to current or conversely, then their roles are interchanged, i.e., (C-1) becomes (L-2) and (C-2) becomes (L-1)*

Duality has been one of our strong driving force. E.g., we have determined[4] all irreducible representations of the icosahedral group—the point group of the fullerene structure C_{60}, without resorting to its dual, the dodecahedral group. We now understand (L-2) means that $L = V_L / \frac{dI_L(t)}{dt}$ and deduce from (C-2) $\frac{dI_L(t)}{dt} = \frac{d^2Q(t)}{dt^2}$. We make a hazard of viewing the currents I_C and I_L are the same on the grounds of duality, which is a required feature to realize a 3-components-in-1 capacitor. Then we obtain

$$L = V / \left(\frac{d^2Q}{dt^2}\right). \tag{2.6}$$

This is consistent with the inductance in Table 1.2 and gives one evidence for a TRSC. Since the negative capacitance implies the presence of inductance, negative capacitance condenser suggests that it can accommodate a two-dimensional inductor device within TRSC without a coil.

We quote the table from p. 16[5] in a slightly different way.

Sys.	General	Electrical	Mechanical
EC	Energy-int. fact. P	Voltage V	Force F
EC	Energy-quant. fact. Q	Charge Q	Displacement y
EC	Energy PQ	Energy $\frac{1}{2}QV$	Energy $\frac{1}{2}Fy$
PEC	Pot. en. reservoir Q	Capacitor	Spring
PEC	Compliance Q/P	Capacitance (1.1)	-
PEC	Kinetic en. reservoir	Inductance	Mass
PEC	Inertance $P/\left(\frac{d^2Q}{dt^2}\right)$	Inductance $L=V/\left(\frac{d^2Q}{dt^2}\right)$	Inertance $M=F/\left(\frac{d^2y}{dt^2}\right)$
PEC	Energy sink	Resistance	Dashpot
PEC	Resistance $P/\left(\frac{dQ}{dt}\right)$	Resistance $R=V/\left(\frac{dQ}{dt}\right)$	Dashpot$R=F/\left(\frac{dy}{dt}\right)$

Table 1.2. Analogous systems

Here EC=Energy and its components, PEC=Passive energy components, int.=intensity, quant.=quantity, fact.=factor, en.=energy, pot.=potential.

By the 3-components-in-1 volatile capacitor, we mean that the function of the capacitor includes that of the inductor within the range of a second order system. In case the inductor is a coil which creates magnetic field and there is magnetic interaction needed, the coil needs to be included in the device.

In the paper[1] some reasoning is given as to why negative capacitance arises. It depends on the virtual differential current which in turn arises from the differential resistance. The key formula is (3.3) which was deduced on the assumption that the resistance is a differentiable function of the potential difference V. In the previous paper[6], graphene oxide (GO) with negative differential resistance was observed, but the existence of negative capacitance was not confirmed in the Au/GO/Au. In[1] it is written that one possibility is that the inequality "the differential current $>$the Ohm current" does not hold in the latter material.

In this note we shall make speculations on a possible conjecture which explains such phenomena.

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3. Volatile TRSC with differential resistance

We shall use some new terminology. A **resistor** is a device consisting of parallel plate electrodes with sufficiently small resistance. It is also referred to as a dielectric. The surface has the nano-structure.

We view resistors as electrodes and vacuum as electrolyte. We construct a device consisting of two resistors whose nano-surfaces are partially in electrical contact. They form an interface across which there flows a current when an outer current is exerted and this may be regarded as a **volatile capacitor** C—volatile because once the current being gone, it stops working as a capacitor. From its structure we may call C a **parallel plate volatile capacitor**. To make clear the role of the dielectric as an electrode, we use the new terminology "resistor". The currents passing through their contacting interface consist of the conduction current I_E, the polarization current I_P, and the tunneling current I_{T_2}. These three flow through the contacting surfaces. There also arises the field emission TC I_{T_1} which flows *in vacuo* without resistance. The TC I_T is the sum of I_{T_1} and I_{T_2}: $I_T = I_{T_1} + I_{T_2}$. For more details, cf. the passages toward the end of this section.

We consider the situation where the constant current I is exerted on C. Then C is equivalent to a parallel circuit consisting of a resistor R and the capacitor C. Let R and V denote the resistance of C and the voltage (potential difference across R) under the exertion of the current. Then both V and R depend on many factors and above all, on the intrinsic circumstances of the interface; in particular, on the distribution of charges which in turn depends on the distance d between two resistors. V and R also depend on each other and we assume that $R = R(V)$, say is a *differentiable function* in V. Then we see that the current going through C is given by

$$I_C = I - I_R = I - \frac{V}{R}, \tag{3.1}$$

where I_R is the Ohm current. By (1.1), the total charge stored in C up to time t is given by

$$Q = \int_{-\infty}^{t} I_C \, dt = \int_{-\infty}^{t} \left(I - \frac{V}{R} \right) dt. \tag{3.2}$$

Differentiating (3.2) with respect to V inside the integral sign (which is possible since the integral may be thought of as a finite one), we deduce that

$$\frac{\partial Q}{\partial V} = \int_{-\infty}^{t} \frac{\partial}{\partial V} \left(I - \frac{V}{R} \right) dt.$$

We simplify the notation and write simply $\frac{d}{dV}$ for the partial derivative $\frac{\partial}{\partial V}$.

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$$\frac{dQ}{dV} = \int_{-\infty}^{t} \frac{1}{R}\left(\frac{V}{R}\frac{dR}{dV} - 1\right) dt. \tag{3.3}$$

We call $\frac{dV}{dR}$ the **differential current**. This is natural since it is the derivative of the Ohm current $\frac{V}{R}$. (3.3) is the key that explains the reason *why* negative capacitance appears. *It shows up when the differential current is bigger than the Ohm current.*

This is a completely new phenomenon not known before nano-material has been invented.

We may speculate that the differential current is the sum of all three currents I_E, I_P, and I_T:

$$I_C = I_E + I_P + I_T. \tag{3.4}$$

In[1], a multi-walled carbon nanotube (MWCNT) material is used as a resistor which has negative differential resistance. As sated above we introduced a vacuum space as electrolyte to facilitate tunneling current through the MWCNT/MWCNT interface, thereby increasing the differential current. This is based on the fact that the tunneling current in a vacuum can result in negative differential resistance. Cf.[7].

Impressing the electromotive current consisting of bias DC and AC on both poles of the condenser, the electromotive force

$$V = V_0 + A\sin\omega t \tag{3.5}$$

arises, where V_0 resp. $A\sin\omega t$ is the electromotive voltage arising from DC resp. AC. The main ingredient of the device is **tunneling current** (TC). The field emission TC I_{T_1} (3.6) occurs, passing through the vacuum area. On the other hand, *due to the randomly distributed MWCNT network in the pellet*, both the (quantum) tunneling current I_{T_2} and the conduction current I_E are possible in the direct contact area. Therefore I_{T_1} is enhanced by I_{T_2}.

The density of I_{T_1} is

$$I_{T_1} = cV^2 \exp\left(-\frac{d}{V}\right), \tag{3.6}$$

where c and d are constants intrinsic to the materia, and V is the potential difference. This makes its effect since it reaches the poles faster than the conduction current.

The intensity of I_{T_2} is

$$I_{T_2} = \frac{q^2 V}{h^2 d}\sqrt{2m\Phi}\exp\left(-\frac{4\pi d}{h}\sqrt{2m\Phi}\right), \tag{3.7}$$

where V is the impressed voltage (3.5), d resp. Φ is the depth resp. the height of the barrier between the plates. Although both TCs are of exponential decay as with the transient current in a circuit in which the electromotive force is gone abruptly by switching off, they are dominating current

4. Possible elucidation of implications of (3.3)

(3.3) is the key for explaining the existence of negative capacitance. But this depends on the rather strong assumption that the resistance is a differentiable function of V. But this may not generally hold since the surfaces of electrodes are not smooth but rather of *fractal nature*. The integration (3.2) is not problematic since it collects small pieces. But differentiability is a rather strong requirement because it requires smoothness at infinitesimal level. But fractal figures are never smooth, cf. e.g., the Takagi tent function. Tus differentiability occurs only in an ideal state. Thus (3.3) is to be interpreted to mean

$$\frac{dQ}{dV} = \int_{-\infty}^{t} \frac{1}{R} \left(\frac{V}{R} \tilde{I}_C^{-1} - 1 \right) dt, \tag{4.1}$$

where $\tilde{I}_C$ is the varying current going through C, which corresponds to the virtual differential current but

$$\tilde{I}_C \approx I_E + I_P + I_T, \tag{4.2}$$

where $\approx$ means approximation. As is stated in[1], the tunneling current in a vacuum can increase the amount of $\tilde{I}_C$ ([7]), so that it exceeds the Ohm current, thus providing negative values for $\frac{dQ}{dV}$. After integration, this gives rise to negative capacitance.

5. Practical use: a 3-components-in-1 device

we assemble remarkable features of the device from[8].

- It is possible to make capacitance either negative or positive.
- The value of resistance can be set satisfying the requirements of various circuits.
- **Volatility.** Charges will be lost when AC electromotive force or bias DC is removed.

These are sufficient to construct the 3-components-in-1 device in which three functions of a circuit (resistance, condenser and coil) can be accumulated, providing a chance for a new integrated circuit. It is hoped that this will give rise to new merits as an integrated circuit with an inductance.

There are many other new features, e.g., “the value of capacitance depends on the frequency and amplitude of the impressed AC current, DC bias, and the structure.” This may be used to control the phase difference in power transmission lines. We shall return to this elsewhere.

References

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