Review of: "Multiplicity of solutions for nonlocal fractional equations with nonsmooth potentials"

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Potential competing interests: No potential competing interests to declare.

In this paper, nonlocal diffusions equations with fractional derivative and nonsmooth potentials are considered. The main contribution is to investigate existence results for weak solutions of the present problem.

This study has some positive points as follows:

- Three different existence results for weak solutions are contructed under distinct assumptions on the data.
- The study here can be considered as an improvement of recent studies related to the mentioned problem.
- Since the model contains some fractional and nolocal operators, this work becomes more complicated than the standard case. Consequently, without sharp estimates, the existence results could not be obtained as desired.

The result is clear and the basic calculations are right as far as I could check. I recommend this paper isaccepted but needs some minor revisions. All of them aim at improving the quality of this paper and making it be more clear.

1. It would be better if the authors add a remark to discuss in detail why “n>2s” is required in the problem.
2. Look at the definition of X_0 in page 3, what is “a.a.”? Please explain in more detail.
3. It would be better if in Definition 2.3, u is defined as a “weak solution” instead of “solution”.
4. A section for conclusion and some discussions should be added at the end of the paper.
5. Proposition 2.1, please explain what is the notation o in

$$h^\circ(u;z)$$

Fractional operator play an important role in modeling physical processes as the authors introduce in Introduction. However, the reference cited for studies on this operator is not much. It would be better if very rencet references are cited further (even in the stochastic case, this operator is studied much). For example, [1] (2017) Stochastic Burgers’ equation with fractional derivative driven by multiplicative noise. *Computers & Mathematics with Applications*, *74*(12), 3195-3208; [2] (2022) Existence and regularity results for stochastic fractional pseudo-parabolic equations driven by white noise. *Discrete & Continuous Dynamical Systems-Series S*, *15*(2).