

Review of: "Periodic second-order systems and coupled forced Van der Pol oscillators"

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In this work, the authors present a general second-order differential coupled system with dependencies on both variables, z, w , their derivatives, z', w' , and on time, t . They proved sufficient conditions for the existence of periodic solutions for problems (1) and (2) without assuming any type of periodicity in the non-linearities. To localize an existing periodic solution, no order requirement is made between lower and upper solutions. Therefore, they increase the range of possibilities when considering well-ordered and not well-ordered localizing functions, α_i, β_i , see Example 5. The methodology is successfully applied to a system of coupled Van der Pol oscillators with forcing terms. Under the conditions of the main result, Theorem 4, the authors prove the existence of a periodic solution for problems (1) and (2) and localize it within a strip bounded by shifted lower and upper solutions, α_0, β_0 , whose requirements follow Definition 3. When applying Theorem 4 to a specific second-order system, one must verify that the non-linearities satisfy the Nagumo-type condition of Definition 1, as well as the required monotonicities. Note that Theorem 4 does not guarantee the existence of a non-trivial solution. That guarantee arises with the nature of the system under study, whether it allows for constant solutions or not.

Nicely written manuscript with full justification.