

Review of: "Circuits, Currents, Kirchhoff, and Maxwell"

Fernando Minotti¹

¹ University of Buenos Aires

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The author presents an interesting study of the concept of conservation of total current (conduction plus displacement currents). In particular, in its application to biochemical effects.

I agree in general with the considerations presented in this work, but mildly disagree on the importance vested in this law by itself. I expand on this in the following considerations:

1) In my view, the important point is the existence of a conservation law for a scalar magnitude in its most general form:

$$\frac{\partial s}{\partial t} + \text{div}(\mathbf{q}) = 0$$

One can then define a vector field \mathbf{U} that satisfies

$$\text{div}(\mathbf{U}) = s$$

so that the conservation law is written as

$$\text{div}\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{q}\right) = 0$$

which further implies that

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{q} = \text{curl}(\mathbf{W})$$

for another vector field \mathbf{W} .

We thus see that the so called non-homogeneous Maxwell equations are the natural expression of an underlying conservation law. What particularize a theory with such conservation law are the expressions of

$$\text{div}(\mathbf{W})$$

and of

$$\text{curl}(\mathbf{U})$$

For instance, by taking

$$\text{div}(\mathbf{W}) = 0$$

and

$$\text{curl}(\mathbf{U}) = -k \frac{\partial \mathbf{W}}{\partial t}$$

with k a positive constant, one obtains a Maxwell-like theory.

If one instead considers

$$\text{div}(\mathbf{W}) = 0$$

and

$$\text{curl}(\mathbf{U}) = -k\mathbf{W}$$

a diffusive theory (with no waves) results. Of course, the possibilities are endless, but the important fact is that the law considered by the author as so fundamental does not seem to be so. What appears as really fundamental is the underlying conservation law, which in the case under study is of course that of charge.

2) The considerations in the previous point also highlights the importance of the “missing” relations, those not implied by the conservation law. For instance, in the case of the circuits mentioned by the author, of GHz frequencies, Faraday’s law of induction is extremely important. The stray capacitances mentioned as needed to design real circuits are enough only in the quasi-stationary regime, at low enough frequencies. Besides, for the design of circuits the Kirchhoff law considered in the paper must in all cases be complemented with Kirchhoff’s second law, not mentioned in the paper.

3) A final point is that the mention in the paper of the interstellar vacuum as incapable of conduction currents is misleading, as the works by Alfvén¹ and others show. In interstellar and even in intergalactic space charge densities exist that sustain huge conduction currents, manifested in the observed large scale, structured magnetic fields in those regions. It is also interesting to mention that Alfvén himself points out the convenience of analyzing cosmic plasmas in terms of the currents, rather than in terms of the fields, quite in line with the ideas of the author.

¹Hannes Alfvén, “Cosmic Plasmas”, D. Reidel Publishing Company, Dordrecht, Holland, 1981.