Research Article

Special Relativity Temperature Transformation for a Monatomic Ideal Gas

Joseph Bevelacqua[1]

1. Bevelacqua Resources, United States

A temperature transformation is derived within the scope of special relativity by examining motion in two translating inertial reference frames. The transformation is based on the four-momentum. A translating monatomic ideal gas is assumed, and the energy component of the momentum four-vector is based on an assumed Maxwell-Boltzmann distribution for the energy. The desired transformation is a complex result that depends on the temperature and momentum of the gas and the relative velocity of the inertial reference frames. In addition, the temperature transformation admits the possibility of negative temperatures.

Corresponding author: J. J. Bevelacqua, bevelresou@aol.com

1. Introduction

The temperature transformation relationship of special relativity has been reviewed extensively, but no consensus has emerged. Landsberg[1] observed that a number of authors suggested that the temperature relationship should have the form of Eq. 1. In Eq. 1, T(v) is the temperature (e.g., of a monatomic ideal gas) in a moving inertial reference frame with velocity (v), relative to the rest frame temperature T(0), and b is an integer. To complete the specification of Eq. 1, γ and β are defined in Eqs. 2 and 3.

Various values for the constant b have been suggested. Without experimental data, the validity of Eq. 1 and the value of b have yet to be determined. Example values have been proposed by various authors including b = -1 by Planck[2], 0 by Landsberg[3], and +1 by Ott[4]. In the Planck formulation a moving body appears to cool, while Ott's result suggests a hotter moving body. Landsberg's result suggests that temperature is an invariant quantity.

These relationships are artifacts of assumptions regarding thermodynamic properties. For example, the Planck, Landsberg, and Ott relationships result from the second law being invariant, thermodynamics being form invariant, and temperature invariance, respectively[5][6][7]. However, a unique temperature relationship has yet to be determined from conventional thermodynamic considerations. Alternative formulations were suggested by Bevelacqua[5][6][7].

$$T(v) = \gamma^{b}\, T(0) \tag{1}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^{2}}} \tag{2}$$

$$\beta = \frac{v}{c}\,. \tag{3}$$

$$\begin{pmatrix} 3kT'/2c \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3kT/2c \\ p_x \\ p_y \\ p_z \end{pmatrix} \tag{4}$$

$$\frac{3kT'}{2c} = \frac{3\gamma kT}{2c} - \beta\gamma p_x \tag{5}$$

$$p'_x = -\frac{3\gamma k\beta T}{2c} + \gamma p_x \tag{6}$$

$$p'_y = p_y = 0 \tag{7}$$

In[5], the author derived a temperature relationship based on kinematic considerations that did not resemble the form summarized in (1). These results suggested that moving bodies appear to cool, but the degree of cooling depended on the form of the equipartition theorem utilized in the formulation[5].

A four-vector with temperature as the fourth component was utilized by Bevelacqua[6]. The four-vector approach suggested that a moving body may exhibit any of the aforementioned temperature relationships[2][3][4] depending on the nature of the four-vector and relative velocity of the inertial reference frames.

Additional work suggested that form invariance of the first and second laws of thermodynamics leads to a temperature transformation that implies that a moving body appears to cool[7]. The constraint of the continuous transformation properties of a 4-vector with temperature as the first component also suggested that temperature does not have a simple interpretation in terms of the components of a four-vector. The difficulties associated with describing black-body radiation and temperature as a component of a four-vector has also been discussed[7]. These issues present challenges to the concept of a continuous temperature transformation within special relativity.

Additional perspectives are provided in[8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24][25][26][27][28][29][30][31][32][33][34][35][36][37][38][39][40][41][42][43][44]. This collection of references reinforces the fact that there is currently no consensus regarding the temperature transformation within the scope of special relativity

The four-vector approaches[6][7] involved *ad hoc* assumptions. This paper utilizes a more physical approach that is based on the momentum four-vector assuming a Maxwell-Boltzmann distribution of the energy of a monatomic ideal gas. The proposed approach has well-defined assumptions that facilitate the derivation of a temperature transformation within the scope of special relativity. As such, both the assumptions and results present the possibility to be investigated experimentally.

2. Assumptions

In order to better define the proposed approach for determining a temperature transformation within the scope of special relativity, it is necessary to specify a set of assumptions. These assumptions and their validity assist in determining the viability and appropriateness of the proposed methodology. These assumptions include:

1. The proposed transformation specifies a relationship between the temperatures of a monatomic ideal gas in two inertial reference frames.

2. In the rest reference frame (K), a monatomic ideal gas, having a Maxwell-Boltzmann energy distribution, has a temperature T with associated energy 3kT/2 where k is the Boltzmann constant[45].
3. In the moving reference frame (K′), a monatomic ideal gas, having a Maxwell-Boltzmann energy distribution, has a temperature T′ with associated energy 3kT′/2.
4. Frame K′ moves with a uniform velocity v in the positive x direction relative to a fixed frame K.
5. The K’ frame initially coincides with K.
6. There is no rotational motion. The only motion is translation.

$$p'_z = p_z = 0 \tag{8}$$

$$T' = \gamma T - \frac{2c\beta\gamma}{3k} p_x \tag{9}$$

$$T' = \gamma\left(T - \frac{2\,v\,p_x}{3k} \right) \tag{10}$$

$$T' = \gamma\, T_{eff} \tag{11}$$

$$T_{eff} = T - \frac{2\,v\,p_x}{3k} \tag{12}$$

$$p_x > \frac{3kT}{2v} \tag{13}$$

3. Theoretical Model

The proposed temperature transformation is based on the momentum four-vector and its transformation properties. For motion of the assumed monatomic gas in the positive x direction, the momentum four-vector transforms as defined in Eq. 4. This transformation leads to the relationships summarized in Eqs. 5-8.

The desired temperature transformation for a monatomic ideal gas is obtained from Eq. 5 and is provided in Eq. 9. Eq. 9 does not have the simple transformation relationship proposed in Eq. 1. The first term in Eq. 9 resembles the Ott relationship[4] with b = 1, but the second term complicates the transformation relationship. The temperature transformation result depends on the relative magnitude of these two terms.

Eq. 9 can be rewritten in the form of Eq. 1 to yield Eq. 10. Eq. 10 is rewritten in the simplified form of Eq. 11 where T_{eff} is defined in Eq. 12. However, T_{eff} is not the same temperature implied by Eq. 1.

4. Results and Discussion

Eq. 10 does not have the simple functional form of Eq. 1 with well-defined integer values for b. As such, the simple form of Eq. 1 is not obtained, but Eq. 11 provides an analogue Ott relationship.

Given the assumptions summarized in Section 2, a definitive temperature transformation does not result, but depends on a number of variables noted in Eq. 10. In addition, Eq. 10 admits the possibility of negative temperatures if the momentum p_x satisfies Eq. 13.

Although negative temperatures are a unique concept, their basis and characteristics have been addressed[46][47][48][49]. The concept of negative temperature and the consequences of Eq. 10 merit additional investigation.

5. Conclusions

A temperature relationship is derived within the scope of special relativity by examining motion in two translating inertial reference frames. A monatomic ideal gas is assumed, and the energy component of the momentum four-vector is

based on an assumed Maxwell-Boltzmann energy distribution. The temperature relationship is more complex than the simple relationship of Eq. 1, and also admits the possibility of negative temperatures.

References

1. ^*Landsberg PT. Thought Experiment to Determine the Special Relativistic Temperature Transformation. Phys Rev Lett. 1980;45:149-150.*
2. a, b*Planck M. Zur Dynamik bewegter Systeme. Annalen der Physik. 1908;26:1-34.*

3. ^a, b^ *Landsberg PT. Does a Moving Body Appear Cool? Nature. 1966;212:571–572.*
4. ^a, b, c^ *Ott H. The Lorentz Transformation of Heat and Work. Z Phys. 1963;175:70-104.*
5. ^a, b, c, d^ *Bevelacqua JJ. Temperature Transformation in Special Relativity. Physics Essays. 1989;2:230-233.*
6. ^a, b, c, d^ *Bevelacqua JJ. The Second Law and Its Impact on Relativistic Thermodynamics. Physics Essays. 1992;5:3-9.*
7. ^a, b, c, d, e^ *Bevelacqua JJ. Challenges to a Temperature Transformation in Special Relativity Derived from Form Invariance of the First and Second Law of Thermodynamics. Physics Essays. 2004;17:288-194.*
8. ^ *Einstein A. On the Principle of Relativity and the Consequences Derived from It. Jahrbuch Radioakt Elektron. 1907;4:441-462.*
9. ^ *Fremlin JH. Does a Moving Body appear Cool? Nature. 1967;213:277-277.*
10. ^ *Landsberg PT. Does a Moving Body Appear Cool? Nature. 1967;214:903-904.*
11. ^ *Noerdlinger PD. A Moving Body must "appear" Cool. Nature. 1967;213:1117-1118.*
12. ^ *Williams IP. Temperature of a Moving Body. Nature. 1967;213:1118-1118.*
13. ^ *Arzelies H. Thermodynamique Relativiste et Quantique. Gauthier-Villars, Paris; 1968.*
14. ^ *van Kampen NG. Relativistic Thermodynamics of Moving Systems. Phys Rev. 1968;173:295-301.*
15. ^ *Landsberg PT, Jones KA. The problem of moving thermometers. Proc Roy Soc London, Series A. 1968;206:477-486.*
16. ^ *Landsberg PT. A Critical Review of Thermodynamics. Mono Books, Baltimore, MD; 1970.*
17. ^ *ter Haar D, Wergeland H. About the Temperature of Moving Bodies. Physics Rep. 1971;1:31-54.*
18. ^ *Grøn O. The Asynchronous Formulation of Relativistic Statistics and Thermodynamics. Il Nuovo Cimento B. 1973;17:141-165.*
19. ^ *Paiva-Veretennicoff I. Is Galilean relativistic thermodynamics well defined? Physica (Utrecht). 1974;75:194-197.*
20. ^ *Buchdahl HA, Simpson MA. Remark on the equilibrium of moving systems. Am J Phys. 1975;43:1041-1045.*
21. ^ *Eimerl D. On relativistic thermodynamics. Annals of Physics. 1975;91:481-498.*
22. ^ *Agmon N. Relativistic transformations of thermodynamic quantities. Found Phys. 1977;7:331-339.*
23. ^ *Treder HJ. Die Strahlungs-Temperatur bewegter Körper. Ann Phys (Leipzig). 1977;34:23-29.*
24. ^ *Treder HJ. Zur Strahlungstemperatur bewegter Körper. Ann Phys (Leipzig). 1978;35:77-78.*
25. ^ *Kriznan JE. Temperature and relativistic thermodynamics. Phys Lett A. 1979;71:174-178.*

26. ^Newburgh RG. Relativistic Thermodynamics: Temperature Transformations, Invariance, and Measurement. Il Nuovo Cimento B. 1979;52:219-228.

27. ^Goodinson PA, Luffman BL. The relativistic transformation law for an ideal-gas scale of temperature. Il Nuovo Cimento B. 1980;60:81-88.

28. ^Newburg RG. Comments on the derivation of the Ott relativistic temperature transformation. Phys Lett A. 1980;78:17-18.

29. ^Ehrlich P. The concept of temperature and the dependence on the laws of thermodynamics. Am J Phys. 1981;49:622–632.

30. ^Isreal W. Thermodynamics of relativistic systems. Physica A. 1981;106:204–214.

31. ^Landsberg PT. Einstein and statistical thermodynamics. Europ. J. Phys. 2 (1982): 203-207.

32. ^Walz DR, Noyes HP, Carezani RL. Calorimetric test of special relativity. Phys. Rev. A 29 (1984): 2110-2114.

33. ^Hsu JP. Four-dimensional symmetry from a broad viewpoint IV – Covariant statistical mechanics with common time. Il Nuovo Cimento B 80 (1984): 201-216.

34. ^Ohsumi Y. Reaction kinetics in special and general relativity and its applications to temperature transformation and biological systems. Phys. Rev. A 36 (1987): 4984-4995.

35. ^Tolman RC. Relativity, Thermodynamics, and Cosmology. Dover, New York (1987).

36. ^Shaffer J. Kinematical and dynamical temperature transformations. Il Nuovo Cimento B 103 (1989): 259-263.

37. ^Menon VJ, Agrawal DC. Concept of relativistic temperature via the Crawford technique. Am. J. Phys. 59 (1991): 258-260.

38. ^Aldrovandi R, Gariel J. On the riddle of moving thermometers. Phys. Lett. A 170 (1992): 5-19.

39. ^Liu C. Einstein and relativistic thermodynamics. Brit J. Hist. Sci. 25 (1992): 185-206.

40. ^Landsberg PT, Matsas GEA. Laying the ghost of the relativistic temperature transformation. Phys. Lett. A 223 (1996): 401-403.

41. ^Trout KP, Greiner AJ. Special relativistic temperature transformation. Il Nuovo Cimento B 113 (1998): 1439-1448.

42. ^Bıro TS, Van P. About the temperature of moving bodies. Foundations of Physics: EPL 89 (2010): 30001-1–30001-6.

43. ^Mares JJ, Hubik P, Sestak J, Spicka V, Kristofik J, Stavek J. Relativistic transformation of temperature and Mosengeil-Ott's antinomy. Physica E-Low-Dimensional Systems & Nanostructures 42 (3) (2016): 484-487.

44. ^*Wallace D. On relativistic temperature. arXiv:2307.00354v2 [cond-mat.stat-mech] 1 Jul 2024 (2024).*

45. ^*Reif R. Fundamentals of Statistical and Thermal Physics. McGraw-Hill, New York (1965).*

46. ^*Klein J. Negative Absolute Temperatures. Phys. Rev. 104 (1956): 589-589.*

47. ^*Ramsey NF. Thermodynamics and Statistical Mechanics at Negative Absolute Temperatures. Phys. Rev. 103 (1956): 20-28.*

48. ^*Braun S et al. Negative Absolute Temperature for Motional Degrees of Freedom. Science 339 (2013): 52-55.*

49. ^*Abraham E. Physics of negative absolute temperatures. Phys. Rev. E 95 (2017): 012125-1-012125-8.*

Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.

 doi.org/10.32388/C20DPQ