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Nuclear Level Density in the X(766, 242) System

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Abstract

Level densities for the X(766, 242) system are calculated using a single particle methodology incorporating the Rost interaction and pairing interaction of Blomqvist and Wahlborn. Energy level fluctuations are smoothed utilizing the constant temperature, power law, and equidistant model functional forms. The constant temperature relationship is used to model the density and total number of energy levels. The total number of levels is also compared with the predictions of the power law and equidistant models. Systematics of level densities for theoretical superheavy nuclei are discussed.

1.0. Introduction

Nuclear level density data are available for many nuclei, but most calculations are performed for nuclei with A ≥ 20[1]. Most calculations utilize realistic single particle levels. Accordingly, shell effects are incorporated into these calculations. The only residual interaction is a pairing force, and the calculations lead to approximate nuclear energy level densities. These approaches or other approximate calculations have not been extended to theoretical superheavy nuclei.

Previous work investigated superheavy nuclei including the Z=204 A=610 system X(610, 204)[2,3], X(636, 204)[4,5], X(692, 214)[4,6] , X(730, 226)[7,8], and X(766, 242)[7]. Such an extension is important because superheavy nuclei, such as X(766, 242) represent an endpoint for energy level density measurements. In addition, the X(766, 242) system is more than twice as heavy as any known nucleus. Previous efforts investigated the other extreme in level systematics by evaluating the ^{4}He system[9].

2.0 Calculational Methodology

Since the method for calculating single particle energies in a spherically symmetric potential is well-established, only salient features are provided. The model used to describe the particle (i) plus core (c) system represents an application of the standard method of Lukasiak and Sobiczewski[10] and Petrovich et. al.[11].

The binding energy E_{NLSJ} of a particle in the field of a nuclear core is obtained by solving the radial Schrödinger Equation

$$\left[\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2}-\frac{L(L+1)}{r^2}\right)-E_{NLSJ}-V_{LSJ}(r)\right]U_{NLSJ}(r)=0(1)$$

where r is the radial coordinate defining the relative motion of the nuclear core and the particle; $V_{LSJ}(r)$ is the model interaction[12,13]; E_{NLSJ} is the core plus particle binding energy; $U_{NLSJ}(r)$ is the radial wave function; and L, S, and J are the orbital, spin, and total angular momentum quantum numbers, respectively. The N quantum number is the radial quantum number, and μ is the reduced mass. For the present application, V_{LSJ} is defined as:

$$V_{LSJ}(r) = V_0(r) + V_{so}(r) + V_C(r)(2A)$$

$$V_o(r) = -\frac{V_o}{1+B_o}(2B)$$

$$V_{so}(r) = -V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2\frac{1}{a_{so}r}\frac{B_{so}}{[1+B_{so}]^2}F(L,S,J)(2C)$$

$$V_C(r) = z_i Z_C e^2 C(r)(2D)$$

where

$$B_j = \exp[(r-R_j)/a_j](3)$$

and

$$R_0 = r_0A^{1/3} = R_C; R_{so} = r_{so}A^{1/3}(4)$$

The parameters V_0, r_0, and a_0 are the strength, radius parameter, and diffuseness for the central potential. Similarly, V_{so}, r_{so}, and a_{so} are the corresponding parameters for the spin-orbit potential. To complete the specification of Eq. 2, the following terms are defined:

$$F(L,S,J) = J(J+1) - L(L+1) - S(S+1)(5)$$

and

$$C(r) = \frac{1}{2R_C}\left(3-\left(r/R_C\right)^2\right); r < R_C(6)$$

$$C(r) = 1/r, r \geq R_C(7)$$

For the Coulomb potential, it is assumed that the particle is a point charge of magnitude z_ie. The core has a charge Z_Ce uniformly distributed through a sphere of radius R_C. Since the potential is not a function of the spherical coordinates, the solution of the angular equation is most easily expressed in terms of spherical harmonics $Y_{LM}(\theta, \varphi)$.

The total bound-state wave function Ψ for the relative motion of the core plus particle, interacting through a spherically symmetric potential, is given by a product of space and spin wave functions:

$$\Psi = \frac{1}{r} U_{NLSJ}(r) \sum_{M_L M_S} C(L, M_L, S, M_S; JM) Y_{LM_L}(\theta, \phi) \chi_{SM_S} \quad (8)$$

where the sum is over M_L and M_S, M_L and M_S are the projections of angular momentum and spin, and χ is the spin wave function. For the calculation of single particle energy levels, N, L, S, and J specify the quantum numbers of the single particle level.

The binding energy of a single particle level is obtained by rewriting the radial Schrödinger equation in the form

$$\left[\frac{d^2}{dr^2} - k(p, r)\right] U(p, r) = 0 \quad (9)$$

where

$$U(p, r) = U_{NLSJ} \quad (10)$$

and

$$k(p, r) = \frac{L(L+1)}{r^2} + \frac{2\mu}{\hbar^2}\left[E_{NLSJ} + V_{LSJ}(r)\right] \quad (11)$$

The model searches for values of p in order to obtain the binding energy E_{NLSJ} for a given potential. The method of searching for p is provided by Brown, Gunn, and Gould[14], and the methodology of Ref. 15 to obtain a converged solution for E_{NLSJ}.

3.0 Nuclear Interaction

Nuclear stability with respect to alpha decay, beta decay, positron decay, electron capture, and spontaneous fission is addressed using the method previously published by the author and coworkers[2,4,7,11] that is similar to the approach of Ref. 10. The single particle level spectrum is generated using a Woods-Saxon potential. The parameters of the potential are obtained from a fit to the particle levels in ^{209}Pb and ^{209}Bi performed by Rost[12]. The central potential strength has a standard form and can be explicitly defined as

$$V_0 = 51.6\left[1 \pm 0.73 \frac{N - Z}{A}\right] \quad (12)$$

where the upper (lower) sign applies to protons (neutrons). The remaining parameters were held constant and are given by Rost[12]: r_o = 1.262 (1.295) fm, r_{so} = 0.908 (1.194) fm, a_o = a_{so} = 0.70 fm, and λ = 17.5 (28.2) for protons (neutrons)[11,12]. V_{so} is related to λ by the relationship[12]:

$$V_{so} = \frac{\lambda V_0}{180} \quad (13)$$

The scaling relationship of Eq. 12 yields reasonable fits to observed single particles levels in ^{120}Sn and ^{138}Ba[11]. The pairing correction term of Blomqvist and Wahlborn[13] is used in the calculations presented herein. The pairing correction improves the predicted energies of occupied levels in ^{120}Sn, ^{138}Ba, and ^{208}Pb[11].

4.0 Single Particle Level Calculations

The X(766, 242) level calculations[7] using the methodology of Section 2.0 are summarized in Fig. 1. These calculations suggest the X(766, 242) system has an alpha decay half-life of 2.1x10^{-5} s, and is stable with respect to beta decay.

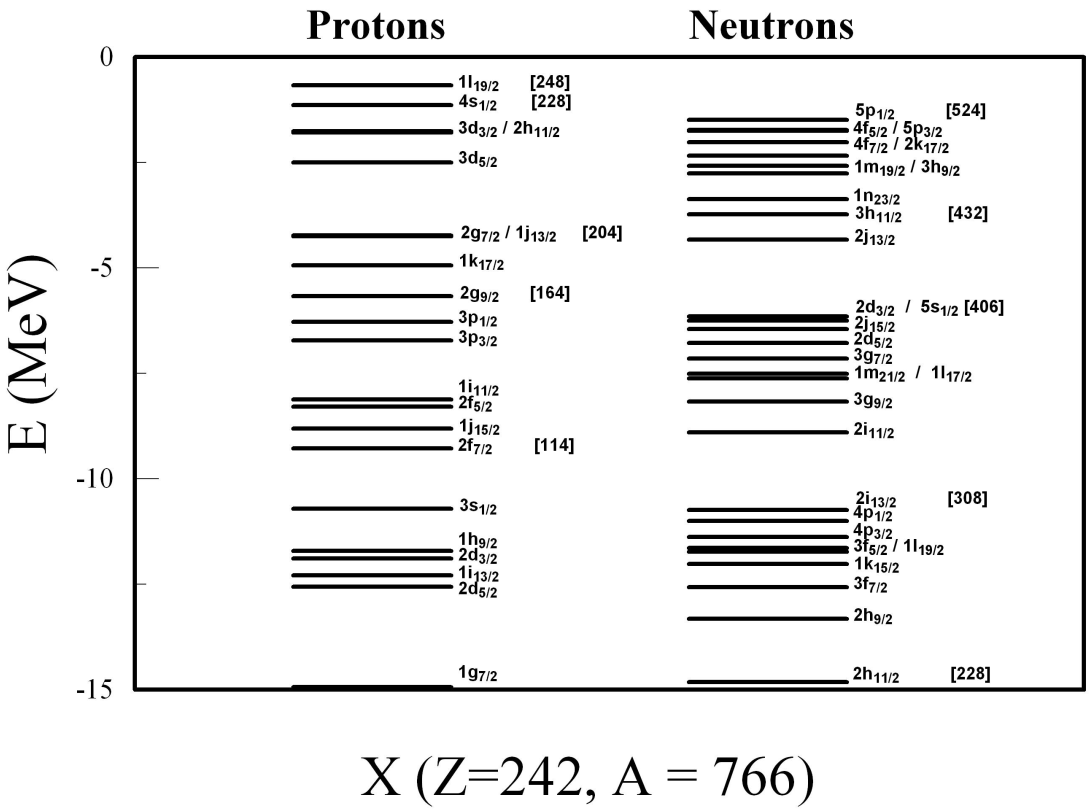

Fig. 1 Calculated single particle energy levels for nucleus X(766, 242). The notation A/B/C is used to indicate adjacent energy levels with level A more tightly bound than level B, which is more tightly bound than level C.

The results summarized in Fig. 1 are used as the basis for the level systematics summarized in Sections 5.0, 6.0, and 7.0. These results are evaluated using the constant temperature model[16] and equidistant model[17-20].

Refs. 21-23 illustrate current applications of the nuclear level density concept. These include determination of energy dependent ratios of level-density parameters in superheavy nuclei[21], nuclear astrophysical reaction rates[22], and machine learning applications[23].

5.0 Constant Temperature Model

Experimental data involving level densities are often analyzed with the constant temperature formula[16]

$$\rho(E) = a\exp(E/T) \quad (14)$$

where ρ(E) is the number of energy levels per MeV, and **a** and T are constants which are determined from a fit to the X(766, 242) single particle levels of Fig. 1. It should be recognized that these simple level density approximations may not accurately reproduce the level density of a nucleus which has marked structure in its single particle levels. As such, the investigation of level density parameters as a function of mass leads to a better understanding of level density systematics as well as individual differences between nuclei.

Within the constant temperature model, the functional form for the level density ρ(E) is expected to be a simple exponential from experimental measurements on A = 36-66 even-even nuclei[16]. The information of Fig. 1 can be fit to the form of Eq. (14), and the values **a** = 0.302412158 and T = 1 / 0.156406614 = 6.3936 are obtained. These values are similar to the fits to the X(610, 204) system (**a** = 5.00473206 and T = 1 / 0.052170492 = 19.1679)[3], X(636, 204) nucleus (**a** = 5.75443276 and T = 1 / 0.045587584 = 21.9358)[5], X(692, 214) (**a** = 4.68385657 and T = 1 / 0.069647127 = 14.3581)[6], and X(730, 226) (**a** = 1.12896845 and T = 1 / 0.104718957 = 9.5494)[8] . Model results for the X(766, 242) system are shown in Fig. 2.

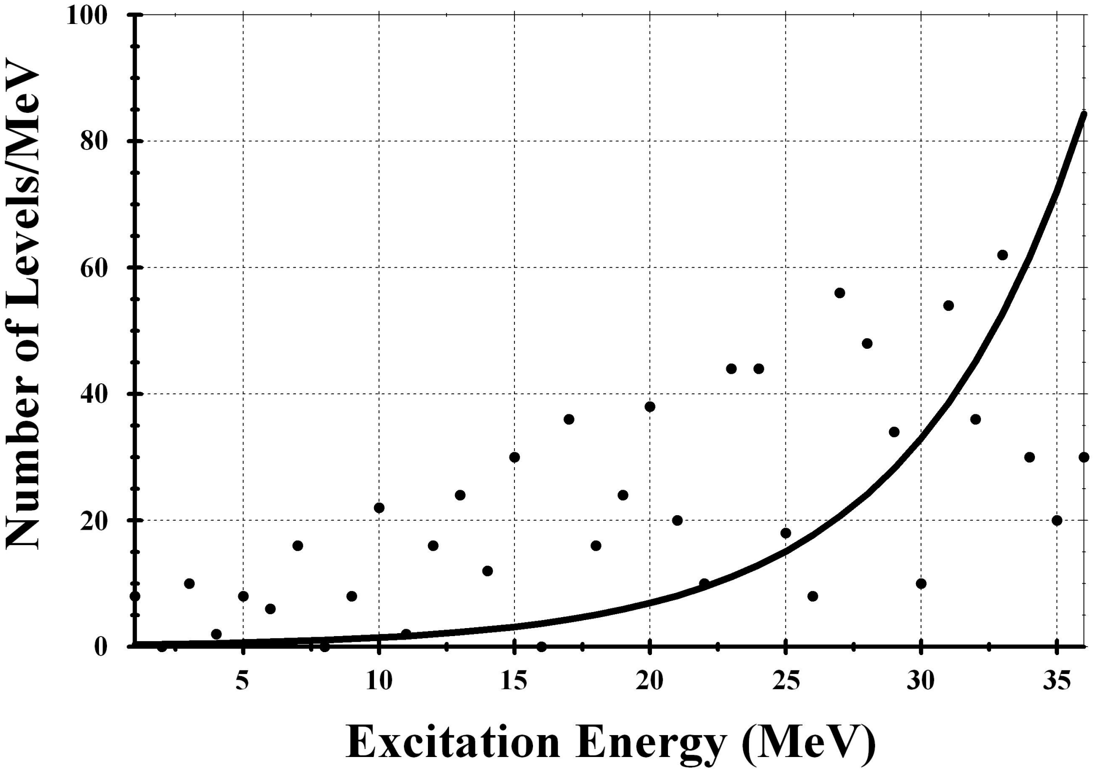

Fig. 2 Energy level density for X(766, 242) using the constant temperature model. The "•" symbol represents the state density for a 1 MeV energy bin. The solid curve is a fit to the constant temperature functional form ρ(E) = a exp(E/T) with **a** = 0.302412158 and T = 1 / 0.156406614 = 6.3936.

As expected there is considerable fluctuation in ρ(E), and this can be minimized by considering the total number of levels N(E) described by the functional form of the constant temperature model

$$N(E) = c\exp(E/d)(15)$$

When the X(766, 242) levels N(E) of Fig. 1 are fit to the constant temperature model of Eq. (15), the curve shown in Fig. 3 is obtained. The parameters c = 17.48621718 and d = 1 / 0.121254076 = 8.2471 are obtained. These values are similar to the parameters derived for the X(610, 204) system (c =8.47138352 and d = 1 / 0.136811694 = 7.3093)[3], X(636, 204) nucleus (c = 8.075645 and d = 1 / 0.139798416 = 7.1532)[5], X(692, 214) (c = 8.47666017 and d = 1 / 0.144936667 = 6.8996)[6], and X(730, 226) (c = 10.53369499 and d = 1 / 0.137789007 = 7.2575)[8].

Eq. 15 underestimates the number of energy levels for the X(766, 242) system below about 27 MeV and overestimates N(E) above about 29 MeV. A comparison of the X(766, 242) system d value to lighter systems using the constant temperature model is summarized in Table 1.

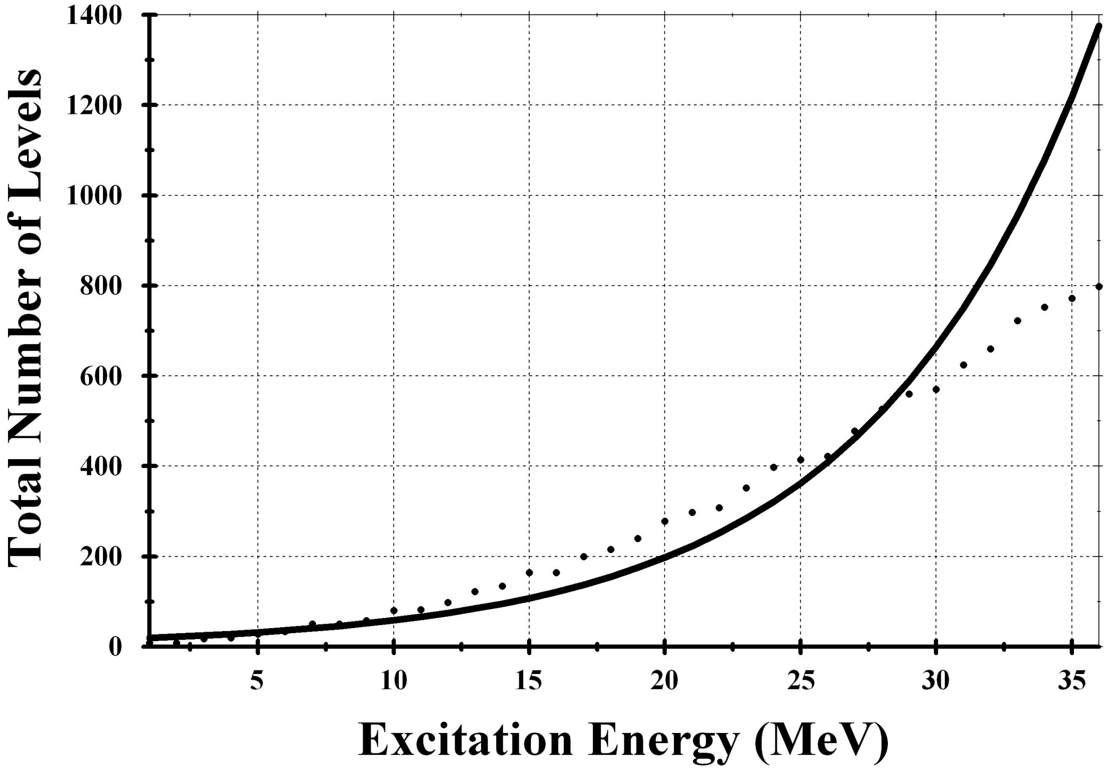

Fig. 3 Total number of energy levels N(E) for X(766, 242) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the constant temperature functional form N(E) = c exp(E/d) where c = 17.48621718 and d = 1 / 0.121254076 = 8.2471.

Table 1 Constant Temperature Model Parameters for Nuclear Densities[a-g]

Nucleus	d (MeV)
^{4}He	2.79[a]
^{36}Ar	1.87
^{38}Ar	1.47
^{40}Ca	1.73
^{50}Cr	1.29
^{52}Cr	1.43
^{54}Cr	1.22
^{54}Fe	1.40
^{56}Fe	1.40
^{58}Fe	1.31
^{68}Zn	0.90
X(610, 204)	7.31[b]
X(636, 204)	7.15[c]
X(692, 214)	6.90[d]
X(730, 226)	7.26[e]
X(766, 242)	8.25[f]

[a] Ref. 6. [b] Ref. 3. [c] Ref. 5. [d] Ref. 6. [e] Ref. 8. [f] This work. [g] All others Ref. 13.

The results of Table 1 suggest the level density d parameter behaves differently in the A > 600 mass region compared with A < 70 nuclear systems. Superheavy d values in the X(610, 204), X(636, 204), X(692, 214), X(730, 226), and X(766, 242) systems are about a factor of four larger than the A < 70 values.

6.0 Power Law Model

The fit to the total number of levels N(E) in X(766, 242) is also modeled using the power law functional form

$$N(E) = aE^{b} \quad (16)$$

where **a** = 2.99080945 and b = 1.50780355. Fig. 4 summarizes the use of Eq. 16 to fit the levels of Fig, 1. Eq. 16 provides an improved fit to that of Eq. 15 that has been traditionally utilized to describe the total number of energy levels as a function of energy. The X(766, 242) parameters are similar to the values derived from the X(610, 204) system (**a** = 1.25223103 and b = 1.69492108)[3], X(636, 204) (**a** = 0.82089983 and b = 1.84100287)[5], and X(692, 214) (**a** = 0.8529546 and b = 1.87210143)[6], and X(730, 242) (**a** = 1.24371715 and b = 1.76223466)[8].

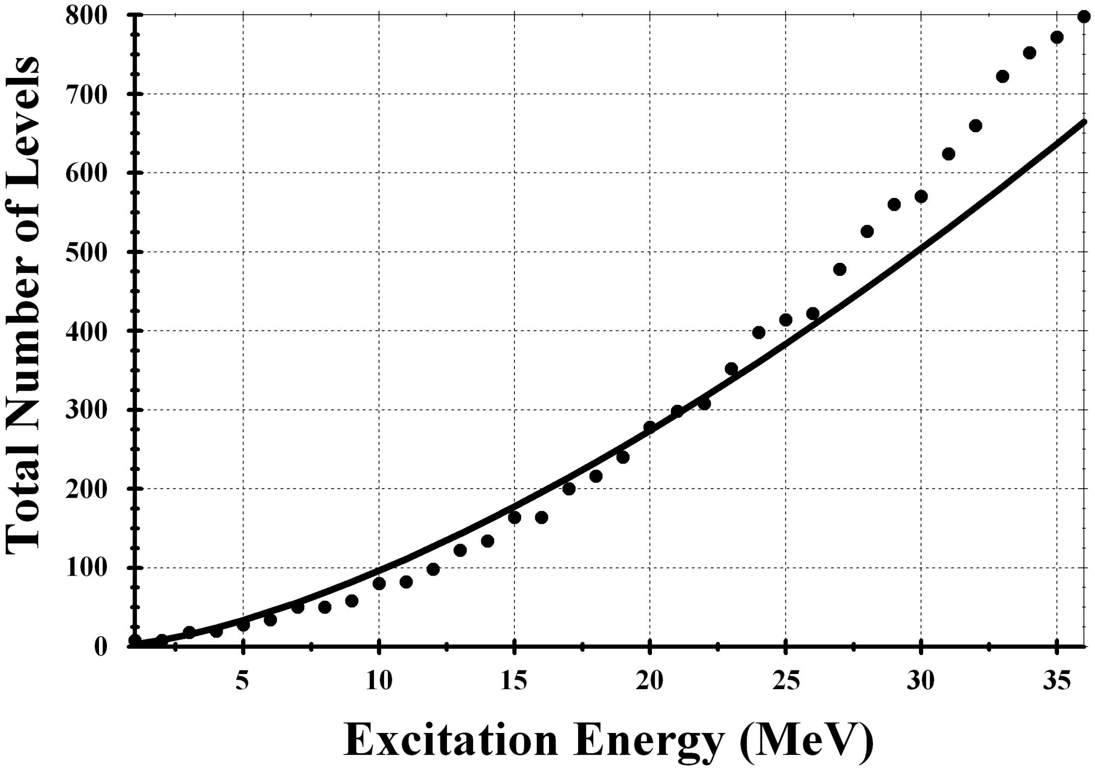

Fig. 4 Total number of energy levels N(E) for X(766, 242) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the power law functional form $N(E) = aE^b$ where **a** = 2.99080945 and b = 1.50780355.

7.0 Equidistant Model

In the equidistant model[17-20], the single particle levels are assumed to be equidistant and nondegenerate. The total state density for a system composed of neutrons and protons is given by

$$N(E) = \frac{\sqrt{\pi}}{12}\frac{\exp(2\sqrt{aE}}{e^{5/4}a^{1/4}} \quad (17)$$

where **a** is a level density parameter. In existing nuclear systems, the parameter **a** has been determined[20] to have the value A/8 where A is the atomic mass. 'The formula is sometimes described as a Fermi gas level density expression, but actually only represents the zeroth order approximation to the level density of a Fermi gas.

The X(766, 242) levels of Fig. 1 have been fit to the functional form of Eq. (17). A fit to the calculated levels is shown in Fig. 5. The X(766, 242) fit is obtained for the value **a** = 1.274 that is similar to the X(610, 204) value (**a** = 1.175), X(636,

204) (**a** = 1.210), and X(692, 214) (**a** = 1.254), and X(730, 226) (a = 1.264). It is important to note that the constant density formula given in Eq. (17) is an approximation that is valid only for energies that are low compared to the energy of the deepest hole that can be made in the nucleus. Most investigations consider excitation energies of less than 15-20 MeV. That is, Eq. (17) is an asymptotic expression, which is valid for an infinite number of occupied levels—i.e. infinite atomic mass A. Accordingly, an improved description of the level density should be obtained as the nucleus mass increases. However, the equidistant model does not reproduce the X(766, 242) level scheme as well as the power law model incorporating Eq. 16.

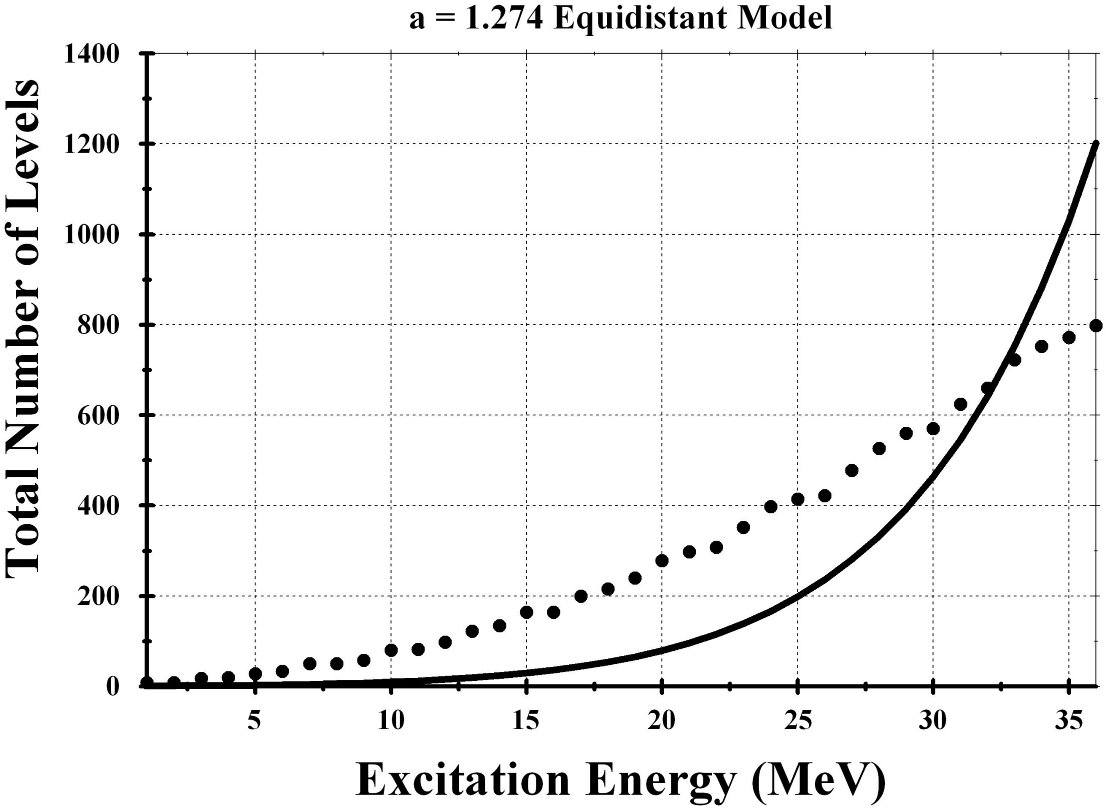

Fig. 5 Total number of energy levels N(E) for X(766, 242) as a function of energy using the equidistant model. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the equidistant model with a = 1.274.

The X(766, 242) **a** = 1.274 value determined by a fit to Eq. 17 is inconsistent with the A/8 value suggested in Ref. 18. This discrepancy suggests that level density systematics may differ in superheavy nuclei. In the A > 600 mass region, **a** exhibits an increasing trend with increasing A (i.e., X(610, 204) (**a** = 1.175), X(636, 204) (**a** = 1.210), X(692, 214) (**a** = 1.254), X(730, 226) (**a** = 1.264), and X(766, 242) (**a** = 1.274)).

8.0 Conclusions

The X(766, 242) single particle level calculations predict a series of levels, and the associated level density. The total number of levels as a function of energy can be fit with a number of functional forms. Given the scatter in the number of energy levels/MeV, the level scheme is roughly fit by the constant temperature model as summarized in Fig. 2.

The total number of energy levels in X(766, 242) as a function of energy is best fit with the power law form N(E) **a** E^b as illustrated in Fig. 4. This power law relationship provides better agreement with N(E) than the constant temperature and equidistant models. The results of these calculations also provide an A = 766 end point for energy level parameters of the constant temperature, equidistant, and power law models.

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