

Peer Review

Review of: "Branching Markov Chains: Survival Thresholds and Applications to Species Navigation"

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This manuscript presents a clear and elegant mathematical result linking the survival of a branching Markov chain to the return probability β of the underlying irreducible Markov chain. The main theorem is concise, conceptually appealing, and effectively generalizes the recent result of Lebensztayn and Pereira. The reduction to a Bienaymé–Galton–Watson process is appropriate, and the proof is technically sound, relying on standard recurrence criteria and properties of power series. The threshold $\beta > 1/2$ as a sharp condition in the transient case is particularly neat and gives the result a strong structural interpretation. The connection to classical examples (e.g., simple random walk on \mathbb{Z}^d) enhances accessibility and situates the theorem within known probability theory.

However, several improvements would strengthen the manuscript. First, the biological interpretation in Section 2 remains largely speculative and metaphorical; either this section should be framed more cautiously as a heuristic illustration, or the evolutionary narrative should be formalized mathematically (e.g., by modeling adaptive changes in transition probabilities). Second, more discussion of how α_c depends quantitatively on β or on structural properties of the chain (spectral radius, Green's function, etc.) would increase mathematical depth. Third, while $\mu(\alpha)$ is shown to be increasing and continuous, a brief justification of strict monotonicity would improve rigor. Fourth, the manuscript would benefit from a short remark clarifying whether the threshold $\beta = 1/2$ has any deeper probabilistic meaning beyond the BGW mean criterion. Finally, minor stylistic tightening and a clearer separation between mathematical results and biological motivation would enhance readability.

Overall, the paper is mathematically solid, concise, and publishable, offering a clean general threshold result. With modest expansion of interpretation and clarification of technical points, it could achieve greater conceptual impact.

Declarations

Potential competing interests: No potential competing interests to declare.