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On the mass of (gravitational) potential energy

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September 2, 2024

Abstract

In classical mechanics, when a body freely moves or is externally forced to move in a conservative force field, such as a planet moving away from a star or a weight lifted from the floor, its kinetic energy or the work done on it is converted and stored as potential energy. The concept of potential energy was developed to uphold the fundamental principle of conservation of energy. According to the widely accepted interpretation of mass-energy equivalence, every form of energy has mass. This leads to the natural questions: does potential energy have mass? And if so, where is that mass located? We will start by briefly reviewing the issue through an examination of some key literature on the topic. The current consensus is that potential energy gets stored in the field energy of the interacting system. As a result of mass-energy equivalence, the equivalent mass is distributed throughout the entire space in some manner. However, this presents some difficulties. Here, like some other scholars in the past, we show that it contradicts the principles of special relativity and argue that potential energy does increase the mass of the bodies composing the system. We present an accessible thought experiment that heuristically corroborates that view specifically for the gravitational potential energy. We finally speculate on how that mass increase is distributed

https://doi.org/10.32388/C96XV7

among the interacting bodies.

Keywords: special relativity; mass-energy equivalence; gravitational potential energy; conservation of energy; conservation of linear momentum; Planck-Einstein relation

1 Introduction

Consider the simple experiment illustrated in Fig. 1. A mass m at rest at point $\mathcal{A}$ in a uniform gravitational field $\mathbf{g}$ is slowly lifted to point $\mathcal{B}$ at a height h above point $\mathcal{A}$. According to classical mechanics, whoever lifts the body performs work on m equal to the force to counteract the gravitational field, mg, times the displacement h. The performed work mgh is, in fact, energy transferred from the lifter to the system. Again, according to classical mechanics, and in order to ensure energy conservation, we say that that energy has not disappeared, but it went in the increase of the *gravitational potential energy* of m, namely $\Delta U = mgh$. Recall that (gravitational) potential energy is defined up to an additive constant and that we can only physically measure its variation. For the sake of simplicity, throughout this paper, when we refer to potential energy, we, in fact, mean variation of potential energy.

Because Einstein's relation $E = mc^2$ prescribes an equivalence between energy and (rest) mass [1], c notoriously being the speed of light in a vacuum, the following questions naturally arise: *does gravitational potential energy have mass? And where is that mass located?*

As far as this author is aware, the above questions date back to at least 1965, when Brillouin published a series of two papers [2, 3]. In the first one [2], he introduced the matter with the following direct and neat words:

> "Einstein's relation between mass and energy is universally known. Every scientist writes
> $$E = Mc^2 \tag{1}$$
> but almost everybody forgets to use this relation for potential energy. The founders of Relativity seemed to ignore the question, although they specified that relation (1) must apply to all kinds of energy, mechanical, chemical, etc. When it comes to mechanical problems, the formulas usually written contain the mass of kinetic energy, but they keep silent about the mass of

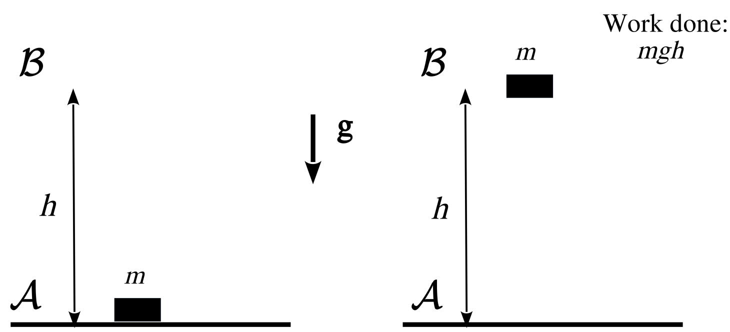

Figure 1: Mass m at rest at point $\mathcal{A}$ in a uniform gravitational field $\mathbf{g}$ gets lifted to point $\mathcal{B}$ at height h above point $\mathcal{A}$. The work externally performed on the mass is mgh, which is also equal to the variation of mass's gravitational potential energy.

> potential energy." "[...] For instance, let us consider a system of two particles interacting together: shall we state that potential energy is located on the first particle? Should it be attributed to the second one? Or split between them? If *energy means mass*, where shall we locate the mass? This is a fundamental question which we have to discuss. The question has very often been ignored, or evaded, because it does not always appear clearly in all problems." [emphasis in the original]

It is quite suggestive that, even 60 years after the seminal paper by Einstein on mass-energy equivalence [1], the question of the mass of potential energy remained poorly investigated. We briefly addressed the topic in [4], but it was not the main subject of that paper.

The common wisdom today agrees with the explanatory framework proposed by Brillouin in 1965. In [2] and the subsequent paper [3], Brillouin explicitly studied the case of electric potential energy between two charges, but by analogy, his results can be extended to every force field and potential energy. According to him, the potential energy is retained in the field, and since energy is equivalent to mass, the new mass distribution is primarily located on the field in the whole space. Although in some calculations [2] Brillouin assumes as a first approximation that the mass of potential energy can be considered as localized in the two interacting bodies and split 50/50 between them, he stresses that [2]:

> "The assumption that the new mass distribution is primarily located on the electric field in the whole space satisfies the obligation for relativistic transformations just as for the electromagnetic field itself. The simplified model with additional mass localized on the particle must be considered only as a simplifying approximation."

The view that potential energy is, in fact, field energy distributed all over space is widely and tacitly agreed today (see, for instance, [5, 6]). However, dissonant voices do not come short. For instance, Hecht [7, 8] criticizes the concept of field energy as "a purely theoretical concept" with no possibility of being experimentally proved/measured. He argues that "to be rigorous, it cannot be said definitively whether energy is stored in the static field or not", adducing, among other arguments, the following agreeable reason:

> "If a property of matter can be measured, we take it to be an empirical quantity having an objective reality; if a property cannot be measured, we take it to be a theoretical quantity, which remains hypothetical. As of now, static field energy is not measurable and should be considered hypothetical."

He also reminds that the concept of potential energy stored in the field energy is at odds with the basic tenets of special relativity (more on that later). In conclusion, Hecht's position is that not only potential energy, although useful, is a purely theoretical concept and does not have objective reality, but when one says that energy is stored as potential energy, as in the example in Fig 1, that energy is not retained in the field energy (another useful but purely theoretical concept) but in the increased masses of the material components of the system. Unfortunately, Hecht does not say exactly how that mass is shared among the components of system. For instance, in Fig 1, does the mass of the body m increase by mgh/c^2 (i.e., gravitational potential energy divided by c^2)? Or is the mass of the source of the gravitational field to increase? Or do they increase both? But, in the last case, in what proportion?

The present paper aims to humbly revive the topic and provide our reasoned answers to the questions at the beginning of this section: *does gravitational potential energy have mass? And where is that mass located?* It will be clear that our view on the matter is closer to Hecht's. We choose not to engage in the debate about whether potential energy is purely a theoretical concept with no objective reality. Instead, we consider it a convenient method for explaining the energy that is unaccounted for after external work is done against a conservative force field or for the kinetic energy that is missing from a body moving in that field. We concur with Hecht's viewpoint that the storage of potential energy in the field poses issues from the standpoint of special relativity. It is more reasonable to assume that the energy gets stored as mass in the material bodies that make up the interacting system (Section 2). In Section 3, we present an accessible thought experiment that heuristically corroborates that view specifically for the gravitational potential energy. In the concluding section, we speculate on how the mass resulting from potential energy is distributed among the material bodies in an interacting system. That is the most speculative part of the paper.

2 Some issues with the field energy explanation

Here, we present two examples supporting that the field energy explanation is somehow problematic.

As already noted by several distinguished scholars (e.g, Brillouin [3], Mills [9], Landau and Lifshitz [10], Hecht [7, 8]), potential energy, as it is defined, depends only on the positions of the interacting bodies at a given instant (and is not expressed as a function of time) and requires that a change in the position of any particle instantaneously affects all the other particles. That is clearly in conflict with special relativity, which maintains that interactions propagate at finite speeds. However, Brillouin [2, 3] insists that the storing of potential energy (and its equivalent mass) in the field energy is special relativity compliant.

Nevertheless, the first example presented here still shows a clear conflict between special relativity and the idea that potential energy and its equivalent mass are stored and primarily located on the field in the whole space. A similar remark has been made by Hecht [7].

Suppose that when the body m in Fig. 1 is raised at the height h, the potential energy mgh and its corresponding mass mgh/c^2 are, in fact, lo-

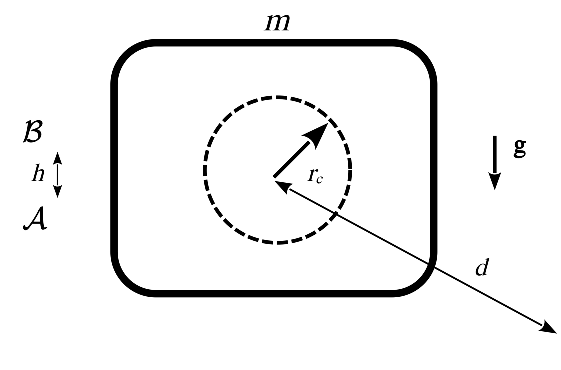

Figure 2: Suppose gravitational potential energy gets stored in the field all over space. Given any gravitational field $\mathbf{g}$ and any downward displacement h, we can always choose a finite distance d from the body's center of mass such that the transfer of energy from the field to the body (namely, potential energy mgh going into kinetic energy) cannot involve the field beyond d since the light distance r_c in the time of the downward displacement is lower than d

cated on the field in the whole space. Now, if, after some time[1], we let the body return to its initial position, its potential energy transforms back (by definition of potential energy) into, say, the kinetic energy of the body. In the approximation of classical mechanics (all velocities much less than the speed of light c), the time taken by the body to get (free-fall back) to its original position and to retrieve the whole potential energy in the form of kinetic energy is

$$\Delta t = \sqrt{\frac{2h}{g}}, \tag{1}$$

and, therefore, owing to special relativity, during that time, the body is causally connected to and could receive energy only from a sphere centered in the body of radius (see, Fig. 2)

[1]Possibly, greater than the time needed by the body to free-fall back to its initial position.

$$r_c = c\Delta t = c\sqrt{\frac{2h}{g}}. \tag{2}$$

In principle, if the magnitude of the uniform gravitational field $\mathbf{g}$ is suitably high and the height h suitably small, the radius r_c could be smaller than any arbitrarily chosen finite distance d from the body's center of mass

$$c\sqrt{\frac{2h}{g}} < d. \tag{3}$$

Condition (3) is not in conflict with the non-relativistic approximation of the above calculation, $v \ll c$. The final velocity v of the body coming back to its initial position is $v = g\Delta t = \sqrt{2gh}$. Hence, we must have $\sqrt{2gh} \ll c$. By multiplying both members of that inequality by $\sqrt{\frac{2h}{g}}$ and using equation (3), we have

$$\sqrt{\frac{2h}{g}}\sqrt{2gh} \ll c\sqrt{\frac{2h}{g}} \quad \rightarrow \quad 2h \ll d, \tag{4}$$

namely, the displacement of the body has to be much smaller than the physical dimension of the body. That condition is not a physical impossibility.

Therefore, since d is arbitrary and finite, how can the potential energy mgh (and its related mass) be located on the field in the whole space if it cannot be *completely* retrieved *in time* without violating special relativity? The fraction of the energy mgh stored in the field beyond d, no matter how much it is, would not be retrieved in time.

Although some recent theoretical and experimental studies seem to indicate non-local properties of force fields (e.g., [11] and references therein), it would make much more sense to imagine that the potential energy of a body is stored *locally* in the form of mass inside the body itself.

Incidentally, wanting to insist on the idea that the potential energy gets stored in the force field, one could imagine that when a body is raised at a given height, h, the potential energy mgh ends up stored in a sort of spherical 'field halo' of radius $c\sqrt{2h/g}$ centered on the body's center of mass. That would be compliant with condition (3) and special relativity. However, this solution is uselessly complicated and hardly justifiable from a physical point of view. The lifting of an object to the height h can be thought of as a series of

N individual lifts, each by an amount of h/N, with N being arbitrarily large. For each of these smaller lifts, the radius of the halo would be arbitrarily small, $c\sqrt{2h/Ng}$, with each halo having the same radius. Therefore, for any given height h, all the potential energy would be stored in N superimposed halos, each with a radius of $c\sqrt{2h/Ng}$. However, since N can be any value, the dimension of the halo is also arbitrary, making this solution meaningless from a physical perspective.

Now, let's get to our second example. Consider the dihydrogen molecule H_2 and its mass m_{H_2}. To separate the molecule into two neutral hydrogen atoms we need to provide some energy (binding energy E_b) and thus perform work on the molecule. Therefore, owing to mass-energy equivalence, no one would be surprised if one says that the mass of two separate hydrogen atoms is greater than the mass of a dihydrogen molecule, $2m_{\mathrm{H}} > m_{\mathrm{H}_2}$, the difference being precisely equal to $(2m_{\mathrm{H}} - m_{\mathrm{H}_2}) = E_b/c^2$.

Given the smallness of the energies involved in chemical reactions, that mass defect would be immeasurable (of the order of 10^{-37} kg [12]). However, for the more intense nuclear binding energies, we have solid experimental proofs that, for instance, when a nucleus captures a neutron and emits a γ-ray, the mass difference Δm between the initial (including unbound neutron) and final nuclear states, multiplied by c^2, should equal the energy of the emitted γ-ray(s) [13]. That also verifies Einstein's mass-energy equivalence formula with a very high degree of confidence.

Therefore, we must assume that the same holds for the much less energetic case of the dihydrogen molecule (chemical bond). Now, in that simple case, we clearly see that the energy of the work done on the dihydrogen molecule goes directly into the two hydrogen atoms as increased mass. No field energy is involved here since, before with H_2 and after with 2H, there are, in principle, no detectable fields and field energy. In that case, the fact that potential (binding) energy is transformed into mass localized in the particles is theoretically unquestioned and taken for granted by anyone. It is even potentially measurable.

3 The gravitational potential energy of a body increases its mass

Here, we provide heuristic proof that the gravitational potential energy of a body contributes to the total mass of the body. Therefore, in the example presented in Fig 1, the total mass of the body after being raised to height h would become $m + mgh/c^2$.

Consider the following ideal experiment. A closed wagon of mass M moves horizontally without friction in a vertical uniform gravitational field $\mathbf{g}$ at a constant velocity v (see Fig. 3). Inside the wagon, attached to floor $\mathcal{B}$, there is a particle of mass $m_{\mathcal{B}}$. At a certain point, mass $m_{\mathcal{B}}$ annihilates into a photon[2] of energy

$$h\nu_{\mathcal{B}} = m_{\mathcal{B}}c^2 \tag{5}$$

(owing to the Planck-Einstein relation $E = h\nu$, where h is the Planck constant).

Then, the photon travels upward toward ceiling $\mathcal{A}$ at a height h and is absorbed and converted by a suitable apparatus into another particle of mass $m_{\mathcal{A}}$. A similar idealized process has been used by Misner, Thorne, and Wheeler in a though experiment on gravitational redshift [14]. The new particle also ends up stuck to the wagon frame. The whole process happens exclusively inside the closed wagon. Owing to the conservation of energy, we must have that

$$h\nu_{\mathcal{B}} = m_{\mathcal{A}}c^2 + m_{\mathcal{A}}gh, \tag{6}$$

namely, the initial energy is equal to the rest energy of the new particle, $m_{\mathcal{A}}c^2$, plus the potential energy of that particle at the height h in the gravitational field $\mathbf{g}$ relative to floor $\mathcal{B}$.

[2]In particle physics, when a single neutral particle decays into photons, momentum conservation dictates that it decays into two photons with equal and opposite momenta. The reference to particle annihilation into a single photon is used loosely here for the sake of derivation, as done, for instance, in [14, 15]. The energy balance analysis, horizontal momentum conservation, and the conclusions of the thought experiment remain unchanged if we imagine the annihilation resulting in two photons, both of which are then deflected upward toward the ceiling. In any scenario, there will be a vertical recoil of the massive coach, which can be made arbitrarily small and can be rigidly absorbed by the railways in accordance with the conservation of vertical linear momentum. The same principle applies in the classical case of a person throwing a ball upward in a moving railway coach.

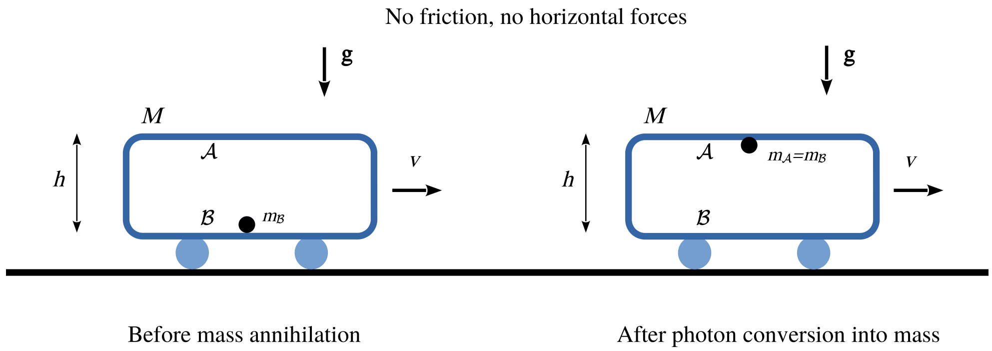

Figure 3: Pictorial representation of the thought experiment described in Section 3.

According to the common understanding today (e.g., [16]), the mass of the generated particle at point $\mathcal{A}$ does not include the equivalent mass of its gravitational potential energy $m_\mathcal{A} gh/c^2$ and then $m_\mathcal{A} < m_\mathcal{B}$.

In reality, we shall show that the total mass of the particle generated at point $\mathcal{A}$, $m_{\text{tot}\mathcal{A}}$, must be equal to $m_\mathcal{B}$, namely, using equations (5) and (6),

$$m_{\text{tot}\mathcal{A}} = m_\mathcal{B} = \frac{h\nu_\mathcal{B}}{c^2} = m_\mathcal{A} + \frac{m_\mathcal{A} gh}{c^2}, \tag{7}$$

and therefore, the total mass of the particle generated at point $\mathcal{A}$ *must include the equivalent mass of its gravitational potential energy* $m_\mathcal{A} gh/c^2$.

Any different scenario seems to violate the conservation of (the horizontal) linear momentum of the closed system wagon+particle. No horizontal external forces act upon the system, and no mass is ejected. Therefore, the total velocity v must be the same before and after the whole process. However, if the mass at point $\mathcal{A}$ is less than the mass at point $\mathcal{B}$, $m_\mathcal{A} < m_\mathcal{B}$, before the annihilation, the total horizontal linear momentum is

$$P_i = (M + m_\mathcal{B})v \tag{8}$$

while, after the conversion of the photon energy into mass, the total horizontal linear momentum becomes

$$P_f = (M + m_{\mathcal{A}})v < P_i. \tag{9}$$

That is quite bizarre. On the other hand, by imposing the conservation of the horizontal linear momentum even with $m_{\mathcal{A}} < m_{\mathcal{B}}$, we would have an equally strange consequence. Without any horizontal external force acting upon the wagon and without any mass ejection, we would see the wagon increase its velocity by itself at the end of the whole process.

At this point, it is possible to derive the exact expressions of the total mass and energy of the body in the uniform gravitational field $\mathbf{g}$ of Fig. 1 after external work is done on that body. When an agent, external to the system body-gravitational field, raises mass m by a distance $\mathrm{d}h$, the infinitesimal work performed on the body is $mg\mathrm{d}h$. Then, according to the result of this section, that energy is stored in the mass of the body, which increases by

$$\mathrm{d}m = \frac{mg\mathrm{d}h}{c^2}. \tag{10}$$

Integrating the differential equation (10) by imposing suitable boundary conditions, the mass m_h of the body at height h is $m_h = me^{\frac{gh}{c^2}}$, where m is the proper mass at height taken as zero. The total energy E_{tot}, proper mass plus gravitational potential energy, at height h is then given by $E_{tot} = m_h c^2 = mc^2 e^{\frac{gh}{c^2}}$. For small distances h, we have $m_h \simeq m + \frac{mgh}{c^2}$ and $E_{tot} \simeq mc^2 + mgh$. Therefore, we recover the classical (and special-relativistic) expressions of the total energy and mass of the (stationary) body, the latter being that given at the beginning of this section.

4 Concluding remarks

In the previous two sections, we believe to have provided some reasonable arguments to support the nowadays less popular (and probably considered wrong by the majority) standpoint that the potential energy of a body in a force field (such as gravitational, electric, magnetic, etc.) increases the mass of the material body, and the energy does not get stored in the field energy. In section 3, we have made a point of corroborating that standpoint specifically for the gravitational potential energy. It should be noted that this particular derivation has been made using the weak gravitational field approximation (near Minkowski flat spacetime). Therefore, any attempt to invalidate the result by claiming that our thought experiment was not formulated within the realm of general relativity is misleading and pointless.

Obviously, even our explanation is not spared from puzzling issues. The first and most important one is how the mass of the potential energy is shared between the interacting bodies. We are back to the previous questions: in the example of Fig 1, is just the mass of the body m to increase by mgh/c^2? Or is the mass of the source of the gravitational field to increase? Or do they increase both? But, in that case, in what proportion?

On this, we are not able to offer a definitive answer. We can only rely on the examples presented in sections 2 and 3 and proceed heuristically. In the thought experiment in section 3, we have shown that the equivalent mass mgh/c^2 of the gravitational potential energy mgh must end up completely in the body inside the wagon. Nothing goes into the mass generating the gravitational field. We must find the reason in the approximation tacitly made in the thought experiment. In that experiment, we implicitly assumed that the source of the uniform gravitational field $\mathbf{g}$ has a mass M_S much greater than that of the body $m_{\mathcal{B}}$, $M_S \gg m_{\mathcal{B}}$. Here, we are ignoring the mass of the wagon, M, since it is only auxiliary to the thought experiment, but we clearly also have $M_S \gg M$.

That means that the center of mass of the system $M_S + [M + m_{\mathcal{B}}]$ is practically located on the source of the gravitational field M_S, and the only displacement relative to the center of mass in the described process is that of mass $m_{\mathcal{B}}$.

Likewise, in the example of the dihydrogen molecule in section 2, when we perform work on both hydrogen atoms to break the molecule apart, they equally move apart relative to the common center of mass. In that case, the equivalent mass of the potential energy equally splits between the two hydrogen atoms, the mass of all free hydrogen atoms being the same in nature.

Therefore, here we see a possible path to answering the question of how the mass of the potential energy is shared between the interacting bodies. We suggest that when energy gets stored as potential energy in the classical sense, the equivalent mass of the potential energy ends up on the body *in proportion to its displacement relative to the center of mass of the interacting system*, namely

$$\Delta m \propto |\Delta \mathbf{r}|, \tag{11}$$

where $\Delta \mathbf{r}$ is the displacement vector relative to the center of mass of the system.

Consider the case of a bound system of two bodies, M and m, with $M \geq m$, orbiting one around the other under a mutual central force. Namely, consider a system with no external work performed on any of the bodies of the system. In that specific case, the kinetic and potential energies of each body periodically transform into one other (partly or totally). Owing to the conservation of linear momentum, we always have that $|M\mathbf{V}| = |m\mathbf{v}|$, where $\mathbf{V}$ and $\mathbf{v}$ are the instantaneous velocities of the bodies relative to the inertial frame of the center of mass of the system. Therefore, after some algebra, the kinetic energy of body M, E_{KM}, is related to the kinetic energy of body m, E_{Km}, as follows

$$E_{KM} = \frac{m}{M} E_{Km}. \tag{12}$$

If, during the motion of the bodies, part or all the kinetic energy of the bodies transforms into potential energy (of the interacting system), we suggest (conjecture) that the equivalent mass of the potential energy ends up in the mass increase of the material bodies according to the same quota

$$\Delta M = \Delta E_{KM}/c^2 = \frac{m}{M} \Delta E_{Km}/c^2 = \frac{m}{M} \Delta m. \tag{13}$$

This last relation can, in fact, be retrieved directly from equation (11). In the case of two isolated and interacting bodies, m and M, the following relation always holds

$$m\mathbf{r} + M\mathbf{R} = \mathbf{0}, \tag{14}$$

where $\mathbf{r}$ and $\mathbf{R}$ are the distance vectors of the bodies from their common center of mass (which is the origin of the inertial reference frame). Therefore, for the displacement vectors, we have

$$M\Delta \mathbf{R} = -m\Delta \mathbf{r}, \tag{15}$$

and by using equation (11), we have

$$\Delta M = \frac{m}{M} \Delta m. \tag{16}$$

If, like in the examples in sections 1 and 3, $M \gg m$, then we have $\Delta M \simeq 0$ and $\Delta m \neq 0$. If, like in the example of the dihydrogen molecule, the masses of the bodies (hydrogen atoms) are the same, $m_1 = m_2$, then we have $\Delta m_1 = \Delta m_2$.

Funding

This research received no external funding.

Data Availability Statement

No new data were created or analysed in this study.

Conflicts of Interest

The author declares no conflict of interest.

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