

# Review of: "Modified Hawking radiation of Schwarzschild-like black hole in bumblebee gravity model"

Gabriel Brandão de Gracia<sup>1</sup>

<sup>1</sup> Federal University of ABC (Universidade Federal do ABC, UFABC)

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The paper is well-written with an informative structure. It provides a good review of the field and introduces extensions concerning the GUP paradigm. It clarifies the origins of the black hole thermodynamics analogies and presents a set of complementary remarks.

However, there are some points that can be improved. First of all, given the importance of the gravitational field solution concerning Bumblebee gravity, it would be interesting to highlight its definitions. For example, mentioning that the Lorentz symmetry violation is due to a non-zero vacuum expectation value of an extra vector field non-minimally coupled to gravity. This field is the one that furnishes new contributions to the energy-momentum tensor. It would be nice to clarify it between eq. 1 and 3 (maybe by presenting the explicit interaction terms between gravity and the mentioned vector field).

After eq. 7, it would be interesting to explain how one can derive angular momentum conservation in the Hamilton-Jacobi framework. It is also not clear why angular momentum and the mass term do not contribute to eq. 10. Regarding this equation, do the minus and plus signs refer to solutions for incoming and outgoing particles?

The eq. 12 should have a proper definition of the Gammas out and in. It would be interesting to add a clarifying paragraph and also a few equations to explain the link between the formalism and the eikonal approximation, and to define a prescription for probability written in terms of the action (which, in this case, has an imaginary part leading to interesting physics).

Regarding the extension to include GUP and the investigation of its associated black hole thermodynamics, it would be interesting to furnish a concise and complete definition of the meaning of the associated beta parameter in eq. 55. Also, a short comment on the generalization of the Hamilton-Jacobi equation in 55 is desirable.

Then, after these improvements to ensure completeness, the paper is suitable for publication in any good theoretical or mathematical physics journal.

