Review of: "Determining Affinity of Social Network using Graph Semirings"

Dario Fasino[1]

1 University of Udine

Potential competing interests: No potential competing interests to declare.

In this paper, the authors introduce a measure to quantify the consistency or strength of connections between subgraphs of an undirected graph. The interesting aspect of this study is that the subgraphs considered may not be pairwise disjoint, that is, the set of subgraphs may constitute a covering rather than a partitioning of the initial graph.
The authors suggest that this measure is an indication of the affinity of the subgraphs and the stability of their interconnections. The measure is defined through an algorithm that produces a condensed version of the starting graph and is based on the number and weight of paths in the condensed graph. Unfortunately, such an algorithm is shown only through a specific example. It would be better if the construction of the reduced graph and the definition of the measure of connection between subgraphs were presented more in the abstract and the example be discussed subsequent to that presentation. In fact, the example is specific to a field of social sciences, while the proposed algorithm is certainly of more general value.

The idea seems original and interesting to me, because it provides a new tool for analyzing the structure of complex networks. For example, I am thinking of the possibility of repeating the operation of graph condensation, in order to obtain a hierarchy of graphs that allows the original network to be described at different levels of abstraction. Below I offer some more insights for revision and expansion of this version of the work.

First, it is unclear to me what is meant by the measure of stability of a path that appears in Section 2.1. In particular, the authors could discuss why they chose the formula $S_p = r + \beta_{C_p}$ shown after Figure 3 over other possible formulas. The discussion provided in the section entitled "Geometrical significance" is actually related to the social network example presented, whereas it would be useful to have a discussion in the language of graph theory.

Finally, in the Introduction the authors state that. the measure $\boldsymbol{\beta}_G$ of a graph G is always greater than or equal to that of its subgraphs. This statement puzzles me. In fact, let G_1 be the complete graph of vertices {1,2,3,4} and let G_2 be the graph obtained by adding to G_1
the vertices {5,6,7,8} and the edges (i, i+4), for $i = 1,\ldots,4$. Evidently G_1 is a subgraph of G_2 but $\boldsymbol{\beta}_{G1} = 4$ and $\boldsymbol{\beta}_{G2} = 3.5$, therefore $\boldsymbol{\beta}_{G1} > \boldsymbol{\beta}_{G2}$.