# Review of: "Can the electromagnetic fields form tensors if $D = \epsilon E$ and $H = B/\mu$?"

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(I use tex notation below)

In this paper the tensorial character of the electromagnetic field is discussed for the case of linear isotropic materials characterized by a permittivity $\epsilon$ and permeability $\mu$. First, the $F^{\alpha\beta}$ field in vacuum is analyzed and it is correctly concluded that it is a rank 2 tensor. The reasoning is that the differential operators $\partial_\mu$ act as covariant vectors, the invariance of the continuity equation $\partial_\mu j^\mu=0$ implies that the 4-current density $j^\mu$ are the components of a contravariant 4-vector, and the wave equation for the potential $A^\mu$ in the Lorentz gauge, with a scalar wave operator, implies that the 4-potential is a contravariant 4-vector. Thus, the Faraday tensor $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$ is a contravariant second rank tensor.

Then the author tries to apply the arguments above to the electromagnetic field $F^{\mu\nu}$ and the field $G^{\mu\nu}$, whose relation to the ordinary vectors $\bm D$ and $\bm H$ is analogous to that of $F^{\mu\nu}$ to $\bm E$ and $\bm B$. Writing $\bm D=\epsilon \bm E$ and $\bm H=(1/\mu) \bm B$, he shows the arguments cannot be applied (for example, the wave operators are not Lorentz scalars), and so, he concludes that the fields $\bm E$, $\bm B$, $\bm D$, and $\bm H$ cannot be combined into relativistic tensors if the fields are related by the consitutive relations above.

The conclusion above is indeed true, but the reason is that the consitutive relations proposed are not written in a properly covariant way. Indeed, $\bm D=\epsilon \bm E$ and $\bm H=(1/\mu) \bm B$ only hold for static isotropic materials. However, after a Lorentz boost, a static material becomes a moving material, for which those relations do not hold. A proper generalization of $G^{\mu\nu}$ may be obtained by first making a transformation from, say, the lab frame to the rest frame of the material, applying there the usual consitutive relations and finally transforming back to the lab frame. This may easily be done using the four velocity $U^\mu$ of the material, which is simply (1,0,0,0) in the rest frame of the material. After some algebra, it is easily shown that the correct constitutive equations are $G^{\alpha\beta}=L^{\alpha\beta}_{\gamma\delta} F^{\gamma\delta}$, where the linear response is characterized by the fourth rank tensor $L^{\alpha\beta}_{\gamma\delta}=(\epsilon \delta^{\alpha\beta}_{\epsilon\gamma}\delta^\eta_\delta +(1/2\mu) \delta^{\alpha\beta\eta}_{\epsilon_\gamma\delta})U^\epsilon U_\eta$, where I used the two-, four- and six-index Kronecker delta functions. In the rest frame of the material the four velocities are $(1,0,0,0)$ and the response above is equivalent to the ordinary constitutive equations, but the fourth rank response is a perfectly relativistic tensor and both $F^{\mu\nu}$ and $G^{\mu\nu}$ are then relativistic second rank tensors. I consider that the results are not correct as they follows from mistaken premises. Thus, I assigned a low score.