

# Review of: "Numerical Evaluation of a Soliton Pair with Long Range Interaction"

Ahmed Refaie Ali<sup>1</sup>

<sup>1</sup> Menoufia University

Potential competing interests: No potential competing interests to declare.

## Review's Report

1. How do the authors numerically evaluate the soliton pair with long-range interaction, and what methods are employed in determining the interaction energy of electric or magnetic monopole pairs?
2. Can you elaborate on how the monopoles are represented by topological solitons of finite size and mass, and how these solitons are described by a field of SO(3) rotations without any divergences?
3. What is the significance of the monopoles feeling a pure Coulombic interaction at large distances, and how does this contribute to the overall understanding of the system?
4. In what ways do the authors test the physical interpretation of these monopoles, particularly in terms of the classical running of the charge at small distances, and what are the expected outcomes due to the finite soliton size?
5. How does the study observe the increase of the effective charge at a few soliton radii in this purely Coulombic system, and what implications does this observation have for the physical characteristics of the solitons?
6. Could you provide more details on the comparison made between the observed increase in effective charge and the running of the coupling in perturbative QED?
7. What are the key findings or insights obtained from the investigation of the soliton pair, and how do these findings contribute to the broader understanding of Coulombic interactions involving monopoles?
8. Small scale results different about large scale in applications, tell us how you can generalize the results?

1. Give more details with validation of the numerical solutions, and you should enhance the discussion with more references like:  
i) Obada, A.-S.F., Abdel-Aty, M. Influence of the stark shift and kerr-like medium on the evolution of field entropy and entanglement in two-photon processes Acta Physica Polonica B, 2000, 31(3), pp. 589–599

ii) K. Al-Heuseen, A.I. Aljameel, M. Kh. Alquran, The Mechanism of Charge Flow and Electric Current in Porous GaN Thin Films during Photo Electrochemical Etching  
Int. J. Thin Film Sci. Tech. Vol. 11, No. 1 (2022) PP: 89-94 doi:10.18576/ijtfst/110111

iii) O. M. Abo-Seida, N. T. M. El-dabe, A. Refaie Ali and G. A. Shalaby, "Cherenkov FEL Reaction With Plasma-Filled Cylindrical Waveguide in Fractional D-Dimensional Space," in IEEE Transactions on Plasma Science, vol. 49, no. 7, pp. 2070-2079, July 2021, doi: 10.1109/TPS.2021.3084904.

- iv) X.J. Yang, A. A. Abdulrahman, A. Refaie Ali, An even entire function of order one is a special solution for a classical wave equation in one-dimensional space. *Therm. Sci.* 27(1B), 491–495 (2023). <https://doi.org/10.2298/TSCI221111008Y>
  - v) Mahmuda Maya, M.U., Alam, M.N. & Refaie Ali, A. Influence of magnetic field on MHD mixed convection in lid-driven cavity with heated wavy bottom surface. *Sci Rep* 13, 18959 (2023). <https://doi.org/10.1038/s41598-023-45707-x>
  - vi) Islam, S., Halder, B. & Refaie Ali, A. Optical and rogue type soliton solutions of the (2+1) dimensional nonlinear Heisenberg ferromagnetic spin chains equation. *Sci Rep* 13, 9906 (2023). <https://doi.org/10.1038/s41598-023-36536-z>
  - vii) Mohamed, H. A., Hadia, N. M. A., Influence of Post Thermal Annealing on the Optical Properties of SnO<sub>2</sub> Films Prepared by Electron Beam Evaporation Technique, *Int. J. of Thin Film Science and Technology* 4 (2015), pp. 1-7
  - viii) Khalida Inayat Noor, Muhammad Aslam Noor, Hamdy M. Mohamed, Quantum Approach to Starlike Functions Appl. Math. Inf. Sci. Volume 15, No. 4 (2021) PP: 437-441 doi:10.18576/amis/150405
10. Are there any limitations or potential areas for further research that the authors acknowledge in their study, and how might these aspects impact the overall conclusions drawn from the numerical evaluation?
  11. How do the results of this study align with or differ from existing theories or experimental observations in the field of soliton physics and Coulombic interactions?
  12. What implications does this research have for practical applications or theoretical advancements in areas related to electric or magnetic monopole pairs and their interactions?