Review of: "On the Application of the Rayleigh-Ritz Method to a Projected Hamiltonian"

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The article by F.M. Fernandez explores the Rayleigh-Ritz variational principle, focusing on the approximation of eigenvalues when restricting the Hamiltonian from an infinite-dimensional Hilbert space to a finite D-dimensional subspace, followed by the diagonalization of the resulting $D \times D$ matrix.

The work is framed as a critique of a recent article by Ding et al. [*Quantum* **8**, 1525 (2024)], which introduced an ensemble variational principle for calculating the eigenvalues of a Hamiltonian in a finite-dimensional Hilbert space. Fernandez characterizes the approach in Ding et al. as “unrealistic” and similar terms, but his critique appears to misunderstand the context and scope of that work. Specifically, Ding et al. explicitly assume a finite-dimensional Hilbert space from the outset, acknowledging that any numerical approximation of eigenvalues inherently requires truncating the Hamiltonian. This essential truncation is a practical necessity for any computational approach and is not a limitation of their method. Fernandez’s critique seems to conflate this deliberate setup with a broader assumption about infinite-dimensional parent Hamiltonians, which Ding et al. do not make.

The main body of Fernandez’s work refers to the eigenvalue interlacing theorem: the eigenvalues x_i of an $m \times m$ submatrix of an $n \times n$ matrix with eigenvalues y_j fulfil the following relation for all $i=1,\ldots,m$: $y_i \leq x_i$ (assuming both sets of eigenvalues are arranged in non-descending order). He elaborates on the question of whether the application of a D-dimensional projection operator to a given Hamiltonian H on an infinite-dimensional Hilbert space would lead to any distinctive relation of a similar type. In particular, he concludes that in that case the eigenvalues of H are approached from below by those of the projected Hamiltonian H_D if the latter are approximated by considering $N \times N$ submatrices for increasing N. This is illustrated for a simple Hamiltonian which happens to be positive semi-definite. Yet, it is rather obvious that this result of the paper is again a direct consequence of the eigenvalue interlacing theorem when applied to the Hamiltonian $-H_D$ and compared to -H. The reason for this is that the projection changes the ordering of the eigenvalues: the smallest D eigenvalues of H become the largest D of H_D. Multiplying then both Hamiltonians by a minus sign and applying the eigenvalue interlacing theorem yields the desired result.

In summary, while the topic addressed by Fernandez is interesting, the work does not present any new insights. Moreover, the critique of Ding et al. is based on a misunderstanding of their assumptions and methodology.