

Peer Review

# Review of: "Inverse Evolution Data Augmentation for Neural PDE Solvers"

Filippo Gatti<sup>1,2</sup>

1. University of Paris-Saclay, Gif-sur-Yvette, France; 2. Laboratoire de Mécanique Paris-Saclay UMR 9026, CentraleSupélec, Gif-sur-Yvette, France

The manuscript entitled "[Inverse Evolution Data Augmentation for Neural PDE Solvers](#)", by [Chaoyu Liu](#), [Chris Budd](#), [Carola-Bibiane Schönlieb](#) focuses on enhancing PDE solution data sets to improve neural operators training. The original contribution resides in the data augmentation strategy the authors proposed: leveraging inverse evolution on standard time-marching schemes to reduce the cost of augmenting the number of snapshots adopted for training purposes. Authors proved that additional data, generated by inverse evolution from random initialization, reinforce neural operator learning, showing promising improvements in a standard benchmark PDEs.

While the idea of inverse evolution seems intriguing, the authors should improve the manuscript by providing further insights on the following points:

- There are 2 constants appearing: one in Eq. (7) and the other in Eq. (14). How do the authors judge the role of such constants on the overall training performance? Isn't pre-processing in Eq. (14) somehow counteracting the effect of constant C in Eq. (7)?
- There a missing figure, according to me, which would elucidate the real improvement of such data augmentation technique based on inverse evolution: after Figure 5, one figure showing the snapshot evolution in time. How the NO perform in predicting on the long-term?
- The authors should better discuss the role of the CFL condition based on the chosen spatial semi-discretization. More in general, it is not clear how the authors preserve the accuracy by mixing snapshots obtained with explicit methods and implicit methods. How would the authors expect the NO would behave? A rigorous error analysis would be definitely beneficial for such a study
- Following my previous question, the authors should discuss whether their data augmentation strategy would help reduce the spectral bias. L2 metrics should be evaluated by frequency bands then.

This would help understanding the benefits on NO training of such data augmentation strategy (if any).

## **Declarations**

**Potential competing interests:** No potential competing interests to declare.