Possible Tetraquark Explanation for the ψ(4230), ψ(4360), and ψ(4415)

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Abstract

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Ji et al. utilize heavy quark spin symmetry, to predict of the hadronic molecular characteristics of the ψ(4230), ψ(4360), and ψ(4415). Using symmetry arguments these states were identified by Ji et al. as DD_1-bar, D^*D_1-bar, and $D^*D_2^*$-bar bound states. The structure of the ψ(4230), ψ(4360), and ψ(4415) states are investigated using a first-order tetraquark mass formula. This mass relationship is based on weakly bound D plus D_1-bar, D^* plus D_1-bar, and D^* plus D_2^*-bar meson clusters. The first-order tetraquark mass formula provides predicted masses within about 3% of the experimental values, and the J^π predictions are in agreement with values suggested by the data. However, the primitive angular momentum structure of the first-order mass model admit additional J^π values.

KEYWORDS: Tetraquark Model for the ψ(4230), ψ(4360), and ψ(4415); Mass Formula; Quark Model; Cluster Model.

1.0 Introduction

The conventional quark model suggests that mesons and baryons are composed of a pair of quark-antiquark pairs (qq-bar) and three quarks (qqq) configurations. However, exotic states beyond the conventional quark model [1,2] have been and continue to be observed. Quantum chromodynamics (QCD), does not exclude the existence of more exotic configurations. These include multiquark states with more than three valence quarks and antiquarks, hybrid configurations with gluonic excitations and quarks, and glueballs that are composed only of gluons.

Ref. 3 notes that to date no $J^{\pi C}$ states having a 0^{--} character have been observed. Theoretical predictions suggest the existence of these states occurring as tetraquarks, hybrid states, glueballs, or a hadronic molecule[3]. Ji et al[3] utilize heavy quark spin symmetry, to predict of the hadronic molecular scenario of the ψ(4230), ψ(4360), and ψ(4415) states. Using symmetry arguments these states were identified as DD_1-bar, D^*D_1-bar, and $D^*D_2^*$-bar bound states[3].

Given the uncertainty in the structure of the ψ(4230), ψ(4360), and ψ(4415) systems, this paper investigates its structure in terms of DD_1-bar, D^*D_1-bar, and $D^*D_2^*$-bar tetraquarks. A first-order tetraquark model has been successful in the description of a number of systems[4-18], and its description using these first-order models is a reasonable approach to investigate these structures.

2.0 Model and Formulation

Zel'dovich and Sakharov[19,20] proposed a semiempirical mass formula that provides a prediction of mesons and baryons in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These meson mass formulas are used as the basis for deriving a first-order tetraquark mass formula. In particular, the model proposed in this paper assumes the tetraquark is partitioned into two meson clusters with the interaction between the clusters providing a minimal contribution to the tetraquark mass.

The meson mass (M_m) formula of Refs. 4 - 18 is:

$$M_m = \delta_m + m_1 + m_2 + b_m \, [m_o^2 / (m_1 \, m_2)] \, \sigma_1 \cdot \sigma_2 \quad (1)$$

where m_1 (m_2) are the mass of the first (second) quark comprising the meson, m_0 is the average mass of a first generation quark[19,20] and the σ_i (i = 1 and 2) are the spin vectors for the quarks incorporated into the meson. The parameters δ_m and b_m are 40 MeV/c^2 and 615 MeV/c^2, respectively[20].

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $\sigma_1 \cdot \sigma_2$ is the scalar product of the quark spin vectors. $\sigma_1 \cdot \sigma_2$ has the value -3/4 and +1/4 for pseudoscalar and vector mesons, respectively[20].

In formulating the tetraquark mass formula, effective quark masses provided by Griffiths[21] are utilized. These effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. The effective masses are utilized in Eq. 1.

These six quarks are arranged in three generations: [d(-1/3), u(+2/3)], [s(-1/3), c(+2/3)], and [b(-1/3), t(+2/3)][22]. The three generations are specified by the square brackets and the quark charges [in elementary charge units (e)] are given within parentheses.

3.0 First-Order Mass Formula for the ψ(4230), ψ(4360), and ψ(4415) Modeled as Tetraquarks

The spin of a tetraquark within the first-order mass formula is determined by coupling the two meson clusters

$$J^\pi = J^\pi(1) \times L \times J^\pi \quad (2)$$

where the first-order mass formula assumes a minimally interacting L=0 configuration[4-18] between the two meson clusters. Eq. 2 provides a primitive J^π assignment using the possible meson clusters.

The first-order mass formula used in this paper partitions the tetraquark into two meson (m_1 and m_2) clusters. Using this structure, the tetraquark mass formula involving ground state meson clusters is assumed to have the form[4-18]

$M(m_1 + m_2) = M_m(m_1) + M_m(m_2) + \Phi$ (3)

where **Φ** defines the interaction between the meson clusters. Within the scope of this mass formula, the meson-meson cluster interaction is assumed to be weak and sufficiently small to be ignored. Accordingly, Eq. 3 represents a quasimolecular four quark systems whose basic character is a weakly bound meson-meson system where the mesons reside in their ground states.

4.0 Results and Discussion

The angular momentum coupling from Eq. 2 and the first-order mass formula of Eqs. 1 and 3 are used to construct the ψ(4230), ψ(4360), and ψ(4415) tetraquark states. As noted previously, the spin and parity assignment for the tetraquark state is derived from Eq. 2. Following Refs. 4 – 18, the first-order mass formula only provides a primitive spin and parity assignment for the meson-meson cluster configuration.

Specific first-order mass formula results for the ψ(4230) modeled as a DD_1-bar tetraquark, ψ(4360) modeled as a D^*D_1-bar tetraquark, and ψ(4415) modeled as a D^*D_2*-bar tetraquark are provided in subsequent discussion.

4.1 ψ(4230) Modeled as a DD_1-bar Tetraquark

The angular momentum coupling from Eq. 2 and the first-order mass formula of Eqs. 1 and 3 are used to construct the DD_1-bar tetraquark states. Eq. 2 yields the spin and parity assignment for the DD_1-bar tetraquark state[22]

J^π (DD_1-bar) = J^π(D) x L x J^π(D_1-bar) = 0^- x 0 x 1^+ = 1^- (4)

The 1^- assignment is in agreement with the data[3,22]. The ψ(4230) has a mass of 4222.7 MeV/c^2.[22]

The tetraquark mass is obtained from Eqs, 1 and 3. For the the DD_1-bar tetraquark. Eq. 3 is rewritten as:

$M(D + D_1\text{-bar}) = M_m(D) + M_m(D_1\text{-bar}) + \Phi + \Delta(D_1\text{-bar} – D)$

$= Z_{mm} + \Phi + \Delta(D_1\text{-bar} – D)$ (5)

where $\Delta(D_1$-bar – D) is the mass difference between the D_1-bar and D mesons, and $Z_{mm} = M_m(D) + M_m(D_1\text{-bar})$. The first-order mass result of 4342 MeV/c^2 is about 3% larger than the experimental value. Model results for the spin and parity and associated mass are in reasonable agreement with data.

4.2 ψ(4360) Modeled as a D^*D_1-bar Tetraquark

The angular momentum coupling from Eq. 2 and the first-order mass formula of Eqs. 1 and 3 are used to construct the D*D1-bar tetraquark state. Eq. 2 yields the spin and parity assignment for the D*D1-bar tetraquark state[22]

J^π (D^*D_1-bar) **= $J^\pi(D^*)$ x L x J^π(D_1-bar) = 1^- x 0 x 1^+ = 0^-, 1^-, 2^- (6)**

The 1^- assignment is in agreement with the data[3,22], However, the primitive coupling structure of the first-order mass model allows 0^-, 1^-, and 2^- states. The ψ(4360) has a mass of 4372 MeV/c^2.[22]

The tetraquark mass is obtained from Eqs, 1 and 3. For the the D*D1-bar tetraquark. Eq. 3 is rewritten as:

$M(D^* + D_1\text{-bar}) = Z_{mm} + \Phi + \Delta(D_1\text{-bar} - D) + \Delta(D^* - D)$ (7)

where $\Delta(D^* - D)$ is the mass difference between the D* and D mesons, and $\Delta(D_1\text{-bar} - D)$ is the mass difference between the D_1-bar and D mesons . The first-order mass result of 4484 MeV/c^2 is about 3% larger than the experimental value. Model results for the spin and parity and associated mass are in reasonable agreement with data.

4.3 ψ(4415) Modeled as a $D^*D_2^*$-bar Tetraquark

The angular momentum coupling from Eq. 2 and the first-order mass formula of Eqs. 1 and 3 are used to construct the $D^*D_2^*$-bar tetraquark states. Eq. 2 yields the spin and parity assignment for the D^* D_2^*-bar tetraquark state[22]

$J^\pi(D^*\ D_2^*\text{-bar}) = J^\pi(D^*) \times L \times J^\pi(D_2^*\text{-bar}) = 1^- \times 0 \times 2^+ = 1^-, 2^-, 3^-$ (8)

The 1^- assignment is in agreement with the data[3,22], However, the primitive coupling structure of the first-order mass model allows 1^-, 2^-, and 3^- states. The ψ(4415) has a mass of 4421 MeV/c^2.[22]

The tetraquark mass is obtained from Eqs, 1 and 3. For the $D^*D_2^*$-bar tetraquark. Eq. 3 is rewritten as:

$M(D^* + D_2^*\text{-bar}) = Z_{mm} + \Phi + \Delta(D_2^*\text{-bar} - D) + \Delta(D^* - D)$ (9)

where $\Delta(D_2^*\text{-bar} - D)$ is the mass difference between the D_2^*-bar and D mesons. The first-order mass result of 4523 MeV/c^2 is about 2% larger than the experimental value. Model results for the spin and parity and associated mass are in reasonable agreement with data.

5.0 Conclusions

The tetraquark structure of the ψ(4230), ψ(4360), and ψ(4415) states are investigated using a first-order tetraquark mass formula. This mass relationship is based on weakly bound D plus D_1-bar, D^* plus D_1-bar, and D^* plus D_2^*-bar meson clusters. The first-order tetraquark mass formula provides predicted masses within about 3% of the experimental values, and the J^π predictions are in agreement with values suggested by the data. However, the primitive angular momentum structure of the first- order mass model admit additional J^π values.

References

1. M. Gell-Mann, A schematic model of baryons and mesons, Phys. Lett.**8**, 214 (1964).

2. G. Zweig, An SU(3) model for strong interaction symmetry and its breaking. Version 2, in Developments in the Quark

Theory of Hadrons. VOL. 1. 1964—1978, edited by D. B. Lichtenberg and S. P. Rosen (Hadronic Press, Nonantum, 1964), pp. 22–101, https://inspirehep.net/literature/4674.

3. T. Ji, X.-K. Dong, F.-K. Guo, and B.-S. Zou, Prediction of a Narrow Exotic Hadronic State with Quantum Numbers $J^{PC}=0^{--}$, Phys. Rev. Lett. **129**, 102002 (2022)

4) J. J. Bevelacqua, First-Order Tetraquark Mass Formula, Physics Essays **29**, 198 (2016).

5) J. J. Bevelacqua, Description of the X(5568) and Proposed 750 GeV/c^2 State in Terms of a First-Order Tetraquark Mass Formula, Physics Essays **29**, 367 (2016).

6) J. J. Bevelacqua, Fusion of Doubly Heavy Mesons into a Tetraquark, Physics Essays **31**, 167 (2018).

7) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed X(3872), Physics Essays **32**, 469 (2019).

8) J. J. Bevelacqua, Description of the X(6900) as a Four Charmed Quark State in Terms

of a First-Order Tetraquark Mass Formula, QEIOS **KLXLKJ**, 1 (2020).

https://doi.org/10.32388/KLXLKJ.

9) J. J. Bevelacqua, Description of the X(2900) as an Open Flavor Tetraquark in Terms of

a First-Order Mass Formula, QEIOS, **OVLMEB**, 1 (2020).

https://doi.org/10.32388/OVLMEB.

10) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed $Z_{cs}(3985)^-$, QEIOS

GLTEU2, 1 (2021). https://doi.org/10.32388/GLTEU2.

11) J. J. Bevelacqua, Possible Tetraquark Explanation for the X(6200), QEIOS **J6AFYW**, 1 (2021). https://doi.org/10.32388/J6AFYW.

12) J. J. Bevelacqua, Possible Tetraquark Explanation for the $T_{cc}{}^+$, Qeios **OMDGAQ**, 1 (2021). https://doi.org/10.32388/OMDGAQ.

13) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed $Z_{cs}(4000)^+$ and

$Z_{cs}(4220)^+$, Qeios **PPLMWV**, 1 (2021). https://doi.org/10.32388/PPLMWV.

14) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed X(3960), Qeios,

O1L0YM, 1 (2022). https://doi.org/10.32388/O1L0YM.

15) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed $T(2900)^{++}$

and T(2900)^0 Structures, Qeios **V6WLTS**, 1 (2022). https://doi.org/10.32388/V6WLTS.

16) J. J. Bevelacqua, Possible K K bar Tetraquark Explanation for the f_0(1370), Qeios

HBDQXV, 1 (2023). https://doi.org/10.32388/HBDQXV.

17) J. J. Bevelacqua, Possible f Quark Model of Tetraquarks and Pentaquarks, Qeios, **8T3IVE**, 1 (2023). https://doi.org/10.32388/8T3IVE.

18) J. J. Bevelacqua, Possible Tetraquark Explanation for the Y(10753), Qeios **NZRGH3**, 1 (2023). https://doi.org/10.32388/NZRGH3.

19) Ya. B. Zel'dovich and A. D. Sakharov, *Kvarkovaia struktura i massy sil'novzaimodeistvuyushchikh chastits*, Yad. Fiz. **4**, 395 (1966).

20) A. D. Sakharov, Mass formula for mesons and baryons, Sov. Phys. JETP **51**, 1059 (1980).

21) D. Griffiths, *Introduction to Elementary Particles*, 2nd ed., (Wiley-VCH, Weinheim, 2008).

22) Particle Data Group, Review of Particle Physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).