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Review Article

Towards a Model-Based Approach: Applications to Historical Demography and Palaeodemography

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There is a large variety of different kinds of models. However, we think that they all have in common that they represent something beyond themselves: they are representations of parts of the world. As scientists, we are driven to select only a few aspects of the phenomena studied. However, these aspects will be characterized with great precision. This explains why models consider only parts of the world, as we are driven to select only a few aspects of the phenomena studied. We will first give a historical presentation of models to show their usefulness in the past. We will then develop agent-based models, which are most used in demography, but also in historical demography. Some of them are not able to solve this incompleteness. Finally, we will show how a deeper philosophical approach to these problems may permit a true scientific treatment of explanation in demography.

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Introduction

I will first try to show here that modelling does not constitute a new approach in the history of demography but has always been occurring in the past. However, some more recent methods, which we will later present, may lead to further development in the understanding of the behaviour of past and contemporary populations.

This paper is dedicated to past populations, for which data are sparser than for contemporary ones. So, for these past populations, modelling will often appear necessary: for example, to reconstruct the missing data or to understand past behaviours. However, my presentation will present a more general view of modelling to understand more clearly its usefulness.

Let us now define what we consider as models.

There is a large variety of different kinds of models and several ways in which they function in the service of science. However, we think that they all have in common to represent something beyond themselves: as suggested by Richard Giere (1988), they are representations of parts of the world by the relationship of similarity.

First, as scientists, we are driven to select only a few aspects of the phenomena studied. However, these aspects will be characterized with great precision to be able to give a good representation. This explains why models consider only parts of the world.

Second, what do we consider as a relationship of similarity? We cannot, in social science, and even in every scientific activity, be able to deduce facts from empirical "laws," but only from a formal structure, which generates explanation within its boundaries. Such a formal structure is only similar to the observations of the real world but represents their functional architecture, as Robert Franck says in his book: *The explanatory power of models* (2002). We will come back to this notion of function in the last part of this presentation.

We will first give, in this talk, a historical presentation of models to show their usefulness in the past.

We will then develop agent-based models, which are most used in demography. Eric Silverman's book on *Methodological investigations in agent-based modelling* (2018) gives a good discussion of the problems they raised, and we will try here to show their incompleteness. Other kinds of simulation models will also be shortly presented. Again, some of them are not able to solve this incompleteness. However, the model using recursive Bayesian networks escapes this criticism.

Finally, we will show how a deeper philosophical approach to these problems may permit a true scientific treatment of explanation in demography, and more generally in social sciences. This approach is already followed in other sciences like natural sciences or biology. I hope that social science will follow it.

History of models

As we previously said, modelling has a long history in demography.

The first researcher using a cross-sectional approach was Euler in 1760, who wrote a memoir on the multiplication of the human species. He made interesting hypotheses about the evolution of human populations, which led to a precise model of stable populations.

His first hypothesis is based on the 'vitality or power of life that is specific to humans'. It leads to equating this vitality with the probability of dying at each age, assumed identical for all persons of the same age.

His second hypothesis rests on 'the principle of propagation, which depends on marriage and fertility'. Again, he identifies this principle with the fact that 'the number of children born every year is always proportional to the number of all living persons'. Even if this definition of a fertility rate may now be considered very rough, it was quite interesting in his time.

His third hypothesis is that 'the two principles of mortality and propagation are independent of each other'. Therefore, he does not need to take into account possible interactions between these two probabilities. Again, this hypothesis needs a new approach.

From these three principles, he was able to reconstruct everything that can be said about such populations, whatever they are. To do this, he will use a large corpus of observations made by Süssmilch, 'that seems adequate to settle most of the questions arising in this research'.

Such an approach, even if it was in many senses approximate, was the first model of population evolution through time. We could speak here not of an axiomatization of demography in the full sense, but of a proto-axiomatization of population sciences.

We had to wait until Lotka in 1939 to continue this work, with what he called Malthusian populations. These populations keep all along the time the same mortality and the same age distribution. Such a population becomes a stable population when its fertility function also remains constant through time. As he showed, if these formerly variable rates become identical at a given moment, the stable population is not reached immediately, but only as a limit.

We can draw a parallel between these results and the first law of Newton's theory of gravitation found in 1687. This inertia principle says: '*Everybody perseveres in his state of rest, or uniform motion in a right line unless it compelled to change this state by forces impressed thereon*.'

Similarly, any population whose fertility and mortality are assumed to become constant from a given instant will end towards the stable population that meets these conditions. But whereas the physical body immediately acquires its uniform motion when no force acts upon it, the stable population is not attained at once, since it is a limit.

However, Bourgeois-Pichat showed in 1994 that, in reality, many observed populations at this time attain without any delay the stable state if their age distribution remains constant through time: he called them semi-stable populations. This is less true today, with the downtrend in fertility in these countries.

It is also easy to flesh out this model of population change with age-specific emigration and immigration rates, expressed as net migration rates. This yields a basic relationship between age structure, mortality, fertility, and migration at a given point in time, as Samuel Preston and Ansley Coale showed in 1982.

However, such a model does not introduce the social, economic, religious, political, and other characteristics of the society or group in which these events occur, which we will now consider.

To do that, we will directly jump to more recent approaches, like the models introduced by Francesco Billari and Alexia Prskawetz in 2003: agent-based modelling.

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Agent-based models and others

The agent-based models are derived from simulation analyses used by mathematicians and physicists. The economist Schelling in 1971 suggested their use to study segregation processes, while the ecologists Botkin et al. at the same time proposed a computer model to predict changes in forest growth. In the nineties, these models spread to several social sciences.

Those who introduced them often took care not to consider each science separately, but to view them as a whole, incorporating the spectrum of social processes —demographic, economic, sociological, political, and so on.

Rather than modelling specific data, this approach models theoretical ideas and is based on computer simulation. It aims to understand how the behaviour of biological, social, or more complex systems arises from the characteristics of the individuals or more general agents composing these systems.

In demography, Billari and Prskawetz clearly said: '*Different to the approach of experimental economics and other fields of behavioural science that aim to understand why specific rules are applied by humans, agent-based computational models pre-suppose rules of behaviour and verify whether these micro-based rules can explain macroscopic regularities*.'

This is, therefore, mainly a bottom-up approach, with population-level behaviour emerging from rules of behaviour of autonomous individuals. There is no necessity to introduce other levels to understand demography: the individual rules of behaviour will explain macroscopic regularities.

As I have already said, such an approach is presented in many papers at this workshop as a way to introduce models in historical demography and palaeodemography.

This approach is very interesting, as it eliminates the need for empirical data on personal and individual characteristics, which are so difficult to introduce in historical demography. It is based on simple decision rules followed by the individuals, which can explain some macro phenomena observed from the rare data observable in these historical sciences.

Thomas Burch, in his contribution in 2002 on Computer modelling of theory, clearly said: '*A model explains some real-world phenomena if; (a) the model is appropriate to the real world system ...; and (b) if the model logically implies the phenomenon.*'

A theoretical model of this kind cannot be validated in the same way as an empirical model. Frequentist inference does not apply to such models, as the probability that an unknown parameter lies in a given interval has no significance in such cases. In Franck's words: '*one had ceased to credit deduction with the power of explaining phenomena. Explaining phenomena means discovering principles which are implied by the phenomena.*'

As the agent-based approach focuses on the mechanisms driving the action of individuals as agents, it will simulate the evolution of such a population from simple rules of behaviour. It may thus use game theory, complex systems theory, evolutionary programming, and—to introduce randomness—Monte Carlo methods. This workshop will show us the preferred methods for historical demography.

It may also use survey data, not to explain the studied phenomenon, but only to verify if the parameters used in the simulation lead to behaviour similar to the one observed in the survey.

I will present shortly here an application of such a model in palaeodemography, from a paper by Robert Axtell et al. in 2002 on the collapse of the Anasazi population, observed from the ninth to the beginning of the fourteenth century. They showed that by using simple household rules for choosing locations for farms and introducing eight adjustable parameters for agents and landscape heterogeneity, they were able to simulate population evolution during this period.

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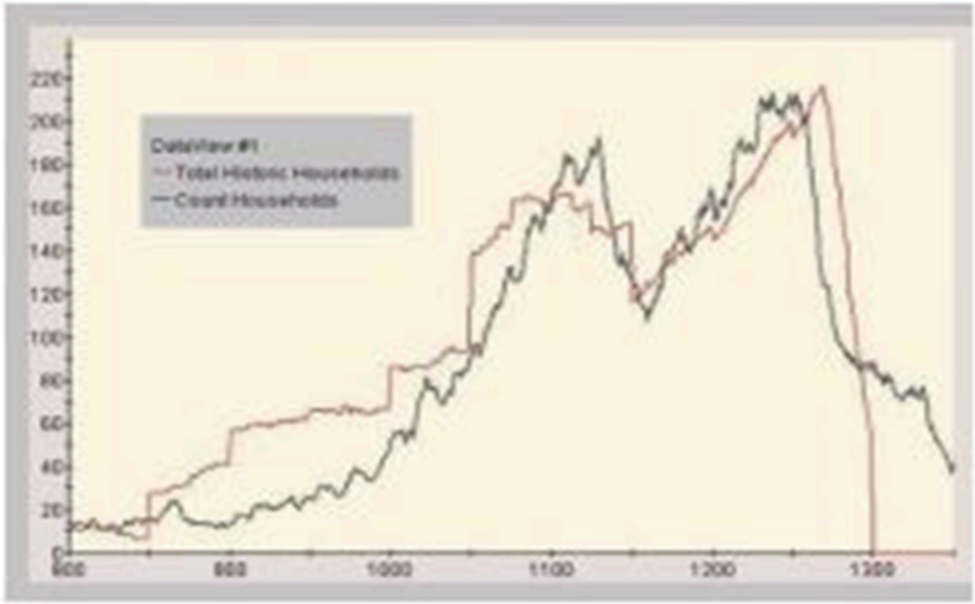

Figure 1. Best single run of the model (in black) compared with the observed population (in red)

To choose the best model, they used the cumulated absolute values of the differences between the observed and simulated populations. In this case, simulated population levels closely follow the historical trajectory. The model analysis also shows that the abandonment of the valley at the beginning of the fourteenth century cannot be explained solely by environmental variations. This model is one of the iconic models in the agent-based community.

However, Marco Janssen, in 2009, showed that this model does not provide much information beyond a comparatively simple model based on two parameters that adjust the carrying capacity of the valley where this population lived: '*The reason for this is that the model acts as a smoothing function of the input data and has limited endogenous dynamics that contribute to the aggregated population data.*'

This last point leads us to some problems raised by agent-based models.

The first problem is that these models are intended to represent the import and impact of individual actions on the macro-level patterns observed in a complex system. This implies that a phenomenon emerging at the aggregate level can entirely be explained by individual behaviour. John Holland, in 2012, while recognizing that agent-based models have been a major tool for studying complex adaptive systems in the last twenty years, insists on these limitations. He said that they '*include a little amount of relevant mathematics and so far little provision for agent conglomerates that provide building blocks and behaviour at a higher level of organization.*'

I can give an example of this in demography in my books of 2002 and 2007, on multilevel analysis. I study the fact of working in the agricultural sector on the migration behaviour of a Norwegian cohort, using different approaches.

We have already spoken of the cross-sectional analysis. However, we have not spoken about the way it introduces different aggregate-level characteristics to explain a phenomenon. This is done by introducing these characteristics in a regression model. The following figure gives the increase in the estimated migration rates with the proportion of farmers present in each Norwegian region.

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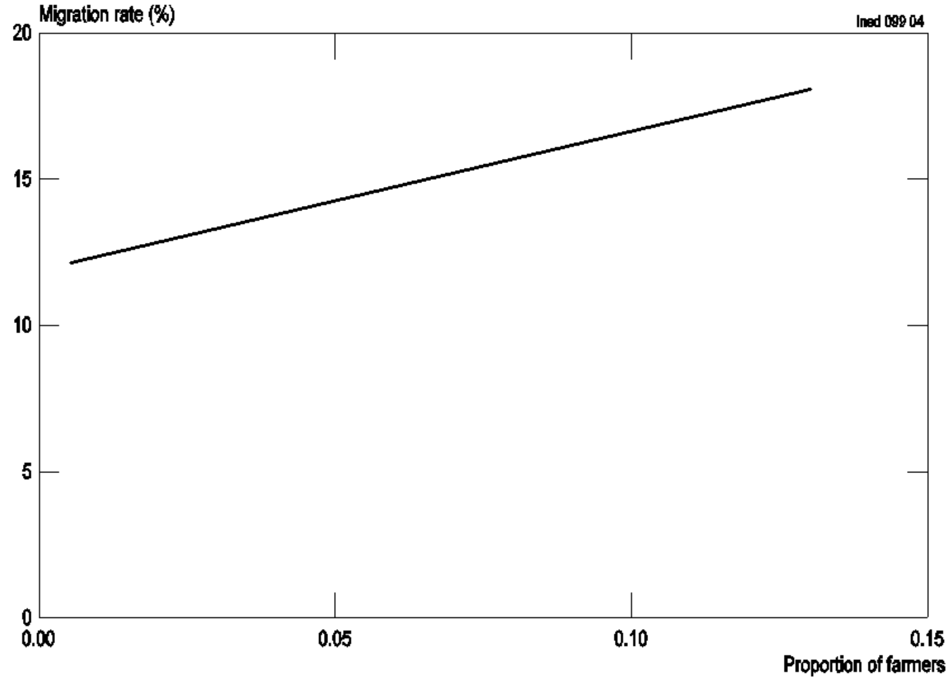

Figure 2. Aggregate level model

Another point of view had been developed since the beginning of the eighties: an event history analysis. This approach uses individual characteristics to explain an individual behaviour. The following figure gives the migration rates of farmers and non-farmers.

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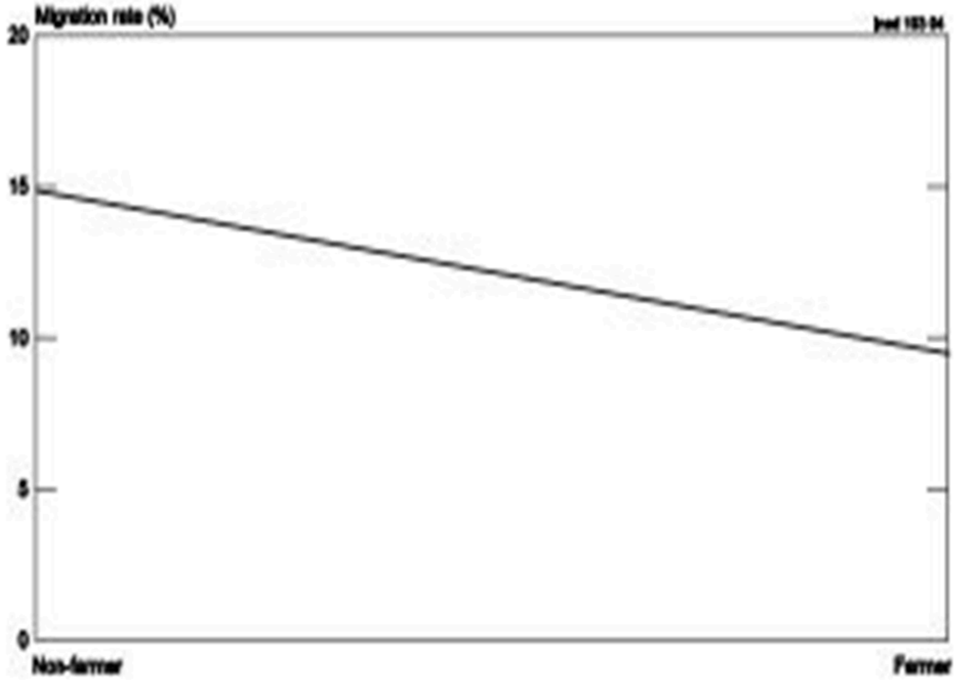

Figure 3. Individual-level model

This gives the opposite result to the previous one: the probability of migrating farmers is now more than a third less than for the other occupations.

To explain these contradictory results, it is necessary to use a multilevel model, which introduces simultaneously the individual behaviour and the aggregate one.

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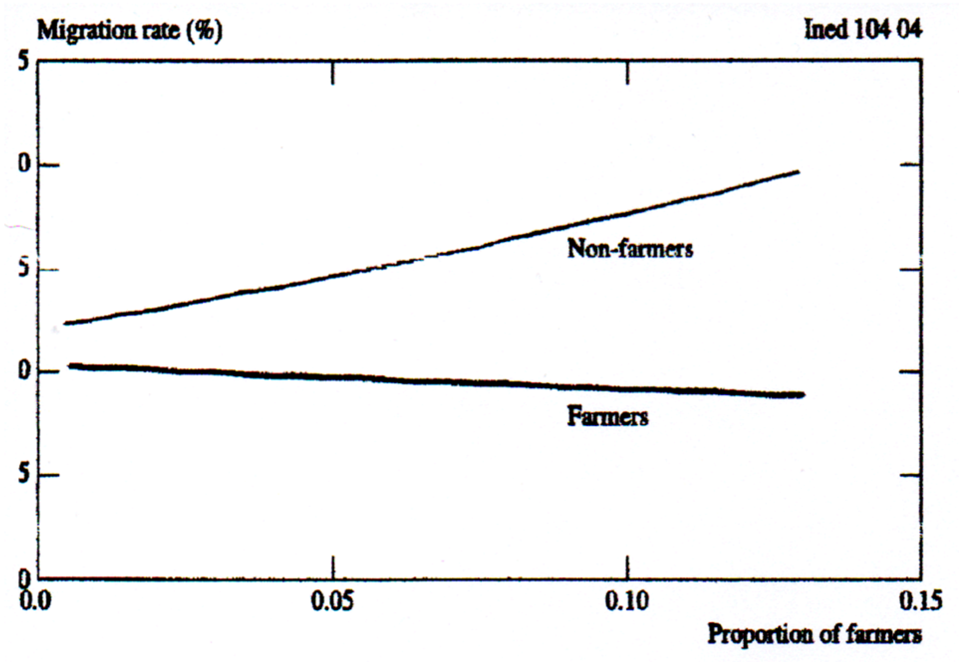

Figure 4. Multilevel model

Such a model permits us to understand the contradictory results obtained with the two previous models: the fact of being a farmer still strongly reduces the probability of migrating, while the fact of living in a region with a large percentage of farmers increases the probability of migration only for non-farmers.

In conclusion, it seems hard, if you consider only the individual level, as in an agent-based model, to explain a contradictory result obtained at the macro-level. I think that often aggregate-level rules cannot be modelled with purely micro-level rules, for they transcend the behaviours of the component agents.

The second problem lies in the search for a good explanation of the observed phenomenon. There are no clear guides to find rules, and their choice often appears arbitrary. Conte et al., in their Manifesto of Computational Social Science in 2012, clearly said about agent-based models: '*First, how to find out the simple local rules? How to avoid ad hoc and arbitrary explanations? As already observed, one criterion has often been used, i.e., choosing the conditions that are sufficient to generate a given effect. However, this leads to a lot of alternative options, all of which are to some extent arbitrary. The construction of plausible generative models is a challenge for the new computational social science.*'

For example, without factoring in the influence of networks on individual behaviour, we can hardly explain a macro-behaviour only by aggregating individual behaviours.

To obtain more satisfactory models, we must introduce, for example, explicit decision-making theories, but also representations, attitudes, strategies, motivations, etc. Unfortunately, the choice of a good theory is influenced by the researcher's discipline and can produce highly different results for the same studied phenomenon. For example, how does one choose between the theory of utility maximization, mainly used by economists, and the theory of planned behaviour, mainly used by sociologists?

The third problem lies in the validation of an agent-based model. We have already said that the usual validation tests for statistical analysis do not apply to those models. How then can we say that the model explains the observed phenomenon?

Günter Küppers and Johannes Lenhard said in 2005, in a paper on the *Validation of simulation*: '*It is consensus in the literature that validation constitutes one of the central epistemological problems of computer simulation methods. Especially in the case of simulations in the social sciences, the answers given by many authors are not satisfactory.*'

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For example, in the case of the Anasazi population, the authors give different ways to compare the number of simulated and measured populations, but they were unable to say which one was the best. They were not able to see that another model using only two parameters replicates the data just as well.

So generally, one typically starts with some data to be explained, and the simulation is said to be successful if the interaction of some interaction rules leads to an approximate reproduction of some structural characteristics of the data. There are no clear ways to say that an agent-based model may be structurally accurate.

We can conclude that there are no clear verification and validation procedures for agent-based models in the social sciences. We need a more clearly designed model to promote a true scientific approach.

As we have previously said, even if agent-based models are prominent in the social sciences, there exist other kinds of simulation models. We restrict, however, our investigation to simulation models used in demography.

Macro-simulation and micro-simulation are some other main alternative methods used in demography for making similar statements about the future. You may find a complete presentation of them in a paper by Evert Van Imhoff and Wendy Post: Microsimulation methods for population projections in 1998. They are subject to the first previous problem, as macro-simulation works only on the aggregate level, and micro-simulation only on the individual level. Also, the validation problem affects these simulations as generally, all projections of population lead to incorrect results: projection errors increase systematically as they look further ahead.

More recently, models based on recursive Bayesian networks have appeared. These models can be applied to modelling the hierarchical structure of observed phenomena. They then avoid the first previous one-level problem. Lorenzo Cassini et al. in 2013 applied such a model to cancer. The higher level of this model contains variables at the clinical level, while the lower level maps the structure of the cell's mechanism for apoptosis. On introducing mechanisms, as we will further see, this approach avoids the validation problem. So, to apply this approach to demography, it needs to be ascertained with a mechanistic approach for demography.

However, given these problems, for the major part of models used in demography, we don't have to throw the baby out with the bathwater! We will have to show why this baby "simulation" is an important one and how to save it.

Towards a model-based synthesis

I have already cited the volume by Franck in 2002, in which you will find a clear explanation of the usefulness of simulation models. We will summarize this explanation here and give its application to demography, which you will find in a paper later published in 2017: *Model based demography: towards a research agenda?*, which I wrote with him and two demographers, Jakub Bijak and Eric Silverman.

First, a semantic approach to theories was developed, for example, by Frederick Suppe in his 1989 book: *The semantic conception of theories and scientific realism*. This book provides some ways to offer a satisfactory epistemological basis for differentiating formal explanatory models from empirical explanatory models.

He begins by saying that: '*A science has not to deal with phenomena in all their complexity; rather it is concerned with certain kinds of phenomena only insofar as their behaviour is determined by, or characteristic of, a small number of parameters abstracted form those phenomena*'.

So, a formal explanatory model is like a filter that retains only a small number of parameters, chosen as the object of research. For demography, the concept of the statistical individual follows perfectly this approach, and the parameters will be the functions of fertility, mortality, and migration, which channel the investigation.

As I said in Probability and Social Science in 2012: '*Under this scenario, two observed individuals, with some identical characteristics, will certainly have different chances of experiencing a given event, for they will have an infinity of other characteristics that can influence the outcome. By contrast, two statistical individuals, seen as units of a repeated random draw, subjected to the same sampling conditions and possessing the same characteristics, will have the same probability of experiencing the event*.'

On the contrary, empirical explanatory models try to seek explanations of social facts from empirical regularities. He showed that this view is wrong, as it obscures much of the epistemic importance other analyses can reveal.

Following the semantic approach, a theory is a formal system, empty of any empirical content. So, the explanation of empirical facts consists of deducing them from a formal system, and not from empirical

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laws. Such a formal structure is here elaborated by the researcher, without any consideration of the empirical data, and its components are conceptual or mathematical.

As Burch said in 2002, such an explanation involves: '*(a) creation of a logical structure of variables and their relationships, a structure which logically implies or entails the event; (b) demonstration that there is correspondence or "isomorphism" between the logical structure and the real-world context in which the event is embedded*.'

The notion of "isomorphism," however, raises some problems for social scientists, while it is possible to apply it to a theoretical physical model. In social sciences, we can only observe real populations, which are not under idealized circumstances as in physical science. So, we may have some doubts about this notion and say that there is no easy way to demonstrate isomorphism. As Burch says: '*This problem of how to assess the relationship between complex simulation models and empirical data has plagued the practice of computer modelling from the beginning and has yet to be adequately resolved*.'

So the main question is the following one: how would one identify the relationship between the theoretical model and the empirical observations, and test the fit of a simulation model? As Burch says: 'correct prediction can result from a model with incorrect assumptions and inputs'.

To go further and to enrich this approach, we will rely on model-based science, which is also known as a mechanistic view. We will follow here the presentation given by Franck for social sciences in two thousand two, and its application to demography we gave with him, Bijak, and Silverman in 2017.

First, such a model permits the generalization of the results obtained by a semantic model to real observations that are not under idealized circumstances. To do that, it introduces some new concepts or some older concepts that had lost their original sense. Let us see them in more detail.

For the pioneers of modern science in the seventeenth century (Bacon, for example, in 1620 in Novum Organon), induction consists of discovering the principle of a phenomenon from the study of its properties. Later, however, from the eighteenth century, philosophers considered its usual sense of generalization (Hume, Mill, or Popper, for example). Deduction, which had often been considered the main scientific instrument, has ceased to have the power of explaining phenomena. Harold Keynes said in his Theory of Probability, in 1939: '*...the tendency to claim that scientific method can be reduced in some way to deductive logic, which is the most fundamental fallacy of all...*'

This research by induction will permit us to find the theoretical explanation, which consists of discovering the combination of concepts without which the observed properties of a phenomenon would be inconceivable or impossible. Simultaneously, it will give us an empirical demonstration of using the factors operative in a given society.

The theoretical explanation represents the conceptual structure of the phenomenon studied. The empirical explanation represents the social factors that give rise to it.

The notion of mechanism has been defined in different ways, according to the considered science, but we prefer the one, more general, given by Franck and following well the previous definitions: 'The formal (conceptual) model is the form of the social mechanism, and the social mechanism is the matter of the formal model.' Such a form will be constructed by an axiomatization of the discipline.

We are now able to give the general method, proposed by Franck, to be able to give a mechanistic explanation: '*(1) Beginning with the systematic observation of certain properties of a given social system, (2) we infer the formal (conceptual) structure that is implied by these properties. (3) This formal structure, in turn, guides our study of the social mechanism that generates the observed properties. (4) The mechanism, once identified, either confirms the advanced formal structure or indicates that we need to revise it.*'

Such a process has already been followed for the study of probability, which is an important tool for demography. From its conception by Pascal in 1654 to its formal axiomatization by Kolmogorov in 1933, a great number of attempts had failed. Similarly, demography's conception occurred with Graunt in 1662, and we have already seen a proto-axiomatization by Euler in 1760. But a full axiomatization, as proposed by Franck, had not yet been accomplished.

Conclusion

For the moment, we have four paradigms that have been followed during the history of demography: first, a cross-sectional one from sixteen sixty-two until the end of the Second World War; followed by a longitudinal version, which went at the beginning of the eighties to a full-fledged event history approach; fourth, a multilevel one beginning in the middle of the eighties. Each

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paradigm did not cancel the previous one but gave a new point of view for demography.

The model-based approach provides us with the means to expand the range of benefits already provided by the four previous paradigms. We gain deeper insight into the interactions between various population systems, and we also gain the capacity to explore the parameter space of the simulations by generating "what-if" scenarios. Simulation parameters—once they result from the functional-mechanistic approach—govern how the complex, interacting social processes in the model work.

We have seen earlier that recursive Bayesian networks can provide a mechanistic hierarchical model for the study of cancer. However, as David Livingstone and Ramesh Shivdasani said in 2006, a mechanism-based theory of cancer may be applied. But for demography, this is not, unfortunately, the case, and we are not yet able to describe functional mechanisms and causal demographic relations.

However, I think that the model-based programme will lead to substantial promise in correctly modeling the social mechanisms at the root of demographic structures. I encourage you, young researchers, to undertake this fundamental task.

References

- Axtell R L, Epstein J M, Dean J S, Gumerman G J, Swedlund A C, Harburger J, Chakravarty S, Hammond R, Parker J, and Parker M. (2002) Population Growth and Collapse in a Multi-Agent Model of the Kayenta Anasazi in Long House Valley. *Proceedings of the National Academy of Sciences* 99(3): 7275-7279.
- Bacon, F. (1620). *Novum Organum*, London: J. Bill.
- Billari, F., Prskawetz, A., ed. (2003). *Agent-based computational demography: using simulation to improve our understanding of demographic behaviour*. Heidelberg, New York: Physica-Verlag.
- Botkin, D.B., Janak, J.F., Wallis, J.R. (1972). Some ecological consequences of a computer model of forest growth. *The Journal of Ecology*, 60, 849-872.
- Bourgeois Pichat, J. (1994). *La dynamique des populations. Populations stables, semi-stables, quasi-stables*. Paris: Editions de l'Institut National d'Etudes Démographiques.
- Burch, T. (2002). Computer modelling of theory: explanation for the 21st century. In *The explanatory power of models. Bridging the gap between empirical and theoretical models in the social sciences*, Franck, R. ed, Methodos Series 1, Boston: Dordrecht / London: Kluwer Academic Publishers, pp. 245-266.
- Casini, L., Illari, P.M., Russo, F., Williamson, J.W. (2011). Models for prediction, explanation and control: recursive Bayesian networks. *Theoria*, 70, 5-33.
- Conte, R., Gilbert, N., Bonelli, G., Cioffi-Revilla, C., Deffuant, G., Kertesz, J., Loreto, V., Moat, S., Nadal, J.-P., Sanchez, A., Nowak, A., Flache, A., San Miguel, M., Helbing, D. (2012). Manifesto of computational social science. *The European Physical Journal Special Topics*, 214, 325-346.
- Courgeau, D., ed. (2003). *Methodology and epistemology of multilevel analysis: approaches from different social sciences.* Methodos Series, vol. 2, Dordrecht / Boston / London: Kluwer Academic Publishers.
- Courgeau, D. (2007). *Multilevel synthesis: from the group to the individual*. Series on demographic methods and population analysis, vol. 18, Dordrecht: Springer.
- Courgeau, D. (2012). *Probability and social science. Methodological relationships between the two approaches*. Methodos Series, vol.10. Dordrecht: Springer.
- Courgeau, D., Bijak, J., Franck, R., Silverman, E. (2017). Model-based demography: towards a research agenda. In A. Grow, J. Van Bavel, (eds), *Agent-based modelling in population studies,* Series on demographic methods and population analysis, vol. 41, Dordrecht: Springer.
- Euler, L. (1760). Recherches générales sur la mortalité et la multiplication du genre humain. *Histoire de l'Académie Royale des Sciences et des Belles Lettres de Berlin*, vol. 16, pp. 144-164.
- Franck, R., Ed. (2002). *The explanatory power of models: bridging the gap between empirical and theoretical research in the social sciences.* Boston / Dordrecht / London: Kluwer Academic Publishers.
- Giere R.N. (1988). Explaining Science: a cognitive approach. *Philosophical Review*, 100 (4), pp. 653-656.
- Graunt, J. (1662). *Natural and political observations mentioned in a following index, and made upon the bills of mortality.* London; Tho. Roycroft.
- Holland, J.H. (2012). *Signals and boundaries. Building blocks for complex adaptive systems*. Cambridge London: The MIT Press.
- Imhoff Van, E., Post, W. (1998) Microsimulation methods for population projections. *Population. An English Selection* (D. Courgeau ed), 10 (1), pp. 97-138).
- Janssen, M.A. (2009). Understanding artificial Anasazi. *Journal of Artificial Societies and Social*

Simulation, 12 (4) 13. http://jasss.soc.surrey.ac.uk/12/4/13.html

- Keynes, H. (1939). *Theory of probability*. Oxford: Clarendon Press.
- Kolmogorov, A. (1933). Grundbegriffe der wahrscheinlichkeitsrenung. In *Ergebnisse der Mathematik*, *vol.2*, Berlin: Springer (English translation, 1950, Morrison N. *Foundations of the theory of probability*, New York: Chelsea).
- Küppers, G., Lenhard, J. (2005). Validation and simulation: patterns in the social and natural science. *Journal of Artificial Societies and Social Simulation, 8, (4),* http://jasss.soc.surrey.ac.uk/8/4/3.html.
- Livingstone, D.M., Shivdasani, K. (2003). Towards mechanism-based cancer care. *Journal of the American Medical Association*, 295 (5), pp. 588-593
- Lotka, A.J. (1939). *Théorie analytique des associations biologiques; Deuxième partie. Analyse démographique avec application particulière à l'espèce humaine*. Paris: Herman.
- Newton, I, (1687). *Philosophiae naturalis principia mathematica*. London: Josephi Streater
- Pascal, B. (1654). *Traité du triangle arithmétique, avec quelques autres traités sur le même sujet*. Paris: Guillaume Desprez.
- Preston, S.H., Coale, A.J. (1982). Age structure/growth, attrition and accession: a new synthesis. *Population Index*, 48 (2), pp. 217-259.
- Schelling, T. (1971). Dynamic models of segregation. *Journal of Mathematical Sociology*, 1, 143-171.
- Silverman, E. (2018*). Methodological investigations in agent-based-modelling*. Methodos Series, vol.13, Dordrecht: Springer.
- Suppe, F. (1989).*The semantic conception of theories and scientific realism*. Urbana: University of Illinois Press.

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