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Analysis of the Spread of Covid-19 via Atangana-Baleanu Fractional Derivatives

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Abstract

We study the spread of the epidemic via Atangana-Baleanu Fractional Derivatives. We present the mathematical analysis and formulation of a fractional model for the epidemic. The existence and uniqueness of the solution for the proposed model are proved. The study also investigates the existence of disease-free equilibrium and analyzes its stability properties. To validate the theoretical results, we provide a numerical scheme for the fractional model and present various simulation results. These results can serve as a valuable resource in developing strategies to mitigate the spread of the epidemic.

Keywords: Atangana-Baleanu's derivative; epidemiological modelling; stability; existence and uniqueness of solution.

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1 Introduction

The coronavirus epidemic emerged in the Chinese city of Wuhan in December 2019, and it rapidly disseminated to other parts of the world. This virus is primarily transmitted through respiratory infection in humans [2]. The disease is characterized by symptoms such as difficulty breathing and fever. The incubation period of the virus ranges from 2 to 14 days before symptoms become evident [2, 3, 13]. In February 2020, Algeria and other countries reported their first confirmed case of the virus in an Italian citizen who had recently arrived. Authorities promptly conducted scans on individuals showing suspicious symptoms or signs of infection. However, despite the daily checks carried out by health authorities, there remains a significant gap between the day of infection and the day of diagnosis. This gap has severe implications for the spread of the disease, as it hinders the accurate assessment of the number of affected individuals and the severity of the disease. Consequently, obtaining precise figures regarding the number of infected and recovering individuals becomes challenging.

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In order to study the clinical development and spread of the epidemic, researchers formulate mathematical models that help assess the severity of the disease and determine necessary interventions to mitigate its impact, such as social isolation, quarantine measures, and travel restrictions [17]. One widely used model for studying disease dynamics is the classical susceptible-infected-recovered (SIR) model [14]. These models provide insights into the dynamics of disease spread, and the timing of outbreaks and declines, and aid in determining when it is appropriate for individuals to return to work. Typically, these models are formulated as differential or stochastic equation sets. The initial results introducing these models date back to the following references [14]. Our model encompasses seven distinct compartments, each representing a different stage or condition related to the epidemic. These compartments include: susceptible individuals, exposed individuals, recovered individuals, quarantined individuals, recovered-exposed individuals and death individuals. By considering these seven compartments, our model aims to provide a comprehensive representation of the different stages and conditions related to the epidemic. In our research, we propose a reformulation of the model presented in reference [13] by incorporating a fractional derivative of the Atangana-Baleanu type. The motivation behind utilizing the Atangana-Baleanu derivative (ABD) in our model is due to its distinct properties, such as a nonlocal and nonsingular kernel. These properties are crucial for accurately capturing the crossover behavior observed in the dynamics of the epidemic. Other fractional derivative operators, such as Riemann-Liouville, Caputo, and Caputo-Fabrizio, do not possess these specific properties, which may or may not adequately describe the future dynamics of the coronavirus. This motivates our choice of the Atangana-Baleanu derivative in our reformulated model [3].

Fractional differential equations have proven to be highly valuable in modeling various biological systems. This can be observed in several research papers that have explored their applications. Some notable papers related to the use of fractional calculus in modeling biological systems include [3, 7, 11, 12, 15]. These studies have highlighted the effectiveness and relevance of fractional differential equations in capturing the complex dynamics and behaviors exhibited by biological phenomena. Additionally, there are specific papers that focus on approximating the Atangana-Baleanu derivative [4, 9]. These works further demonstrate the significance of fractional calculus in accurately describing and understanding the dynamics of biological systems.

This paper presents a comprehensive analysis of the epidemic by introducing a mathematical model that captures the dynamics of disease spread, as described in reference [13, 18]. Subsequently, we propose a fractional model of the epidemic and investigate the existence and uniqueness of solutions for our system. Additionally, we examine the existence of a disease-free equilibrium and analyze its stability properties.

The structure of the article is as follows: In Section 2, we provide definitions and explanations pertaining to fractional calculus. We then proceed to present the mathematical model of the coronavirus, followed by its formulation using fractional derivatives. In Section 3, we conduct a detailed analysis of the fractional model for the coronavirus, focusing specifically on the existence and uniqueness of solutions. We delve into the mathematical properties and characteristics of the model in order to gain deeper insights into its behavior. Moving on to Section 4, we explore the equilibrium points of the model and examine their stability properties. This analysis allows us to understand the long-term behavior of the epidemic and assess the potential for disease control and mitigation strategies. In Section 5, we present a numerical scheme for the fractional model, providing a practical approach to simulate and study its dynamics. We include illustrative results from numerical simulations conducted with various values of the fractional order paramete. These simulations help us visualize and analyze the impact of different parameters on the spread and control of the epidemic.

Finally, in the conclusion, we summarize our findings and offer recommendations based on the insights gained from the study. These recommendations aim to guide future research and inform potential strategies for managing and combating an epidemic.

2 Preliminaries

In this section, we will cover some of the most important basic concepts and definitions of fractional calculus and epidemic models.

Definition 1. [6] Let $z \in H^1(a,b)$, where $b > a$. The Atangana-Baleanu's derivative (ABD) of order $\gamma \in (0,1]$ is defined as follows:

$$ {}^{ABD}_{a}D^{\gamma}_{t} z(t) = \frac{B(\gamma)}{1-\gamma} \int_a^t E_\gamma \left((t-v)^\gamma \frac{\gamma}{\gamma - 1} \right) z'(v) dv, $$

in which E_γ is the one-parameter Mittag-Leffler function and $B(\gamma) = 1 - \gamma + \frac{\gamma}{\Gamma(\gamma)}$ is called the normalization function featuring $B(0) = B(1) = 1$.

Definition 2. [6] The Riemann-Liouville's integral (RLI) of order γ is defined as follows:

$$ {}^{RLI}I^{\gamma}(z(t)) = \frac{1}{\Gamma(\gamma)} \int_a^t (t-v)^{\gamma-1} z(v) dv, \gamma > 0. $$

Definition 3. [6] The Antangana-Baleanu's integral(ABI) of order $\gamma \in (k, k+1]$ and $k \in \mathbb{Z}^+$ is defined by:

$$ {}^{ABI}I^{\gamma}(z(t)) = \frac{\gamma}{B(\gamma)} {}^{RLI}I^{\gamma}(z(t)) + \frac{1-\gamma}{B(\gamma)} z(t), $$

in which

$$ B(\gamma) = 1 - \gamma + \frac{\gamma}{\Gamma(\gamma)}, $$

is called the normalization function featuring $B(0) = B(1) = 1$.

Property 1. [2] The Atangana-Baleanu's integral of order $\gamma \in (k, k+1]$ and $k \in \mathbb{Z}^+$ is stated as :

$$ {}^{ABI}I^{\gamma}(z(t)) = \frac{\gamma - k}{B(\gamma - k)} {}^{RLI}I^{\gamma}(z(t)) + \frac{1 + k - \gamma}{B(\gamma - k)} {}^{RLI}I^{k} z(t). $$

Remark 1. For $\gamma (k \in \mathbb{Z}^+)$, and $t > a$, the following properties are satisfied:

1. ${}^{ABD}D^{\gamma\, ABI}I^{\gamma}(z(t)) = z(t)$,
2. ${}^{ABI}I^{\gamma\, ABD}D^{\gamma}(z(t)) = z(t) - \sum_{d=0}^{k} z^{(d)}(a) \frac{(t-a)^d}{d!}$.

2.1 Model description of corona-virus with Atangana-Baleanu fractional derivative

We analyze the corona–virus model discussed by Ben Fradj and Cherif [13] . Let us consider the total population at time t as $N(t)$. We divide the population $N(t)$ into seven distinct subgroups: susceptible individuals $S(t)$, exposed individuals $E_1(t)$, infected individuals $I(t)$, recovered-exposed individuals $E_2(t)$, recovered individuals $R(t)$, deceased individuals $D(t)$, and quarantined individuals $Q(t)$.

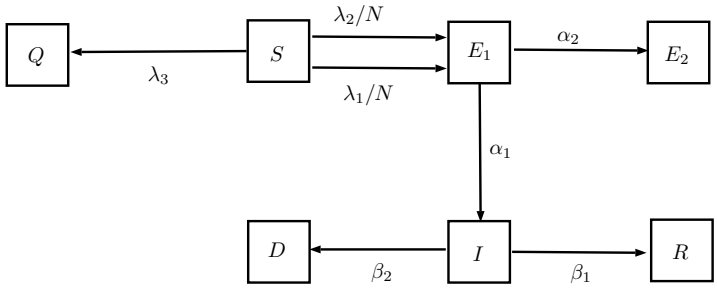

Figure 1: Schematic diagram of the model [13].

The relationships and interactions between these subpopulations can be visually represented through the following compartmental diagram: The model is developed based on a system of non-linear differential equations, which can be expressed as follows:

$$\begin{cases} \dot{S}(t) = -\frac{\lambda_1}{N} S(t)E_1(t) - \frac{\lambda_2}{N} S(t)I(t) - \lambda_3 S(t), \\ \dot{E}_1(t) = \frac{\lambda_1}{N} S(t)E_1(t) + \frac{\lambda_2}{N} S(t)I(t) - (\alpha_1 + \alpha_2)E_1(t), \\ \dot{I}(t) = \alpha_1 E_1(t) - (\beta_1 + \beta_2)I(t), \\ \dot{E}_2(t) = \alpha_2 E_1(t), \\ \dot{R}(t) = \beta_1 I(t), \\ \dot{D}(t) = \beta_2 I(t), \\ \dot{Q}(t) = \lambda_3 S(t), \end{cases} \tag{1}$$

with the initial conditions

$$S(0) = S_0, E_1(0) = E_{01}, I(0) = I_0, E_2(0) = E_{02}, R(0) = R_0, D(0) = D_0, Q(0) = Q_0,$$

where λ_1 is the contact rate between S and E_1, λ_2 is infection rate, λ_3 is the rate at which susceptible people enter in quarantine, α_1 is the inverse of the average latent time, α_2 is the recovered rate of E_1, β_1 is the recovered rate, β_2 is the death rate and the total constant population N is give by

$$N = S(t) + E_1(t) + I(t) + E_2(t) + R(t) + D(t) + Q(t).$$

To incorporate the aforementioned effect into the mathematical representation, we modify the system by replacing the traditional time derivative with the Atangana-Baleanu ordered derivative [6]. The resulting formulation is as follows:

$$\begin{cases} {}_0^{ABD}D_t^{\gamma} S(t) = -\frac{\lambda_1}{N} S(t)E_1(t) - \frac{\lambda_2}{N} S(t)I(t) - \lambda_3 S(t), \\ {}_0^{ABD}D_t^{\gamma} E_1(t) = \frac{\lambda_1}{N} S(t)E_1(t) + \frac{\lambda_2}{N} S(t)I(t) - (\alpha_1 + \alpha_2)E_1(t), \\ {}_0^{ABD}D_t^{\gamma} I(t) = \alpha_1 E_1(t) - (\beta_1 + \beta_2)I(t), \\ {}_0^{ABD}D_t^{\gamma} E_2(t) = \alpha_2 E_1(t), \\ {}_0^{ABD}D_t^{\gamma} R(t) = \beta_1 I(t), \\ {}_0^{ABD}D_t^{\gamma} D(t) = \beta_2 I(t), \\ {}_0^{ABD}D_t^{\gamma} Q(t) = \lambda_3 S(t), \end{cases} \tag{2}$$

where ${}_0^{ABD}D_t^{\gamma}$ the Antangana-Baleaneau fractional derivative of order $\gamma \in (0, 1]$, the model variables in (1) are non negative, and has appropriate initial conditions.

3 Existence and uniqueness of solutions via Atangana-Baleanu fractional derivative

The initial value problem (2) can be written as the following matrix from :

$$ {}_{a}^{ABD}D_{t}^{\gamma}X'(t) = MX(t) + f(X), $$

where $X(t) = (S, E_1, I, E_2, R, D, Q)$,

$$ M = \begin{pmatrix} -\lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\alpha_1+\alpha_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -(\beta_1+\beta_2) & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, $$

$$ f(x) = \left(-\frac{\lambda_1}{N}SE_1 - \frac{\lambda_2}{N}SI,\ \frac{\lambda_1}{N}SE_1 + \frac{\lambda_2}{N}SI,\ 0,\ 0,\ 0,\ 0,\ 0 \right). $$

In order to establish the nonnegative of solutions with the initial conditions, we need also the following lemmas.

3.1 Bounded and non-negative solutions

For the proof of the theorem about non-negative solutions and bounded we shall need the following lemma

Lemma 1. *[3] Let $0 < \gamma \leq 1$ and $f : [a, b] \to \mathbb{R}$ be a differentiable function such that $f' \in L^1[a, b]$ and ${}^{ABD}D_{a^+}^{\gamma} f \in C[a, b]$. Then, for any $t \in [a, b]$, there exists $\xi \in [a, b]$ such that*

$$ f(t) = f(a) + \frac{1-\gamma}{B(\gamma)}\,{}^{ABD}D_{a^+}^{\gamma} f(t) + \frac{(t-a)^{\gamma}}{B(\gamma)\Gamma(\gamma)}\,{}^{ABD}D_{a^+}^{\gamma} f(\xi). $$

Remark 2. Let $f(t) \in C[a, b]$ and ${}^{ABD}D^{\gamma} f \in C[a, b]$ for $0 < \gamma \leq$. It is clear from Lemma(1) that if ${}^{ABD}D^{\gamma} f \geq 0$ for all $t \in (0, b)$, then the function f is non-decreasing, and if ${}^{ABD}D^{\gamma} f \leq 0$ for all $t \in (0, b)$, then the function f is non-increasing.

Definition 4. [11] Let $C[0, T]$ be the class of continuous functions $x(t)$ defined on the interval $[0, T]$, satisfying the condition $sup|x(t)| < \infty$ for $t \in [0, T]$. Additionally, let $\mathfrak{C}([0, T])$ be the class of continuous column vectors $X(t)$ defined as:

$$ X(t) = \begin{pmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) & x_5(t) & x_6(t) & x_7(t) \end{pmatrix}^T, $$

where $x_i(t) \in C[0, T], i = 1, \ldots, 7$. The norm of $X(t) \in \mathfrak{C}[0, T]$ is given by :

$$ \|X\| = \sum_{i=1}^{7} sup|x_i(t)|, t \in [0, T]. $$

Lemma 2. *[11] Consider the interval $E = [0, T] \times [X^{(0)}(0) - \epsilon, X^{(0)}(0) + \epsilon]$, where $T > 0$ and $\epsilon > 0$. Let $f : E \to \mathbb{R}$ be a function. Then, the fractional differential equation*

$$ {}_{a}^{ABD}D_{t}^{\gamma}X(t) = f(t, X(t)), \gamma > 0, X^{(0)}(0) = X_0,\ k = 0, 1, 2, \ldots, m-1, $$

has a solution, and this solution is unique if the following two conditions hold simultaneously:

(1) *f is bounded and continuous on E.*

(2) *f satisfies the Lipschitz condition with respect to the second variable, i.e.,*

$$|f(t, X(t)) - f(t, Y(t))| \leq L|X(t) - Y(t)|,$$

where $L > 0$ is a constant independent of t, X and Y.

Theorem 1. *The initial value problem (2) possesses a unique solution. Moreover, this solution remains nonnegative and bounded.*

Proof. We will divide the proof into three steps:
Step 1 Firstly, we will prove that the solution is nonnegative. From equation (2), we have:

$$\begin{aligned}
{}^{ABD}D\gamma S(t)|_{S=0} &= 0,\\
{}^{ABD}D^{\gamma}E_1(t)|_{E_1=0} &= \frac{\lambda_2}{N}SI \geq 0,\\
{}^{ABD}D^{\gamma}I(t)|_{I=0} &= \alpha_1 E_1 \geq 0,\\
{}^{ABD}D^{\gamma}E_2(t)|_{E_2=0} &= \alpha_2 E_1 \geq 0,\\
{}^{ABD}D^{\gamma}R(t) = |_{R=0} &= \beta_1 I \geq 0,\\
{}^{ABD}D^{\gamma}D(t)|_{D=0} &= \beta_2 I \geq 0,\\
{}^{ABD}D^{\gamma}Q(t)|_{Q=0} &= \lambda_3 S \geq 0.
\end{aligned}$$

Based on Lemma (1) and Remark (2), we can conclude that the solution of equation (2) is nonnegative.
Step 2 Secondly we prove that solution is bouned. From (2) by using the first equation and adding the first two equations, we get :

$${}^{ABD}D\gamma S(t) \leq \lambda_3 S(t),$$

$${}^{ABD}D^{\gamma}N(t) \leq -\xi_1 N(t),$$

with $N(t) = S(t) + E_1(t)$ and $\xi_1 = min(\lambda_3, \alpha_1 + \alpha_2)$, by integration of (ABD) we get :

$$S(t) \leq \left(\frac{B(\gamma)}{B(\gamma) + \lambda_3(1-\gamma)}S(0) - \frac{\lambda_3\gamma B(\gamma)}{B(\gamma) + \lambda_3(1-\gamma)}\,{}^{ABI}I^{\gamma}s(t)\right) \leq S(0),$$

an in other hand

$$N(t) \leq \left(-\xi_1\frac{B(\gamma)}{B(\gamma) + \xi 1(1-\gamma)}N(0) - \frac{\xi 1\gamma B(\gamma)}{B(\gamma) + \xi_1(1-\gamma)}\,{}^{ABI}I^{\gamma}s(t)\right) \leq N(0),$$

Hence

$$S(t) + E_1(t) \leq S(0) + E_1(0).$$

Then

$$E_1(t) \leq S(0) + E_1(0).$$

By adding the first three equations of (2), and the same procedure we get :

$$^{ABD}D^{\gamma}M(t) \leq -\xi_2 M(t),$$

with $M(t) = S(t) + E_1(t) + I(t)$ and $\xi_2 = min(\lambda_3, \alpha_2, \beta_1 + \beta_2)$, by following the same procedure, we obtain:

$$M(t) \leq \left(-\xi_2 \frac{B(\gamma)}{B(\gamma) + \xi 2(1-\gamma)} N(0) - \frac{\xi 2\gamma B(\gamma)}{B(\gamma) + \xi_2(1-\gamma)} {}^{ABI}I^{\gamma} s(t) \right) \leq M(0).$$

Hence, we have:

$$S(t) + E_1(t) + I(t) \leq S(0) + E_1(0) + I(0).$$

Therefore,

$$I(t) \leq S(0) + E_1(0) + I(0).$$

We can follow the same approach for the remaining variables E_2, R, D, and Q, and obtain:

$$\begin{aligned}
E_2(t) &= E_2(0) + \frac{\alpha_2(1-\gamma)}{B(\gamma)}(E_1(0) + S(0)), \\
R(t) &= E_2(0) + \frac{\beta_1(1-\gamma)}{B(\gamma)}(S(0) + E_1(0) + I(0)), \\
D(t) &= D(0) + \frac{\beta_2(1-\gamma)}{B(\gamma)}(S(0) + E_1(0) + I(0)), \\
Q(t) &= Q(0) + \frac{\lambda_3(1-\gamma)}{B(\gamma)}(S(0)).
\end{aligned}$$

Step 3 We define the matrix norm as $|||.|||$, where $|||M||| = \rho(M)$ and $\rho(M)$ represents the largest eigenvalue of the matrix M. Let us consider the bounded and continuous function

$$F(X) = MX + f(X).$$

Assume that $X(t)$ and $Y(t)$ are two distinct solutions of the initial value problem (2), such that:

$$X = (S, E_1, I, E_2, R, D, Q),\ Y = (S', E_1', I', E_2', R', D', Q') \text{ and } X, Y \in \mathfrak{C}[0, T],$$

then

$$\begin{aligned}
\|F(X) - F(Y)\| &= \|MX + f(X) - MY - F(Y)\| \\
&\leq \|M(X(t) - Y(t))\| + \|f(X) - f(Y)\| \\
&\leq |||M|||\|X(t) - Y(t)\| + \frac{\lambda_1}{N}|S||E_1 - E_1'| + \frac{\lambda_1}{N}|E_1'||S - S'| \\
&+ \frac{\lambda_2}{N}|S||I - I'| + \frac{\lambda_2}{N}|I'||S - S'| \\
&\leq \sup\{\lambda_3, \alpha_1 + \alpha_2, \beta_1 + \beta_2\} \|X - Y\| \\
&+ \frac{\lambda_1 + \lambda_2}{N} sup\left\{|S|, |E_1'|, |I'|\right\} \|X - Y\| \\
&\leq \max\{\lambda_3, \alpha_1 + \alpha_2, \beta_1 + \beta_2\} \|X - Y\| \\
&+ \frac{\lambda_1 + \lambda_2}{N} \max\left\{|S|, |E_1'|, |I'|\right\} \|X - Y\|.
\end{aligned}$$

Let

$$L = max\left\{\lambda_3, \alpha_1+\alpha_2, \beta_1+\beta_2\right\} + \frac{\lambda_1+\lambda_2}{N}\max\left\{|S|, |E_1'|, |I'|\right\}.$$

It is clear that $L > 0$. Then

$$\|F(X) - F(Y)\| \leq L\|X(t) - Y(t)\|.$$

By applying Lemma (2), and since $F(X(t))$ is continuous and satisfying Lipschitz condition, the initial value problem (2) has a unique solution. □

4 The stability result of the equilibrium point

To determine the equilibrium points of the system (2), we set the Atangana-Baleanu arbitrarily ordered derivatives to zero, i.e.,

$${}^{ABD}D^{\gamma}S = {}^{ABD}D^{\gamma}E_1 = {}^{ABD}D^{\gamma}I = {}^{ABD}D^{\gamma}E_2 = {}^{ABD}D^{\gamma}R = {}^{ABD}D^{\gamma}D = {}^{ABD}D^{\gamma}Q = 0,$$

From this, we can deduce that the equilibrium point is given by

$$E^* = (0,0,0,0,0,0,0)^T.$$

Now, we proceed to evaluate the equilibrium point for the system (2). The Jacobian matrix of the system (2) is determined as follows:

$$J = \begin{pmatrix} -\frac{\lambda_1}{N}E_1 - \frac{\lambda_2}{N}I - \lambda_3 & -\frac{\lambda_1}{N}S & -\frac{\lambda_2}{N}S & 0 & 0 & 0 & 0 \\ \frac{\lambda_1}{N}E_1 + \frac{\lambda_2}{N}I & \frac{\lambda_1}{N}S - (\alpha_1+\alpha_2) & \frac{\lambda_2}{N}S & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -(\beta_1+\beta_2) & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Substituting by the equilibrium point E^* in the above Jacobian matrix, we get :

$$J(E^*) = \begin{pmatrix} -\lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\alpha_1+\alpha_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -(\beta_1+\beta_2) & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The Jacobian matrix obtained from the system (2) has eigenvalues equal to zero. Therefore, the equilibrium point E^* is determined to be unstable.

5 Numerical results and discussions

Upon applying the fractional integral operator ${}^{ABI}I$ to equation (2) and utilizing the property (1), we obtain the following numerical results for our fractional model with the ABD derivative:

$$
\begin{aligned}
S(t) &= S(0) + \frac{1-\gamma}{B(\gamma)}K_1(t,S) + \frac{\gamma}{B(\gamma)}\int_0^t (t-v)^{\gamma-1}K_1(v,S)dv,\\
E_1(t) &= E_1(0) + \frac{1-\gamma}{B(\gamma)}K_2(t,E_1) + \frac{\gamma}{B(\gamma)}\int_0^t (t-v)^{\gamma-1}K_2(v,E_1)dv,\\
I(t) &= I(0) + \frac{1-\gamma}{B(\gamma)}K_3(t,I) + \frac{\gamma}{B(\gamma)}\int_0^t (t-v)^{\gamma-1}K_3(v,I)dv,\\
E_2(t) &= E_2(0) + \frac{1-\gamma}{B(\gamma)}K_4(t,E_2) + \frac{\gamma}{B(\gamma)}\int_0^t (t-v)^{\gamma-1}K_4(v,E_2)dv,\\
R(t) &= R(0) + \frac{1-\gamma}{B(\gamma)}K_5(t,R) + \frac{\gamma}{B(\gamma)}\int_0^t (t-v)^{\gamma-1}K_5(v,R)dv,\\
D(t) &= D(0) + \frac{1-\gamma}{B(\gamma)}K_6(t,D) + \frac{\gamma}{B(\gamma)}\int_0^t (t-v)^{\gamma-1}K_6(v,D)dv,\\
Q(t) &= Q(0) + \frac{1-\gamma}{B(\gamma)}K_7(t,Q) + \frac{\gamma}{B(\gamma)}\int_0^t (t-v)^{\gamma-1}K_7(v,Q)dv.
\end{aligned}
$$

The kernel functions for the fractional epidemic model (2) are defined as follows:

$$
\begin{aligned}
K_1(t,S) &= -\frac{\lambda_1}{N}SE_1 - \frac{\lambda_2}{N}SI - \lambda_3 S,\\
K_2(t,E_1) &= \frac{\lambda_1}{N}SE_1 + \frac{\lambda_2}{N}SI - (\alpha_1+\alpha_2)E_1,\\
K_3(t,I) &= \alpha_1 E_1 - (\beta_1+\beta_2)I,\\
K_4(t,E_2) &= \alpha_2 E_1,\\
K_5(t,R) &= \beta_1 I,\\
K_6(t,D) &= \beta_2 I,\\
K_1(t,Q) &= \lambda_3 S.
\end{aligned}
$$

To solve the fractional epidemic model (2), we employ the numerical procedure outlined in

reference [3]. The model can be represented as follows:

$$
\begin{aligned}
S(t_{n+1}) &= S(t_0) + \frac{1-\gamma}{B(\gamma)}K_1(t_n, S) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t_{n+1}-v)^{\gamma-1}K_1(v, S)dv,\\
E_1(t_{n+1}) &= E_1(t_0) + \frac{1-\gamma}{B(\gamma)}K_2(t_n, E_1) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t_{n+1}-v)^{\gamma-1}K_2(v, E_1)dv,\\
I(t_{n+1}) &= I(t_0) + \frac{1-\gamma}{B(\gamma)}K_3(t_n, I) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t_{n+1}-v)^{\gamma-1}K_3(v, I)dv,\\
E_2(t_{n+1}) &= E_2(t_0) + \frac{1-\gamma}{B(\gamma)}K_4(t_n, E_2) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t_{n+1}-v)^{\gamma-1}K_4(v, E_2)dv,\\
R(t_{n+1}) &= R(t_0) + \frac{1-\gamma}{B(\gamma)}K_5(t_n, R) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t_{n+1}-v)^{\gamma-1}K_5(v, R)dv,\\
D(t_{n+1}) &= D(t_0) + \frac{1-\gamma}{B(\gamma)}K_6(t_n, D) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t_{n+1}-v)^{\gamma-1}K_6(v, D)dv,\\
Q(t_{n+1}) &= Q(t_0) + \frac{1-\gamma}{B(\gamma)}K_7(t_n, Q) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t_{n+1}-v)^{\gamma-1}K_7(v, Q)dv.
\end{aligned}
$$

Discretizing the above equation yields the following system:

$$
\begin{aligned}
S(t_{n+1}) &= S(t_0) + \frac{1-\gamma}{B(\gamma)}K_1(t_n, S) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} (t_{n+1}-v)^{\gamma-1}K_1(v, S)dv,\\
E_1(t_{n+1}) &= E_1(t_0) + \frac{1-\gamma}{B(\gamma)}K_2(t_n, E_1) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} (t_{n+1}-v)^{\gamma-1}K_2(v, E_1)dv,\\
I(t_{n+1}) &= I(t_0) + \frac{1-\gamma}{B(\gamma)}K_3(t_n, I) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} (t_{n+1}-v)^{\gamma-1}K_3(v, I)dv,\\
E_2(t_{n+1}) &= E_2(t_0) + \frac{1-\gamma}{B(\gamma)}K_4(t_n, E_2) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} (t_{n+1}-v)^{\gamma-1}K_4(v, E_2)dv,\\
R(t_{n+1}) &= R(t_0) + \frac{1-\gamma}{B(\gamma)}K_5(t_n, R) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} (t_{n+1}-v)^{\gamma-1}K_5(v, R)dv,\\
D(t_{n+1}) &= D(t_0) + \frac{1-\gamma}{B(\gamma)}K_6(t_n, D) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} (t_{n+1}-v)^{\gamma-1}K_6(v, D)dv,\\
Q(t_{n+1}) &= Q(t_0) + \frac{1-\gamma}{B(\gamma)}K_7(t_n, Q) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} (t_{n+1}-v)^{\gamma-1}K_7(v, Q)dv.
\end{aligned}
$$

By utilizing the general formula for Adams' method [16, 2], we derive the subsequent numerical

approach for the Corona virus model 2:

$$S(t_{n+1}) = S(t_0) + \frac{1-\gamma}{B(\gamma)}K_1(t_n, S) + \frac{\gamma h^\gamma}{B(\gamma)\Gamma(\gamma+2)}\sum_{j=0}^{n}\sigma_{j,n+1}K_1(t_j, S),$$

$$E_1(t_{n+1}) = E_1(t_0) + \frac{1-\gamma}{B(\gamma)}K_2(t_n, E_1) + \frac{\gamma h^\gamma}{B(\gamma)\Gamma(\gamma+2)}\sum_{j=0}^{n}\sigma_{j,n+1}K_2(t_j, E_1),$$

$$I(t_{n+1}) = I(t_0) + \frac{1-\gamma}{B(\gamma)}K_3(t_n, I) + \frac{\gamma h^\gamma}{B(\gamma)\Gamma(\gamma+2)}\sum_{j=0}^{n}\sigma_{j,n+1}K_3(t_j, I),$$

$$E_2(t_{n+1}) = E_2(t_0) + \frac{1-\gamma}{B(\gamma)}K_4(t_n, E_2) + \frac{\gamma h^\gamma}{B(\gamma)\Gamma(\gamma+2)}\sum_{j=0}^{n}\sigma_{j,n+1}K_4(t_j, E_2),$$

$$R(t_{n+1}) = R(t_0) + \frac{1-\gamma}{B(\gamma)}K_5(t_n, R) + \frac{\gamma h^\gamma}{B(\gamma)\Gamma(\gamma+2)}\sum_{j=0}^{n}\sigma_{j,n+1}K_5(t_j, R),$$

$$D(t_{n+1}) = D(t_0) + \frac{1-\gamma}{B(\gamma)}K_6(t_n, D) + \frac{\gamma h^\gamma}{B(\gamma)\Gamma(\gamma+2)}\sum_{j=0}^{n}\sigma_{j,n+1}K_6(t_j, D),$$

$$Q(t_{n+1}) = Q(t_0) + \frac{1-\gamma}{B(\gamma)}K_1(t_n, Q) + \frac{\gamma h^\gamma}{B(\gamma)\Gamma(\gamma+2)}\sum_{j=0}^{n}\sigma_{j,n+1}K_7(t_j, Q),$$

where

$$\sigma_{j,n+1} = \begin{cases} (n^{\gamma+1} - (n-\gamma)(n+1)^\gamma), & \text{if } k = 0, \\ ((n-k+2)^{\gamma+1} + (n-k)^{\gamma+1} - 2(n-k+1)^{\gamma+1}), & \text{if } 1 \le k \le n, \\ 1, & \text{if } k = n+1. \end{cases}$$

5.1 Data fitting and numerical simulations

We present a set of numerical findings. Firstly, we draw upon data from the National Office of Statistics (NOS) in Algeria to provide key demographic insights for the year 2019. The total population during that period stood at 43,400,000, and the average life expectancy was 76.79 years. Additionally, based on the available information, the estimated mortality rate is $\mu = 0.0048$.

Subsequently, we analyze data collected in Algeria from March 24, 2020, to June 30, 2020, and deduce the following parameter values: $\lambda_1 = 0.8$, $\lambda_2 = 0.02$, $\lambda_3 = 0.155$, $\alpha_1 = 0.01$, $\alpha_2 = 0.2$, $\beta_1 = 0.023$, and $\beta_2 = 0.0041$.

For the simulation, we have chosen the following initial conditions: The total population at time $t = 0$ is $N(0) = 43,400,000$, which can be expressed as

$$N(0) = S(0) + E_1(0) + I(0).$$

To determine the number of susceptible individuals at $t = 0$, we use the formula

$$S(0) = N(0) - E_1(0) - I(0).$$

Moreover, we consider the initial conditions for the infected and exposed populations as follows: $I(0) = 70$ and $E_1(0) = 200$.

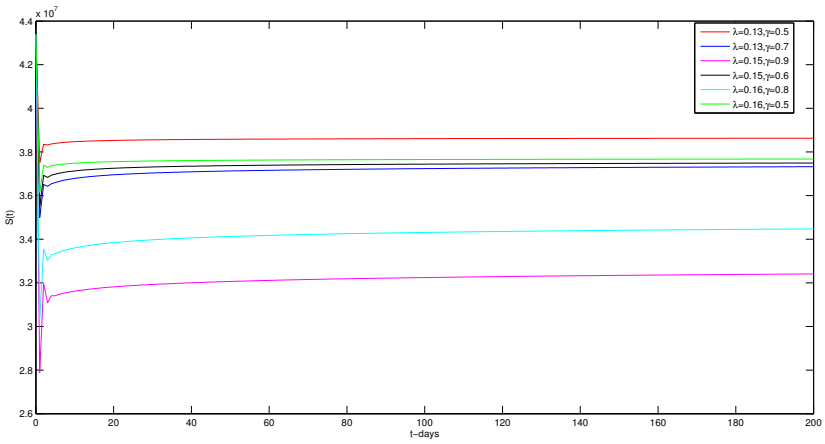

Figure 2: Susceptible population

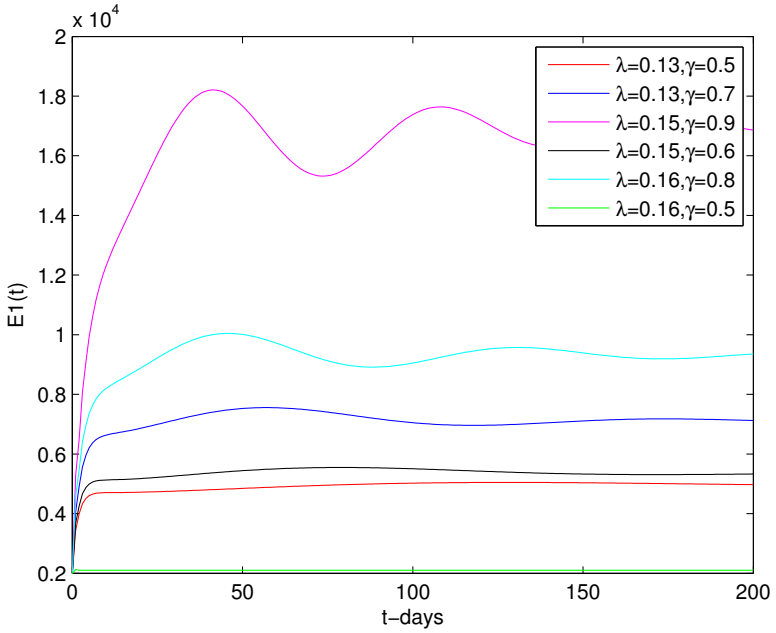

Figure 3: Exposed population

Figure 3 displays the plotted curves that represent the number of individuals who are exposed to the infection. A noteworthy observation from the graph is that an escalation in the number of people exposed to the infection directly corresponds to an increase in the number of individuals who subsequently become infected.

This observation highlights the crucial role of the exposed population in driving the spread of the infection. As more individuals are exposed to the virus, there is a higher likelihood of transmission to susceptible individuals, leading to a rise in the number of infections. Understanding and monitoring the dynamics of the exposed population are crucial for assessing the potential risk and impact of an infectious disease outbreak. In Figure 4, the plotted curves illustrate the evolution of the number of infected individuals over time. It is evident from the graph that the number of infected people

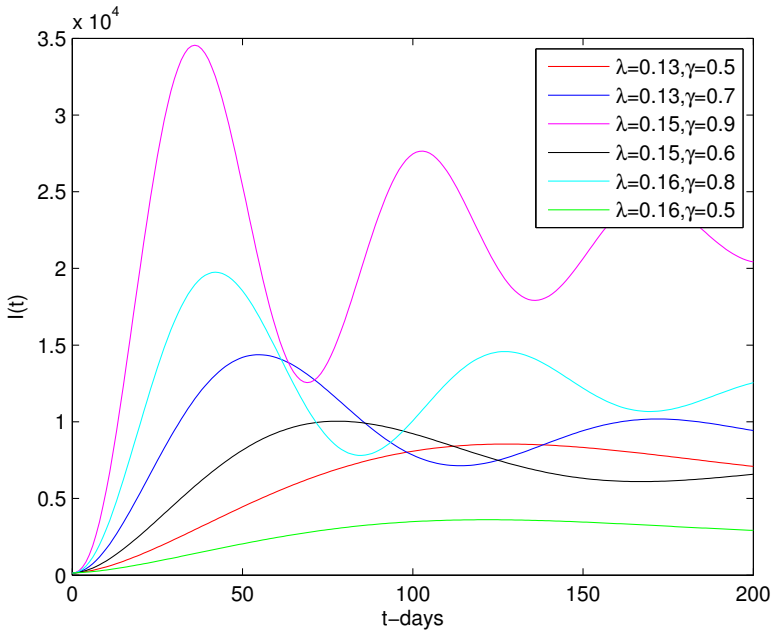

Figure 4: Infected population

reached its peak approximately 50 days ago and has since started to decrease.

The dynamics of infection can be influenced by altering the values of two key parameters: the fractional order γ and the quarantine rate λ_3. By varying these parameters, different patterns in the infection dynamics can be observed. Adjusting γ affects the rate at which infected individuals recover, while modifying λ_3 influences the effectiveness of the quarantine measures in limiting the spread of the infection.

Understanding the impact of these parameters is essential for devising effective strategies to control and manage the spread of the infection. The simulation results provide valuable insights into the behavior of the infected population and the potential outcomes under different parameter settings.

In Figure 7, the graph showcases the evolution of the number of deceased individuals over time. The number of deaths is significantly influenced by changes in the number of infected individuals, as expected. As the infection spreads and the number of infected individuals rises, the number of deaths also increases due to the severity of the disease.

Moreover, the values assigned to the two parameters, the fractional order γ and the quarantine rate λ_3, have a notable impact on the number of deceased individuals. Altering these parameters can lead to different outcomes in terms of the mortality rate and the overall impact of the infection on the population.

Understanding the relationship between the number of infected individuals and the resulting number of deaths, as well as the role of parameter values, is crucial for predicting and mitigating the consequences of an infectious disease outbreak. This information can aid policymakers and public health officials in making informed decisions to implement effective measures for reducing the impact of the disease on the population.

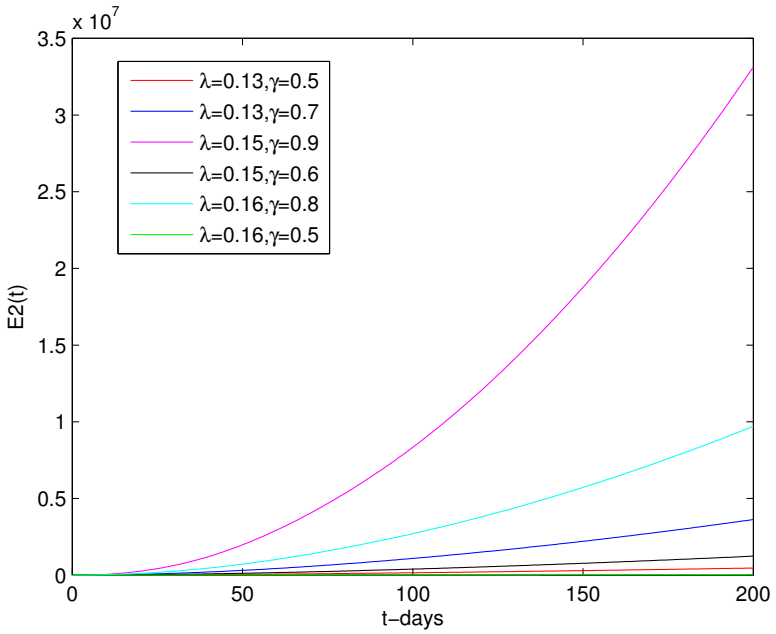

Figure 5: Exposed-Recovered population

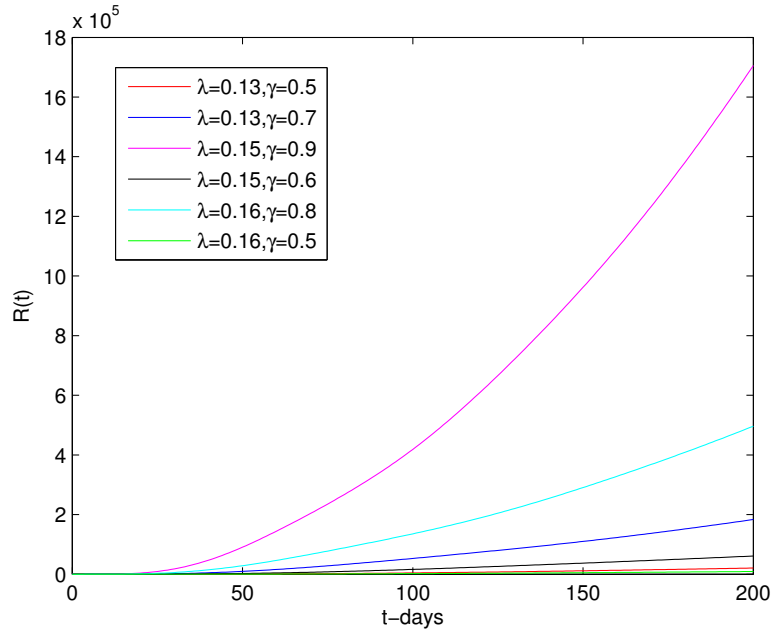

Figure 6: Recovered population

6 Conclusions

This paper presented a fractional order model of Covid-19 using the Atangana-Baleanu's fractional derivative. The mathematical analysis of the model demonstrated the existence of bounded and nonnegative solutions. Furthermore, a numerical scheme was developed to solve the fractional model of Covid-2019, allowing for variations in the fractional order and quarantine rate. The obtained

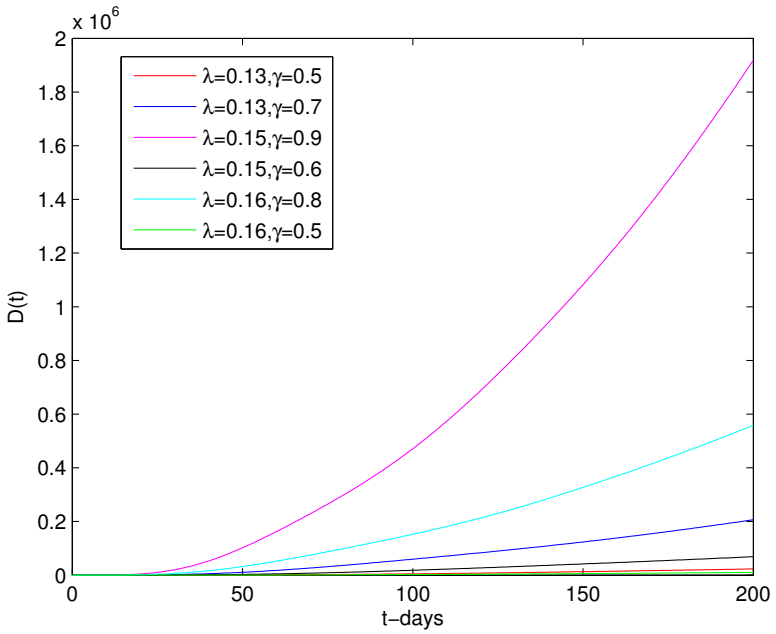

Figure 7: Death population

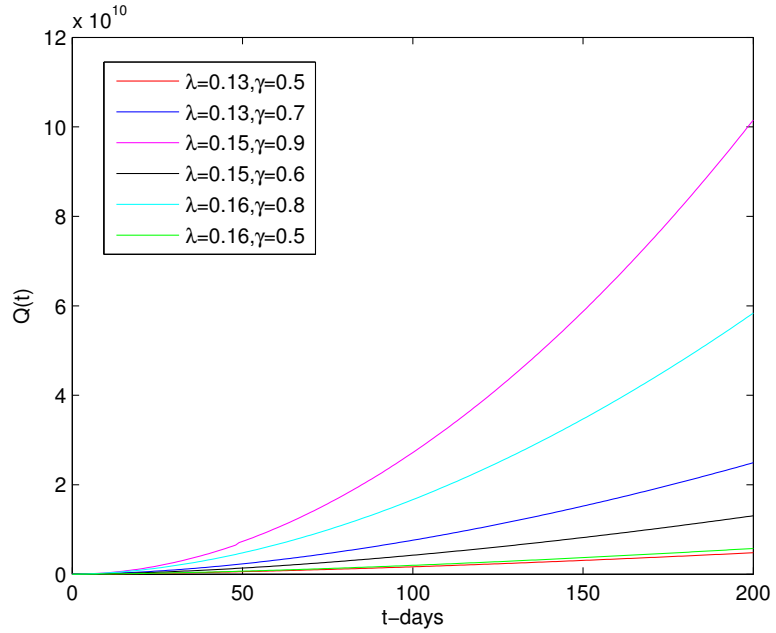

Figure 8: Quarantine Population

graphical results clearly indicated that decreasing the fractional order and quarantine rate leads to a decrease in the infection population. These findings contribute to a better understanding of the dynamics of the Covid-19 pandemic and can aid in the development of effective control measures.

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