Research Article

Revisiting the Origin of Neutrino Flavour Transformations

Shi-Biao Zheng[1]

1. Department of Physics, Fuzhou University, China

To account for neutrino oscillations, it was postulated that the neutrino has nonvanishing mass and each flavor eigenstate is formed by a quantum superposition of three distinct mass eigenstates, whose probability amplitudes interfere with each other during its propagation. However, I find that a neutrino or antineutrino produced by the decay of an unstable particle cannot be in such a superposition, as different mass eigenstates, if they exist, are necessarily correlated with different momentum states of the composite system produced by the decay, which would destroy the quantum coherence among these mass eigenstates. I further find that the states of a neutrino and an electron become nonseparable after their charged-current interaction. This nonseparability leads to decoherence for neutrinos propagating in matter, but was not taken into consideration in previous investigations of the matter effect. Due to this decoherence, the deficit of solar electron neutrinos cannot exceed 1/2 based on the aforementioned postulation even if the corresponding superposition of mass eigenstates can be produced. These results unambiguously show that the origin of neutrino flavor transformations needs to be revisited. I propose an alternative mechanism that can reasonably account for neutrino transformations. It is based on virtual excitation of the Z bosonic field diffusing over the space. During the propagation, the neutrino can continually excite and then immediately re-absorb a virtual Z boson. This virtual bosonic excitation produces a backaction on the neutrino, enabling it to oscillate among three flavors. When the neutrino propagates in matter, its behavior is determined by the competition between the coherent flavor transformation and decoherence effect resulting from scatterings.

Corresponding author: Shi-Biao Zheng, t96034@fzu.edu.cn

 doi.org/10.32388/DA9B0G

I. Introduction

The discovery of neutrino flavor transformations represents one of the most striking advancements of physical science achieved during the past few decades[1][2][3][4][5][6][7][8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24][25][26][27]. Such phenomena are commonly believed to show that neutrinos have nonvanishing mass, which is beyond the standard model[28][29]. To account for these phenomena, it is assumed that there exist three distinct neutrino mass eigenstates. Each flavor eigenstate corresponds to a quantum-mechanical superposition of these mass eigenstates, and vice versa[30]. During the propagation of the neutrino, the probability amplitude associated with each mass eigenstate accumulates a phase that is approximately proportional to the square of the corresponding mass. The time-evolving phase differences among these probability amplitudes give arise to flavor oscillations, as a consequence of their quantum interference.

This interpretation is valid only when the neutrino can be in the superposition state at the production. Previously, it was realized that the neutrino emitted by an unstable particle with a definite momentum is necessarily entangled with the particles accompanying the neutrino at the production[31][32][33]. This entanglement would destroy the coherence between the neutrino's mass eigenstates. To overcome this problem, it was argued that the neutrino can be disentangled with the accompanying particles when the momentum uncertainty of the unstable particle is sufficiently large[31][32][33]. However, I find that different mass eigenstates of such a neutrino, if they exist, are necessarily correlated with different joint momentum states of the entire system produced by the decay, including the neutrino and the accompanying particles, which prohibits occurrence of interference effects between the mass eigenstates. I further find that the previous interpretation of the matter effect has overlooked the critical fact that the states of the neutrino and electron become nonseparable after their charged-current (CC) interaction[34][35][36]. Due to this nonseparability, the solar ^{8}B neutrino cannot adiabatically evolve from a quantum superposition of different mass eigenstates to a pure mass eigenstate, which was assumed to be responsible for the observed 2/3 deficit. I finally propose an alternative mechanism, based on the virtual excitation of the Z bosonic field, to interpret neutrino flavor transformations observed in different experiments.

doi.org/10.32388/DA9B0G

II. No-go theorem regarding production of superpositions of mass eigenstates

This section is devoted to the proof of the no-go theorem that the neutrino or antineutrino emitted by an unstable particle cannot be in a quantum-mechanical superposition of different mass eigenstates. For the sake of clarity, I will illustrate this theorem with the β decay of a neutron, but the conclusion holds for other weak CC decays.

Lemma 1: For the β decay, when there is no mass-momentum entanglement, the produced electron antineutrino has a definite mass.

The wavefunction of the neutron, before undergoing the β decay, can be expanded as

$$|\psi_n\rangle = \int \varphi(\mathbf{P}_n) d^3\mathbf{P}_n |\mathbf{P}_n\rangle, \tag{1}$$

where $|\mathbf{P}_n\rangle$ denotes the momentum eigenstate of the neutron with the eigenvalue $\mathbf{P}_n$. Suppose that the electron antineutrino produced by the β decay possesses three different mass eigenstates, which are not entangled with different momentum eigenstates of the antineutrino, proton, and electron. Then the state of the composite system can be written as

$$|\psi_{\nu+p+e}\rangle = \int d^3\mathbf{P}_\nu d^3\mathbf{P}_p d^3\mathbf{P}_e F(\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e) |\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e\rangle \left(\sum_{j=1}^{3} C_j |\bar{\nu}_j\rangle \right), \tag{2}$$

where $|\bar{\nu}_j\rangle$ denotes the jth eigenstate for the electron antineutrino. $|\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e\rangle$ represents joint momentum eigenstate of the entire system, where the antineutrino, proton, and electron are all in their momentum eigenstates with the eigenvalues $\mathbf{P}_\nu$, $\mathbf{P}_p$, and $\mathbf{P}_e$ respectively. The joint probability amplitude distribution $F(\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e)$ satisfies the normalization condition

$$\int d^3\mathbf{P}_\nu d^3\mathbf{P}_p d^3\mathbf{P}_e |F(\mathbf{P}_\nu, \mathbf{P}_p, \mathbf{P}_e)|^2 = 1. \tag{3}$$

The joint momentum of the entire antineutrino-proton-electron system is essentially in a superposition of infinitely many components, which implies that both the total momentum and energy are undeterministic. Despite these uncertainties, the energy and momentum conservation laws are still satisfied for each momentum component of the wave function, as correctly pointed out in Ref. [31].

doi.org/10.32388/DA9B0G

We here consider a specific component, denoted as $|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\rangle$. The momentum conservation law implies that this momentum component originates from the neutron momentum component $|\mathbf{P}_n^0\rangle$, with

$$\mathbf{P}_n^0 = \mathbf{P}_\nu^0 + \mathbf{P}_p^0 + \mathbf{P}_e^0. \tag{4}$$

The energies of the neutron, proton, and electron associated with the component $|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\rangle$ are given by

$$\begin{aligned} E_n^0 &= \sqrt{m_n^2 + (P_n^0)^2}, \\ E_p^0 &= \sqrt{m_p^2 + (P_p^0)^2}, \\ E_e^0 &= \sqrt{m_e^2 + (P_e^0)^2}, \end{aligned} \tag{5}$$

where m_n, m_p, and m_e are the masses of the neutron, proton, and electron, respectively. According to the energy conservation law, the antineutrino's energy associated with the component $|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\rangle$ is definite, given by $E_\nu^0 = E_n^0 - E_p^0 - E_e^0$. Consequently, the antineutrino's mass is also definite, which is equal to $m_\nu = \sqrt{(E_\nu^0)^2 - (P_\nu^0)^2}$. This leads to $m_j = m_\nu$ when $C_j \neq 0$. Such a conclusion is inconsistent with the postulation that each flavor eigenstate is a linear superposition of three different mass eigenstates. If the flavor oscillations are caused by nonzero mass differences, this state cannot exhibit any oscillatory behavior.

Lemma 2: For the β decay, different mass eigenstates of the produced electron antineutrino are necessarily correlated with different joint antineutrino-proton-electron momentum states that are orthogonal to each other.

Generally, the wavefunction of the entire system produced by the β decay can be written in the form of

$$|\psi\rangle = \sum_j \int_{\sigma_j} d^3\mathbf{P}_{\nu,j} d^3\mathbf{P}_{p,j} d^3\mathbf{P}_{e,j} G(\mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j}) |\mathbf{P}_{\nu,j}, \mathbf{P}_{p,j}, \mathbf{P}_{e,j}\rangle |\bar{\nu}_j\rangle, \tag{6}$$

where σ_j denotes the distribution region of the joint antineutrino-proton-electron momentum associated with $|\bar{\nu}_j\rangle$. In order to satisfy the condition $m_1 \neq m_2 \neq m_3$, there should not be any overlapping between the momentum distribution regions associated with different mass eigenstates, i.e., $\sigma_j \cap \sigma_k = \oslash$ for $j \neq k$. This can be interpreted as follows. Suppose that there is an overlapping between the regions σ_j and σ_k with $j \neq k$. Then, according to the aforementioned analysis, both m_j and m_k can be uniquely determined by a specific joint momentum component $|\mathbf{P}_\nu^0, \mathbf{P}_p^0, \mathbf{P}_e^0\rangle$ that falls within the overlapping regime. This implies $m_j = m_k$ when $\sigma_j \cap \sigma_k \neq \oslash$.

doi.org/10.32388/DA9B0G

For such an entangled state, when the momentum states are traced out, the mass degree of freedom is left in a classical mixture, described by the density operator

$$\begin{aligned}\rho_\nu &= \mathrm{Tr}_{\mathbf{P}_\nu,\mathbf{P}_n,\mathbf{P}_e}|\psi\rangle\langle\psi| \\ &= \int d^3\mathbf{P}_\nu d^3\mathbf{P}_p d^3\mathbf{P}_e \langle\mathbf{P}_\nu,\mathbf{P}_p,\mathbf{P}_e|\psi\rangle\langle\psi|\mathbf{P}_\nu,\mathbf{P}_p,\mathbf{P}_e\rangle \\ &= \sum_{j,k} D_{j,k}|\bar{\nu}_j\rangle\langle\bar{\nu}_k|,\end{aligned} \tag{7}$$

with

$$\begin{aligned}D_{j,k} = \int_{\sigma_j} & d^3\mathbf{P}_{\nu,j}d^3\mathbf{P}_{p,j}d^3\mathbf{P}_{e,j}\int_{\sigma_k} d^3\mathbf{P}_{\nu,k}d^3\mathbf{P}_{p,k}d^3\mathbf{P}_{e,k} \\ & G(\mathbf{P}_{\nu,j},\mathbf{P}_{p,j},\mathbf{P}_{e,j})G^*(\mathbf{P}_{\nu,k},\mathbf{P}_{p,k},\mathbf{P}_{e,k}) \\ & \langle\mathbf{P}_{\nu,k},\mathbf{P}_{p,k},\mathbf{P}_{e,k}|\mathbf{P}_{\nu,j},\mathbf{P}_{p,j},\mathbf{P}_{e,j}\rangle.\end{aligned} \tag{8}$$

Since $\sigma_j \cap \sigma_k = \oslash$ for $j \neq k$, each of the joint momentum eigenstates in the region σ_j is orthogonal to all the momentum eigenstates in σ_k. This implies $\langle\mathbf{P}_{\nu,k},\mathbf{P}_{p,k},\mathbf{P}_{e,k}|\mathbf{P}_{\nu,j},\mathbf{P}_{p,j},\mathbf{P}_{e,j}\rangle = 0$ throughout the integral region $\sigma_j \otimes \sigma_k$ for $j \neq k$. Therefore, we have

$$\rho_\nu = \sum_j D_{j,j}|\bar{\nu}_j\rangle\langle\bar{\nu}_j|, \tag{9}$$

where

$$D_{j,j} = \int_{\sigma_j} d^3\mathbf{P}_{\nu,j}d^3\mathbf{P}_{p,j}d^3\mathbf{P}_{e,j}|G(\mathbf{P}_{\nu,j},\mathbf{P}_{p,j},\mathbf{P}_{e,j})|^2. \tag{10}$$

In other words, the quantum coherence among the mass eigenstates is destroyed by their quantum entanglement with different joint momentum states.

The aforementioned proof indicates that when the antineutrino is not entangled with its accompanying particles, its mass eigenstates, if they exist, would be necessarily correlated with different momenta of the antineutrino itself. This mass-momentum correlation would destroy the quantum coherence between the neutrino's mass eigenstates, which was overlooked by other authors in previous investigations of entanglement and coherence associated with neutrino oscillations[31][32][33]. This decoherece effect can be further interpreted in terms of complementarity[37][38][39][40][41][42][43][44][45]. The information about the mass eigenstate of the antineutrino is encoded in its momentum. Consequently, one can determine which eigenstate the antineutrino is in by measuring its momentum in principle, which is sufficient to destroy the coherence among the mass eigenstates. Such a decoherence effect does not depend upon whether or not the momentum is actually measured.

This decoherece effect can also be illustrated in the position representation. When the antineutrino is not entangled with other particles, its wavefunction in this representation evolves as

$$|\varphi_\nu(t)\rangle = \sum_j \int d^3\mathbf{r} f_j(\mathbf{r},t)|\mathbf{r}\rangle|\bar{\nu}_j\rangle, \quad (11)$$

where

$$f_j(\mathbf{r},t) = (2\pi)^{-3/2}\int_{\sigma_j} d^3\mathbf{P}_{\nu,j} G(\mathbf{P}_{\nu,j}) e^{i(\mathbf{P}_{\nu,j}\cdot\mathbf{r}-E_j t)}, \quad (12)$$

$E_j = \sqrt{p_j^2+m_j^2}$ with m_j being the mass of the jth mass eigenstate, and $|\mathbf{r}\rangle$ denotes the position eigenstate. The coherence between $|\bar{\nu}_j\rangle$ and $|\bar{\nu}_k\rangle$ manifested on the detection of the antineutrino is given by

$$\begin{aligned} C_{j,k} =& \int_D d^3\mathbf{r} f_j(\mathbf{r},t) f_k^*(\mathbf{r},t) \\ =& (2\pi)^{-3}\int_{\sigma_j} d^3\mathbf{P}_{\nu,j}\int_{\sigma_k} d^3\mathbf{P}_{\nu,k} G(\mathbf{P}_{\nu,j}) G^*(\mathbf{P}_{\nu,k})\int_D d^3\mathbf{r} e^{i[(\mathbf{P}_{\nu,j}-\mathbf{P}_{\nu,k})\cdot\mathbf{r}-(E_k-E_j)t]}, \end{aligned} \quad (13)$$

where D is the detection region of the antineutrino. When the size of the detector is much larger than that of the antineutrino's wavepacket, $\int_D d^3\mathbf{r} e^{i(\mathbf{P}_{\nu,j}-\mathbf{P}_{\nu,k})\cdot\mathbf{r}}$ can be well approximated by taking the integral over the whole space. As $\mathbf{P}_{\nu,j} \neq \mathbf{P}_{\nu,k}$, such an integral is zero, which implies that the coherence $C_{j,k}$ vanishes. This result can also be understood in terms of the position-dependent phase difference between $\left|\bar{\nu}_j\right\rangle$ and $\left|\bar{\nu}_k\right\rangle$ owing to the associated momentum difference. We note that such phase differences were also included in previous investigations, exemplified by the statement "*Since* $p_{x,i} \neq p_{x,j}$*, phase differences exist between the components at the point of detection*" in Ref. [32]. However, the detection of neutrinos or antineutrinos cannot be restricted to a single point. The interference effects of mass eigenstates (internal degree of freedom) would be averaged out when integrating the position (external degree of freedom) over the large volume of the detector, as the position-averaged value of the phase factor caused by the corresponding momentum difference is zero.

III. Neutrino-Electron Mixture Produced by the Charged-Current Interaction

The no-go theorem regarding production of superpositions of mass eigenstates is applicable to neutrinos or antineutrinos produced in the weak charged-current decay of other unstable particles, including mesons and muons. It also holds solar ^{8}B neutrinos, which are produced by the reaction [2]

doi.org/10.32388/DA9B0G

$$^{8}\mathrm{B} \rightarrow^{8} \mathrm{Be} + e^{+} + \nu_e. \tag{14}$$

I further note that even if solar ^{8}B neutrinos can be initially in a superposition of mass eigenstates, they cannot adiabatically evolve into a pure mass eigenstate, where the population of the electron flavor eigenstate was assumed to be about 1/3 [2], as will be detailedly interpreted below.

Previously, the flavor transformation of solar ^{8}B neutrinos was attributed to the matter effect proposed by Mikheyev and Smirnov by extending the idea of Wolfenstein, referred to as the MSW effect [34][35][36]. This effect originates from the CC interaction between the electron neutrino and the background electrons in matter, which can be described by the effective Hamiltonian,

$$H_{cc} = \frac{G_F}{\sqrt{2}} \nu_e^{+} \gamma_4 \gamma_\lambda (1+\gamma_5) \mathbf{e} \mathbf{e}^{\dagger} \gamma_4 \gamma_\lambda (1+\gamma_5) \nu_e. \tag{15}$$

where ν_e and $\mathbf{e}$ denote the fields associated with the electron neutrino and electron, respectively. Using the Fierz transformation, the Hamiltonian was rewritten in the form of

$$H_{cc}' = \frac{G_F}{\sqrt{2}} \nu_e^{+} \gamma_4 \gamma_\lambda (1+\gamma_5) \nu_e \mathbf{e}^{\dagger} \gamma_4 \gamma_\lambda (1+\gamma_5) \mathbf{e}. \tag{16}$$

Then the electron field was considered as a static background, whose state is not affected by the CC interaction, so that $\mathbf{e}^{\dagger} \gamma_4 \gamma_\lambda (1+\gamma_5) \mathbf{e}$ can be replaced with $\delta_{\lambda,4} N_e$, where N_e is the number density of electrons. With this treatment, the Hamiltonian H_{cc}' is effectively equivalent to an external potential for the neutrino, given by $V = \sqrt{2} N_e G_F$.

To derive the MSW effect, it was further supposed that each neutrino flavor eigenstate is formed by a linear superposition of three mass eigenstates

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U_{\alpha j} |\nu_j\rangle, \tag{17}$$

where j labels the mass eigenstate, and $\alpha = e, \mu, \tau$ denotes the flavor of the neutrino. $|\nu_e\rangle$ was assumed to be approximated by a superposition of $|\nu_1\rangle$ and $|\nu_2\rangle$, i.e., $U_{e3} \simeq 0$ [2]. Under this assumption, neither H_{cc}' nor the free Hamiltonian can couple $|\nu_e\rangle$ to $|\nu_3\rangle$, and thus the population of $|\nu_3\rangle$ can be neglected for the initial state $|\nu_e\rangle$. Then the dynamics can be described in a two-dimensional subspace $\{|\nu_e\rangle, |\nu_\beta\rangle\}$, where

$$|\nu_\beta\rangle = \mathcal{N}_\beta (U_{\tau 3} |\nu_\mu\rangle - U_{\mu 3} |\nu_\tau\rangle), \tag{18}$$

with $\mathcal{N}_\beta = \left(|U_{\tau 3}|^2 + |U_{\mu 3}|^2\right)^{-1/2}$. Within this subspace, the Hamiltonian can be approximately expressed as

 doi.org/10.32388/DA9B0G

$$H \simeq V \left|\nu_e\right\rangle \left\langle\nu_e\right| + \sum_{\eta,\xi=e,\beta} M_{\eta,\xi}(p) \left|\nu_\eta\right\rangle \left\langle\nu_\xi\right| \tag{19}$$

where

$$M_{\eta,\xi}(p) \simeq \sum_{j=1,2} \frac{m_j^2}{2p} U_{\eta j}^* U_{\xi j} \tag{20}$$

with $U_{\beta j} = \mathcal{N}_\beta (U_{\tau 3} U_{\mu j} - U_{\mu 3} U_{\tau j})$. Here p denotes the neutrino momentum, which is much larger than the mass (m_j) associated with each mass eigenstate. The trivial common energy, described by $p \sum_{j=1,2} \left|\nu_j\right\rangle \left\langle\nu_j\right|$, has been discarded. When $V \gg m_j^2/2p$, the electron flavor approximately coincides with the eigenstate of the Hamiltonian with the larger eigenenergy. If the electron number density is changed sufficiently slowly, the neutrino adiabatically follows the corresponding Hamiltonian eigenstate during its propagation. On the solar surface, V can be neglected as compared to $m_j^2/2p$ so that the eigenstates of the Hamiltonian coincide with the mass eigenstates. This implies that the initial electron flavor eigenstate evolves to the mass eigenstate with the larger mass ($\left|\nu_2\right\rangle$) when the neutrino reaches the solar surface. This mass eigenstate remains invariant until being detected on the Earth. The resulting probability $P_{\left|\nu_e\right\rangle \to \left|\nu_e\right\rangle}$ is approximately equal to $\left|U_{2e}^\dagger\right|^2$, which was assumed to be about $1/3$ [2].

This treatment has overlooked the crucial fact that the CC reaction leads to neutrino-electron entanglement when the neutrino is in a superposition of the electron flavor eigenstate and the other two flavor eigenstates before the reaction. As the neutrino and the electron can be transformed into each other by their CC reaction, it helps to make the presentation more clear to refer the original neutrino and the original electron to as particle 1 and particle 2, respectively. The CC reaction transforms the state $|\nu_e, \mathbf{p}_1\rangle_1 |e, \mathbf{p}_2\rangle_2$ into $|e, \mathbf{p}_3\rangle_1 |\nu_e, \mathbf{p}_4\rangle_2$, where the subscripts "1" and "2" outside the kets label the two particles, and $\mathbf{p}_1$ ($\mathbf{p}_3$) and $\mathbf{p}$ ($\mathbf{p}_4$) are their momenta before (after) the reaction. If particle 1 is initially in the flavor eigenstate $|\nu_e\rangle$ and $\mathbf{p}_1 = \mathbf{p}_4$, the Fierz rearranging is equivalent to relabeling the two particles, which does not cause any problem. However, when it is initially in a superposition of $|\nu_e\rangle$ and $|\nu_\beta\rangle$, it will be entangled with particle 2 by the CC reaction. To illustrate this point, we suppose that the two-particle system is initially in the state

$$|\psi_0\rangle = \left(C_e |\nu_e, \mathbf{p}_1\rangle_1 + C_\beta |\nu_\beta, \mathbf{p}_1\rangle_1\right) |e, \mathbf{p}_2\rangle_2. \tag{21}$$

In this case, the CC reaction actually corresponds to a conditional dynamics, by which particle 1 exchanges its state with particle 2 when it is initially in the electron flavor eigenstate, but nothing occurs if it is initially in the other two flavor eigenstates. This conditional state swapping evolves the system to the entangled state

$$|\psi\rangle = C_e|e,\mathbf{p}_3\rangle_1|\nu_e,\mathbf{p}_4\rangle_2 + C_\beta|\nu_\beta,\mathbf{p}_1\rangle_1|e,\mathbf{p}_2\rangle_2. \tag{22}$$

It should be noted that the electron transformed from the neutrino does not have the same momentum as the original electron, i.e., $\mathbf{p}_3 \neq \mathbf{p}_2$. Such momentum differences have been used to identify neutrino-electron scattering events in SNO experiments[2]. This quantum entanglement is masked by the Fierz rearranging and the subsequent replacement of the electron part in the Hamiltonian with a number. We further note that the Fierz rearranging is valid for calculation of the e -ν_e scattering amplitude, which is irrelevant to the quantum coherence between $|\nu_e\rangle$ and $|\nu_\beta\rangle$. However, it overlooks the fact that $|\nu_e\rangle$ and $|\nu_\beta\rangle$ are carried by different particles after the CC reaction, which is essential for correct description of the neutrino state evolution in matter. In other words, the states of the two particles are no longer separable after their CC interaction, so that the electrons participating in such interactions cannot be treated as a static background for the neutrino, and their effects cannot be modeled as a potential for the propagating neutrino.

Due to the quantum entanglement, each of the two particles is essentially in a mixture of the neutrino and electron states. This critical point can be illustrated more clearly by the reduced density operators for these particles, each obtained by tracing out the degree of freedom of the other particle, given by

$$\begin{aligned} \rho_1 &= \mathrm{Tr}_2(|\psi\rangle\langle\psi|) \\ &= |C_e|^2|e,\mathbf{p}_3\rangle_1\langle e,\mathbf{p}_3| + |C_\beta|^2|\nu_\beta,\mathbf{p}_1\rangle_1\langle\nu_\beta,\mathbf{p}_1|, \\ \rho_2 &= \mathrm{Tr}_1(|\psi\rangle\langle\psi|) \\ &= |C_e|^2|\nu_e,\mathbf{p}_4\rangle_2\langle\nu_e,\mathbf{p}_4| + |C_\beta|^2|e,\mathbf{p}_2\rangle_2\langle e,\mathbf{p}_2|. \end{aligned} \tag{23}$$

Under the subsequent free Hamiltonian dynamics, ρ_1 and ρ_2 evolve as

$$\begin{aligned} \rho_1' &= |C_e|^2|e,\mathbf{p}_3\rangle_1\langle e,\mathbf{p}_3| + |C_\beta|^2|\varphi_1,,\mathbf{p}_1\rangle_1\langle\varphi_1,\mathbf{p}_1|, \\ \rho_2' &= |C_e|^2|\varphi_2,\mathbf{p}_4\rangle_2\langle\varphi_2,\mathbf{p}_4| + |C_\beta|^2|e,\mathbf{p}_2\rangle_2\langle e,\mathbf{p}_2|, \end{aligned} \tag{24}$$

where

$$\begin{aligned} |\varphi_1\rangle_1 &= u_{p_1}|\nu_\beta\rangle_1 + v_{p_1}|\nu_e\rangle_1, \\ |\varphi_2\rangle_2 &= u_{p_4}^*|\nu_e\rangle - v_{p_4}^*|\nu_\beta\rangle. \end{aligned} \tag{25}$$

u_p and v_p depend on time as

doi.org/10.32388/DA9B0G

$$
\begin{aligned}
u_p &= \cos(\lambda_p t) - i\frac{\Delta_p}{\sqrt{\lambda_p^2+\Delta_p^2}}\sin(\lambda_p t),\\
v_p &= \frac{-i\lambda_p}{\sqrt{\lambda_p^2+\Delta_p^2}}e^{i\theta_p}\sin(\lambda_p t),
\end{aligned}
\tag{26}
$$

where $\Delta_p = [M_{e,e}(p) - M_{\beta,\beta}(p)]/2$, $\lambda_p = |M_{e,\beta}(p)|$, and $\theta_p = \arg[M_{e,\beta}(p)]$. After this free evolution, the total $|\nu_e\rangle$-state population is

$$
\begin{aligned}
P_{|\nu_e\rangle} &= \int d^3\mathbf{p}({}_1\langle\nu_e,\mathbf{p}|\,\rho_1'|\nu_e,\mathbf{p}\rangle_1 +{}_2\langle\nu_e,\mathbf{p}|\,\rho_2'|\nu_e,\mathbf{p}\rangle_2)\\
&= |C_e u_{p_4}|^2 + |C_\beta v_{p_1}|^2.
\end{aligned}
\tag{27}
$$

Such a probability does not present the cross terms proportional to $C_e^* C_\beta$ and $C_e C_\beta^*$. This is due to the fact that the state components $|\varphi_1\rangle_1$ and $|\varphi_2\rangle_2$ have different momenta and are carried by different particles, so that quantum interference cannot occur.

If one only concerns about the neutrino part in the two-particle system, its behavior just after the CC reaction can be effectively described by the classically mixed state

$$
\rho_\nu = |C_e|^2\,|\nu_e,\mathbf{p}_4\rangle\,\langle\nu_e,\mathbf{p}_4| + |C_\beta|^2\,|\nu_\beta,\mathbf{p}_1\rangle\,\langle\nu_\beta,,\mathbf{p}_1|\,. \tag{28}
$$

However, it should be born in mind that the two mixed state components are essentially carried by two different particles. The validity of this description can be illustrated by examining the subsequent free Hamiltonian dynamics, by which ρ_ν evolves to

$$
\rho_\nu' = |C_e|^2\,|\varphi_2,\mathbf{p}_4\rangle\,\langle\varphi_2,\mathbf{p}_4| + |C_\beta|^2\,|\varphi_1,\mathbf{p}_1\rangle\,\langle\varphi_1,\mathbf{p}_1|\,. \tag{29}
$$

The resulting neutrino's $|\nu_e\rangle$-state probability is also equal to $|C_e u_{p_4}|^2 + |C_\beta v_{p_1}|^2$. This equivalence further confirms that the CC reaction indeed destroys the quantum coherence between $|\nu_e\rangle$ and $|\nu_\beta\rangle$. When a second CC reaction occurs, these two particles will be further entangled with a third particle. Under the competition between the coherent coupling and CC-reaction-induced decoherence, the population of $|\nu_e\rangle$ is progressively decreased while that of $|\nu_\beta\rangle$ is increased until reaching the steady state

$$
|\nu_e\rangle\,\langle\nu_e| + |\nu_\beta\rangle\,\langle\nu_\beta|)/2. \tag{30}
$$

For simplicity, we here have discarded the momentum degrees of freedom. For this mixed state, the gain of the $|\nu_e\rangle$-state population originating from the $|\nu_\beta\rangle \to |\nu_e\rangle$ transition cancels out the loss due to the $|\nu_e\rangle \leftrightarrow |\nu_\beta\rangle$ transition. Therefore, the probability $P_{|\nu_e\rangle\to|\nu_e\rangle}$ should not be smaller than 1/2, which is inconsistent with the solar ^{8}B neutrino experiments[1][2].

doi.org/10.32388/DA9B0G

IV. Flavor Transformations Mediated by the Z Bosonic Field

In Ref.[34], Wolfenstein proposed a neutral-current interaction that can transform the flavor of a neutrino propagating in matter. Such an interaction can occur only when the mediating Z bosonic field can connect different flavor eigenstates of the neutrino. I here show that such a Z bosonic field can lead to flavor transformations even when the neutrino propagates in the vaccum. The dynamics of the system combined by the neutrino field and such a Z bosonic field is governed by the Lagrangian[28][29]

$$\mathcal{L}_{n+Z} = \mathcal{L}_f + \mathcal{L}_m + \mathcal{L}_I. \tag{31}$$

$\mathcal{L}_f$ is the free part without considering any coupling, which, to the second order, can be written as

$$\mathcal{L}_f = -\frac{1}{4}\left(\frac{\partial \mathbf{Z}_\lambda}{\partial x_\nu} - \frac{\partial \mathbf{Z}_\nu}{\partial x_\lambda}\right)^2 - \phi_\alpha^\dagger \gamma_4 \gamma_\lambda \frac{\partial}{\partial x_\lambda}\phi_\alpha, \tag{32}$$

where ϕ_α represents the field for the neutrino with flavor α ($\alpha = e, \mu, \tau$), and $\mathbf{Z}_\lambda$ denotes the Z bosonic field. $\mathcal{L}_m$ is the mass part, given by

$$\mathcal{L}_m = -\frac{1}{2}M_Z^2 \mathbf{Z}_\mu^2, \tag{33}$$

where M_Z is the mass of the Z boson, gained by coupling to the Higgs field. The interaction between the neutrino field and the Z bosonic field is described by

$$\mathcal{L}_I = i\eta \mathbf{Z}_\lambda U_{\alpha,\beta} \phi_\alpha^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5)\phi_\beta. \tag{34}$$

The deviation of $U_{\alpha,\beta}$ from $\delta_{\alpha,\beta}$ characterizes to what extent the flavor conservation is violated due to coupling to the Z bosonic field. The interactions associated with the W bosonic fields are irrelevant for the neutrino flavor transformation, and not shown here.

When $U_{\alpha,\beta} \neq 0$, the neutrino can transform between α - and β-types of flavors by emitting and then immediately re-absorbing a virtual Z boson. To quantitatively describe thus-realized flavor transformation, we expand $\mathbf{Z}_\lambda$ in terms of the complete set of mode functions within a box of volume V[29],

$$\begin{aligned}
\mathbf{Z}_T &= \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega V}} \mathbf{a}_T(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} + H.c.,\\
\mathbf{Z}_3 &= \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega V}} \frac{\omega}{M_Z} \mathbf{a}_L(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} + H.c.,\\
\mathbf{Z}_4 &= \sum_{\mathbf{k}} \frac{i}{\sqrt{2\omega V}} \frac{k}{M_Z} \mathbf{a}_L(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} + H.c.,
\end{aligned} \tag{35}$$

where $\omega=\sqrt{M_Z^2+k^2}$, $T=1,2$ denotes the transverse polarization degrees of freedom, and $\mathbf{a}_L(\mathbf{k})$ and $\mathbf{a}_T(\mathbf{k})$ represent the annihilation operators for the longitudinal and transverse polarizations with the wavevector $\mathbf{k}$. The neutrino field can be expressed as

$$\phi_\alpha=\sqrt{\frac{1}{V}}\sum_{\mathbf{k}}\mathbf{b}_\alpha(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{36}$$

where $\mathbf{b}_\alpha$ denote the annihilation operator of the neutrino field with the flavor α. As we here focus on the left-handed neutrino, the right-handed anti-neutrino part is not included in ϕ_α. Then the Hamiltonian for neutrino and the Z bosonic field is given by

$$\mathcal{H}_{n+Z}=\mathcal{H}_0+\mathcal{H}_I, \tag{37}$$

Here $\mathcal{H}_0$ corresponds to the "free" part without considering the neutrino-boson interaction, given by[29]

$$\mathcal{H}_0=\sum_{\mathbf{k}}k\mathbf{b}_\alpha^\dagger(\mathbf{k})\mathbf{b}_\alpha(\mathbf{k})+\sum_{\mathbf{k}}\omega\left[\mathbf{a}_L^\dagger(\mathbf{k})\mathbf{a}_L(\mathbf{k})+\sum_{T=1,2}\mathbf{a}_T^\dagger(\mathbf{k})\mathbf{a}_T(\mathbf{k})\right]. \tag{38}$$

The interaction part $\mathcal{H}_I$ is

$$\begin{aligned}\mathcal{H}_I&=-\int dx\mathcal{L}_I\\&=-\frac{i\eta}{\sqrt{2\omega V}}U_{\alpha,\beta}\sum_{\mathbf{k},\mathbf{k}'}\mathbf{X}_\lambda\mathbf{b}_\alpha^\dagger(\mathbf{k})\gamma_4\gamma_\lambda(1+\gamma_5)\mathbf{b}_\beta(\mathbf{k}'),\end{aligned} \tag{39}$$

where

$$\begin{aligned}\mathbf{X}_T&=\mathbf{a}_T(\mathbf{k}-\mathbf{k}')+\mathbf{a}_T^\dagger(\mathbf{k}'-\mathbf{k}),\\\mathbf{X}_3&=\frac{\omega'}{M_Z}\left[\mathbf{a}_L(\mathbf{k}-\mathbf{k}')+\omega'\mathbf{a}_T^\dagger(\mathbf{k}'-\mathbf{k})\right],\\\mathbf{X}_4&=\frac{i\left|\mathbf{k}-\mathbf{k}'\right|}{M_Z}\left[\mathbf{a}_L(\mathbf{k}-\mathbf{k}')-\mathbf{a}_T^\dagger(\mathbf{k}'-\mathbf{k})\right],\end{aligned} \tag{40}$$

with $\omega'=\sqrt{M_Z^2+\left|\mathbf{k}-\mathbf{k}'\right|^2}$.

As the energy of the neutrino is much smaller than M_Z, the neutrino cannot emit any real Z gauge boson. However, the virtual excitation of the Z boson can induce effective couplings among different neutrino flavors. Suppose that the system is initially in the state $|\Psi_i\rangle=|\nu_\alpha(\mathbf{p})\rangle|0\rangle$, where $\mathbf{p}$ denotes the momentum of the neutrino, and $|0\rangle$ is the vacuum state of the Z boson field. This initial state can be coupled to the final state $|\Psi_f\rangle=|\nu_\beta(\mathbf{p}),0\rangle$ through the intermediate states $|\Psi_m^j(\mathbf{k})\rangle=\mathbf{a}_j^\dagger(\mathbf{p}-\mathbf{k})|\nu_\gamma(\mathbf{k}),0\rangle$, where $j=T,L$. Based on the second-order perturbation theory, $\mathcal{H}_{int}$ can be replaced by the effective Hamiltonian

doi.org/10.32388/DA9B0G

$$\mathcal{H}_{eff} = \xi_{\alpha,\beta}|\nu_\alpha(\mathbf{p}),0\rangle\langle\nu_\beta(\mathbf{p}),0|, \tag{41}$$

where

$$\begin{aligned}\xi_{\alpha,\beta} &\simeq -\sum_{\mathbf{k}} \frac{\langle\Psi_i|\mathcal{H}_{int}|\Psi_m^j(\mathbf{k})\rangle\langle\Psi_m^j(\mathbf{k})|\mathcal{H}_{int}|\Psi_f\rangle}{M_z} \\ &\simeq -\sum_{\mathbf{k}} \frac{6\eta^2 U_{\alpha,\gamma}U_{\gamma,\beta}}{M_z^2 V} \\ &= -\frac{3\eta^2}{4\pi^3 M_z^2}\int d^3\mathbf{k} U_{\alpha,\gamma}U_{\gamma,\beta}.\end{aligned} \tag{42}$$

The integral divergence can be eliminated if $U_{\alpha,\gamma}$ contains a decaying factor, such as $e^{-\Gamma|\mathbf{k}-\mathbf{p}|^2}$, which implies that the coupling between the initial and intermediate neutrino states exponentially decreases with the increasing momentum-phase distance. The result shows that the neutrino can change its flavor through virtual excitation of Z bosons, which can be understood as a backaction of the gauge field on the neutrino.

V. Interpretation of Experimental Results Based on the New Mechanism

We now proceed to qualitatively interpret previous neutrino experiments based on this physical mechanism. Discarding the degrees of freedom of the Z bosonic field and performing the transformation $e^{i(p+\xi_{\tau,\tau})t}$, the effective Hamiltonian $\mathcal{H}_{eff}$ is transformed to

$$\begin{aligned}\mathcal{H}'_{eff} = \Delta|\nu_e\rangle\langle\nu_e| &+ (\Delta+\delta)|\nu_\mu\rangle\langle\nu_\mu| \\ &+ (\xi_{e,\mu}|\nu_e\rangle\langle\nu_\mu| + \xi_{\tau,\mu}|\nu_\tau\rangle\langle\nu_\mu| \\ &+ \xi_{\tau,e}|\nu_\tau\rangle\langle\nu_e| + H.c.),\end{aligned} \tag{43}$$

where $\Delta = \xi_{e,e} - \xi_{\tau,\tau}$ and $\delta = \xi_{\mu,\mu} - \xi_{e,e}$. We here suppose all the Hamiltonian parameters are real. Under the condition $\delta, \xi_{e,\mu} \ll \xi_{\tau,\alpha} \ll \Delta$ with $\alpha = e, \mu$, the neutrino evolution exhibits both fast and slow oscillations. For the short timescale $T \ll 1/\lambda$ with $\lambda = \xi_{e,\mu} + \xi_{\tau,e}\xi_{\tau,\mu}/\Delta$, the transition probability from the initial state $|\nu_\alpha\rangle$ $(\alpha = e, \mu)$ to $|\nu_\tau\rangle$, caused by the fast oscillation, is approximately given by

$$P_f(|\nu_\alpha\rangle \to |\nu_\tau\rangle) \simeq \frac{1}{2}A_f^\alpha[1-\cos(\Delta t)], \tag{44}$$

where $A_f^\alpha = 4\xi_{\tau,\alpha}^2/\Delta^2$. When $A_s^\alpha \ll 1$, such an oscillation has a small amplitude. The electron-antineutrino disappearance observed in the reactor neutrino experiments[17][18][19] can be described by this equation. The slow oscillation occur between $|\nu_e\rangle$ and $|\nu_\mu\rangle$. The resulting transition probability is well approximated by

$$P_s(|\nu_e\rangle \leftrightarrow |\nu_\mu\rangle) \simeq \frac{1}{2}A_s[1-\cos(\Omega_l t)]. \tag{45}$$

where $\Omega = \sqrt{4\lambda^2+\delta'^2}$ and $A_s = |2\lambda/\Omega|^2$ with $\delta' = \delta + (\xi^2_{\tau,\mu} - \xi^2_{\tau,e})/\Delta$. For $\lambda \gtrsim |\delta'|$, we have $A_s \sim 1$, which qualitatively coincides with the KamLAND experimental result[20][21], although the quantitative dependences of the Hamiltonian parameters $\xi_{\alpha,\beta}$ on the neutrino energy are still unclear.

The observed deficit of solar electron neutrinos can be well explained in terms of the competition between the coherent couplings induced by the Z bosonic field and the decoherence effect due to the CC interaction, which can be modeled as the incoherent transformation

$$\rho \to S_e\rho S_e + S_{\mu+\nu}\rho S_{\mu+\nu}, \tag{46}$$

where $S_e = |\nu_e\rangle\langle\nu_e|$, $S_{\mu+\nu} = |\nu_\mu\rangle\langle\nu_\mu| + |\nu_\tau\rangle\langle\nu_\tau|$, and ρ denotes the density operator of the neutrino before the CC reaction. Phenomenally, the competition between the coherent flavor coupling and incoherent scattering associated with $|\nu_e\rangle$ can be described by the master equation

$$\frac{d\rho}{dt} = i[\rho, \mathcal{H}'_{eff}] + \frac{\gamma}{2}(2S_e\rho S_e - S_e\rho - S\sigma_e). \tag{47}$$

The decoherence rate γ depends upon the neutrino energy and the properties of the matter. As the scattering cross-section scales with the neutrino energy, the high-energy neutrinos would have evolved into a steady state before reaching the solar surface, described by the density operator $\rho = \frac{1}{3}\sum_\alpha |\nu_\alpha\rangle\langle\nu_\alpha|$, which remains invariant during the subsequent propagation. Consequently, the proportion of detected $|\nu_e\rangle \to |\nu_e\rangle$ events is $1/3$, as revealed in solar ^{8}B neutrino experiments[1][2]. If the neutrino is free of such incoherent scatterings, the average fraction of the $|\nu_e\rangle$-outcomes after a long-time propagation is $1-\frac{1}{2}A_s$. For the intermediate case, the value is between these two extremes[1].

The atmospheric neutrinos originate from the pion decay and subsequent muon decay, which produce both the μ- and e-type neutrinos with the numbers $N^0_\mu \simeq 2N^0_e$. According to the Super-Kamiokande experiments[10][11], when the Zenith angle (Θ) ranges from 0° to 80°, the disappearance probability of the μ-type events for multi-GeV neutrinos exhibits slight oscillations, reflecting the comprehensive effect of the fast oscillation to $|\nu_\tau\rangle$ and the slow $|\nu_e\rangle \longleftrightarrow |\nu_\mu\rangle$ oscillation. Due to the imbalance between N^0_μ and N^0_e, for the number of μ-type events (N_μ) the gain from the $|\nu_e\rangle \to |\nu_\mu\rangle$ transition is less than the loss due to $|\nu_\mu\rangle \to |\nu_e\rangle$ transition. For N_e, the loss due to $|\nu_e\rangle \to |\nu_\tau\rangle$ transfer is partly compensated for by the net gain from the $|\nu_e\rangle \longleftrightarrow |\nu_\mu\rangle$ oscillation, so that its deficit is much smaller than that of N_μ. With the extension of the propagation distance in the vacuum, the slow $|\nu_e\rangle \longleftrightarrow |\nu_\mu\rangle$ oscillation plays an increasingly important role. Within the regime $80^\circ < \Theta < 90^\circ$, the number of net $|\nu_\mu\rangle \to |\nu_e\rangle$ transfer events surpasses that of $|\nu_e\rangle \to |\nu_\tau\rangle$ ones. When $\Theta > 90^\circ$, the neutrino evolves under the competition between the coherent Hamiltonian dynamics and decoherence effect due to interactions with matter of the Earth. For multi-GeV neutrinos, the reactions $\nu_{\mu/\tau} + e \to \mu/\tau + \nu_e$ are allowed, but their occurrence probabilities would be much smaller than that for $\nu_e + e \to e + \nu_e$, since the transformations from $\nu_{\mu/\tau}$ into μ need to cost much more kinetic energy than the $\nu_e \to e$ transformation. As the sensitivities of the charged- and neutral-current reactions strongly depend upon the type of the neutrino, they will deteriorate the coherence between the neutrino flavors. For $90^\circ < \Theta < 100^\circ$, the neutrino retains some quantum coherence after passing through the Earth, exhibiting slight oscillations. With the increase of Θ, the slow $|\nu_e\rangle \longleftrightarrow |\nu_\mu\rangle$ oscillation has a higher probability of being interrupted by scattering events, which prevents monotonous increase of the net gain of N_e from such an oscillation. When $100^\circ < \Theta < 180^\circ$, the neutrino has approximately evolved to the maximally mixed state ρ after crossing the Earth, so that the three flavors roughly have the same population, independent of the initial state.

VI. Discussions

As our theoretical model allows occurrence of oscillations even if the neutrino is massless, one may argue that the state of a massless particle cannot be changed during its propagation in the vacuum. This is the case if the particle remains unperturbed, as exemplified by the free propagation of a photon. However, the photonic state can be changed by some scattering process. For example, a photon can transform between modes a and b with the assistance of Λ-type three-level atoms, which have an excited state $|e\rangle$ and two ground states $|g\rangle$ and $|f\rangle$ [46]. The transitions $|e\rangle \to |g\rangle$ and $|e\rangle \to |f\rangle$ are respectively coupled to these two modes with the same detuning. When the detuning is much larger than the coupling strengths, the photon would oscillate between these two modes through a Raman scattering process, mediated by virtual excitation of the atoms. The flavor oscillations of the neutrino bear some similarity to such a Raman process. As the neutrino is inevitably disturbed by the vacuum fluctuations of the gauge fields pervading the space, its propagation cannot be really free even in the vacuum. The backaction of the virtually excited Z gauge field on the neutrino is analogous to that of the virtually excited atoms on the photon in the Raman process. One may further ask: Does there exist any other mechanism that can mediate the neutrino flavor transformation? The answer is presumably negative for the following reason. The W and Z bosonic fields are the only elements that can interact with the neutrino during its propagation in the vacuum. As the neutrino flavor is defined by the charged current that is mediated by the W bosonic field, the vacuum fluctuation of the Z bosonic field seems to be the only mechanism that can change the neutrino flavor. Finally, it should be noted that the virtual excitation of Z bosons cannot lead to flavor transformations between different flavors of charged leptons owing to their large mass differences.

References

1. [a, b, c, d] *R. Davis, Jr., Nobel Lecture: A half-century with solar neutrinos, Rev. Mod. Phys. 75, 985-994 (2003).*
2. [a, b, c, d, e, f, g, h] *A. B. McDonald, Nobel Lecture: The Sudbury Neutrino Observatory: Observation of flavor change for solar neutrinos, Rev. Mod. Phys. 88, 030502 (2016).*
3. ^ *S. Fukuda et al. (Super-Kamiokande Collaboration), Solar ^{8}B and hep neutrino measurements from 1258 days of Super-Kamiokande data, Phys. Rev. Lett. 86, 5651 (2001).*
4. ^ *Q. R. Ahmad et al. (SNO Collaboration), Measurement of the rate of ν_e + d → p + p + e^- interactions produced by ^{8}B solar neutrinos at the Sudbury Neutrino Observatory, Phys. Rev. Lett. 87, 071301 (2001).*
5. ^ *Q. R. Ahmad et al. (SNO Collaboration), Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory, Phys. Rev. Lett. 89, 011301 (2002).*
6. ^ *Q. R. Ahmad et al. (SNO Collaboration), Measurement of day and night neutrino energy spectra at SNO and constraints on neutrino mixing parameters, Phys. Rev. Lett. 89, 011302 (2002).*
7. ^ *S. N. Ahmed et al. (SNO Collaboration), Measurement of the total active ^{8}B solar neutrino flux at the Sudbury Neutrino Observatory with enhanced neutral current sensitivity, Phys. Rev. Lett. 92, 181301 (2004).*
8. ^ *B. Aharmim et al. (SNO Collaboration), Independent measurement of the total active ^{8}B solar neutrino flux using an array of ^{3}He poportional counters at the Sudbury Neutrino Observatory, Phys. Rev. Lett. 101, 111301 (2008).*
9. ^ *B. Aharmim et al. (SNO Collaboration), Combined analysis of all three phases of solar neutrino data from the Sudbury Neutrino Observatory, Phys. Rev. C 88, 025501 (2013).*
10. [a, b] *M. Koshiba, Nobel Lecture: Birth of neutrino astrophysics, Rev. Mod. Phys. 75, 1011 (2003).*

11. ^a, b^ T. Kajita, Nobel Lecture: Discovery of atmospheric neutrino oscillations, Rev. Mod. Phys. 88, 030501 (2016).

12. ^ Y. Fukuda et al. (Super-Kamiokande Collaboration), Evidence for oscillation of atmospheric neutrinos, Phys. Rev. Lett. 81, 1562 (1998).

13. ^ Y. Fukuda et al. (The Super-Kamiokande Collaboration), Measurement of the flux and zenith-angle di stribution of upward throughgoing muons by Super-Kamiokande, Phys. Rev. Lett. 82, 2644 (1999).

14. ^ Y. Ashie et al. (The Super-Kamiokande Collaboration), Evidence for an oscillatory signature in atmosp heric neutrino oscillations, Phys. Rev. Lett. 93, 101801 (2004).

15. ^ M. Koshiba, Nobel Lecture: Birth of neutrino astrophysics, Rev. Mod. Phys. 75, 1011 (2003).

16. ^ M.G. Aartsen et al. (IceCube Collaboration), Measurement of Atmospheric Neutrino Oscillations at 6–5 6 GeV with IceCube DeepCore, Phys. Rev. Lett. 120, 071801 (2018).

17. ^a, b^ F. P. An et al., Observation of electron-antineutrino disappearance at Daya Bay, Phys. Rev. Lett. 108, 171803 (2012).

18. ^a, b^ J. Ahn et al. (RENO Collaboration), Observation of reactor electron antineutrinos disappearance in th e RENO experiment, Phys. Rev. Lett. 108, 191802 (2012).

19. ^a, b^ Y. Abe et al. (Double Chooz Collaboration), Indication of reactor $\bar{\nu}_e$ disappearance in the Double Ch ooz experiment, Phys. Rev. Lett. 108, 131801 (2012).

20. ^a, b^ K. Eguchi et al. (KamLAND Collaboration), First results from KamLAND: Evidence for reactor antineu trino disappearance, Phys. Rev. Lett. 90, 021802 (2003).

21. ^a, b^ T. Araki et al. (KamLAND Collaboration), Measurement of neutrino oscillation with KamLAND: Evid ence of spectral distortion, Phys. Rev. Lett. 94, 081801 (2005).

22. ^ M. H. Ahn et al., Indications of neutrino oscillation in a 250 km long-baseline experiment, Phys. Rev. L ett. 90, 041801 (2003).

23. ^ E. Aliu et al. (The K2K Collaboration), Evidence for muon neutrino oscillation in an accelerator-based experiment, Phys. Rev. Lett. 94, 081802 (2005).

24. ^ P. Adamson et al. (MINOS Collaboration), Measurement of neutrino oscillations with the MINOS detect ors in the NuMI Beam, Phys. Rev. Lett. 101, 131802 (2008).

25. ^ P. Adamson et al. (MINOS Collaboration), Measurement of the neutrino mass splitting and flavor mixi ng by MINOS, Phys. Rev. Lett. 106, 181801 (2011).

26. ^P. Adamson et al. (MINOS+ Collaboration), Precision constraints for three-flavor neutrino oscillations from the full MINOS+ and MINOS dataset, Phys. Rev. Lett. 125, 131802 (2020).

27. ^M.A. Acero et al. (NOvA Collaboration), First measurement of neutrino oscillation parameters using neutrinos and antineutrinos by NOvA, Phys. Rev. Lett. 123, 151803 (2019).

28. a, bS. Weinberg, A model of leptons, Phys. Rev. Lett. 19, 1264 (1967).

29. a, b, c, dT. D. Lee, Particle physics and field theory (in Chinese), Shanghai Science and Technology Press (2006).

30. ^Z. Maki, N. Nakagawa, and S. Sakata, Remarks on the Unified Model of Elementary Particles, Prog. Theor. Phys. 28, 870 (1962).

31. a, b, c, dE. Kh. Akhmedov and A. Yu. Smirnov, Neutrino oscillations: Entanglement, energy-momentum conservation and QFT, arXiv:1008.2077.

32. a, b, c, dR. G. Hamish Robertson, Neutrino Oscillations and Entanglement, arXiv:1004.1847.

33. a, b, cA. G. Cohen, S. L. Glashow, and Z. Ligeti, Disentangling neutrino oscillations, Phys. Lett. B 678, 191 (2009).

34. a, b, cL. Wolfenstein, Neutrino oscillations in matter, Phys. Rev. D 17, 2369 (1978).

35. a, bS.P. Mikheyev and A.Yu. Smirnov, Resonance amplification of oscillations in matter and spectroscopy of solar neutrinos, Sov. J. Nucl. Phys. 42, 913 (1985).

36. a, bT. K. Kuo and James Pantaleone, Neutrino oscillations in matter, Rev. Mod. Phys. 61, 937 (1989).

37. ^M. O. Scully, B.-G. Englert, and H. Walther, Quantum optical tests of complementarity, Nature 351, 111 (1991).

38. ^B.-G. Englert, Fringe visibility and which-way information: an inequality, Phys. Rev. Lett. 77, 2154 (1996).

39. ^S.-B. Zheng, A simplified scheme for testing complementarity and realizing quantum eraser. Opt. Commun. 173, 265 (2000).

40. ^M. Brune et al., Observing the progressive decoherence of the “meter” in a quantum measurement, Phys. Rev. Lett. 77, 4887 (1996).

41. ^E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and V. Umansky, Dephasing in electron interference by a ‘which-path’ detector, Nature 391, 871 (1998).

42. ^S. Dürr, T. Nonn, and G. Rempe, Origin of quantum-mechanical complementarity probed by a ‘which-way’ experiment in an atom interferometer, Nature 395, 33 (1998).

43. ^*P. Bertet et al., A complementarity experiment with an interferometer at the quantum-classical boundary, Nature 411, 166 (2001).*

44. ^*K. Liu et al., A twofold quantum delayed-choice experiment in a superconducting circuit, Sci. Adv. 3, e1603159 (2017).*

45. ^*X.-J. Huang et al., Entanglement-interference complementarity and experimental demonstration in a superconducting circuit, npj Quantum Information 9, 43 (2023).*

46. ^*B. W. Shore and P. L. Knight, The Jaynes-Cummings Model, J. Mod. Opt. 40, 1195 (1993).*

Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.