Review of: "New Approximate Symmetry Theorems and Comparisons With Exact Symmetries"

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Potential competing interests: No potential competing interests to declare.

This is my report on the paper ``New approximate symmetry theorems and comparisons with exact symmetries" by M. Pakdemirli.
I worked in the area of ordinary differential equations but not in the particular area of this paper. So the following comments are made by a non-expert in this area of mathematics.

The paper studies an ordinary differential equation of kth order

$$F(x, y, y', \ldots, y^{(k)}, \epsilon) = 0,$$

where ϵ is a perturbation parameter. The goal is to find approximate solutions $y(x)$ determined by

$$\frac{dy}{dx} = \frac{\eta(x, y)}{\xi(x, y)}.$$

In some cases, ξ and η also depend on ϵ or some other parameters. Three proposals (called methods IV, V, VI) are made to find appropriate ξ and η. As far as I can tell, the notions of approximate solution and approximate symmetry do not have precise definitions.
An important contribution made by the paper consists of two tables (Tables 1 and 2) where the proposed methods are applied to a list of example differential equations, and corresponding functions ξ and η are given. The paper also includes a long list of 52 references that is useful for researchers working in this area.

I add some observations that might lead to an improvement of the paper.

\begin{enumerate} \item Equations (2.3) and (2.8) (which are the same) contain the term $\mu\frac{\partial}{\partial \epsilon}$ twice. One of these terms should be removed. \item In equations (2.5), (2.10), and (2.15) (which are the