Review of: "The edge rings of compact graphs"

Rick Norwood[1]

1 East Tennessee State University

Potential competing interests: No potential competing interests to declare.

My only comments are some suggestions for minor improvements in the grammar and punctuation.

Introduction, paragraph two: “We call a simple graph to be*compact* if it not only satisfies the odd-cycle condition but also contains no even cycles.” Better: “A simple graph is *compact* if … .”

Introduction, paragraph two: “In this paper, we devote to investigating the properties of the edge rings of compact graphs.” Better: “We devote this paper to investigating … .”

1. Preliminaries, 1.2, paragraph one: “Let *G* be a simple graph, i.e., a finite graph without loops and multiple edges, with vertex set *V*(*G*) and edge set *E*(*G*).” Better: “Let *G* be a simple graph, i.e., a finite graph without loops or multiple edges, … ."

1. Preliminaries, 1.2, paragraph one: “For a subset *W* of *V*(*G*), the *induced subgraphGW* is the graph with vertex set *W* and for every pair *x*,*y*∈ *W*, they are adjacent in *GW* if and only if they are adjacent in*G*.” Two suggestions: “*…induced subgraph GW…” (space between “subgraph” and “GW”), and “… vertex set W, where for xy∈ W the vertices are adjacent … .”*

1. Preliminaries, 1.2, paragraph two: “The generators of the toric ideal of*IG* are binomials which are tightly related to even closed walks in *G*.” Better: “… are closely related … ”

1. Preliminaries, 1.2, paragraph two: “It is known that the set

{*fW*:*W* is a primitive even closed walks of *G*}

is the universal Gröbner base of *IG* by e.g. [16, Proposition 10.1.10] or [4, Proposition 5.19]." Better: " … W is a primitive even closed walk of G} … "

Lemma 1.2, ii: “Γ=(*C*1,*C*2)*, where each of C*1 *and C*2 *is an odd cycle of G having exactly one common vertex;” Better:*Γ=(*C*1,*C*2)*, where C*1 *and C*2 *are odd cycles of G having exactly one common vertex;"*

Lemma 1.2, iii: two suggestions: “ …*where each of C*1 *and C*2 *is an odd cycle of G with … ." Better: “… where C*1 *and C*2 *are odd cycles of G with … .”*And *“…appears in each…”*Better: “… *appears in either … ”*

Below Lemma 1.2: "… to ensure it is indeed an even closed walk … " Better: " … to ensure that Γis indeed an even closed walk … "

1. Preliminaries, 1.3, paragraph one: "For any $f=\{i,j\} \in E(G)$ denote $v_f=\mathbf{e}_i+\mathbf{e}_j$, where $\mathbf{e_i}$ is the ith unit vector of R_n." Better: "… where $\mathbf{e_k}$ is the kth unit vector of R_n."

1. Preliminaries, 1.3, paragraph one: "… generated all the monomials… " Better: "… generated by all the monomials…"

I hope these notes on the Introduction and the Preliminaries are of some use.