Research Article

Properties of Elementary Particles, Dark Matter, and Dark Energy

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This paper suggests new elementary particles, a specification for dark matter, and modeling regarding dark-energy phenomena. Thereby, this paper explains data that other modeling seems not to explain. Suggestions include some methods for interrelating properties of objects, some catalogs of properties, a method for cataloging elementary particles, a catalog of all known and some method-predicted elementary particles, neutrino masses, quantitative explanations for observed ratios of non-ordinary-matter effects to ordinary-matter effects, qualitative explanations for gaps between data and popular modeling regarding the rate of expansion of the universe, and insight regarding galaxy formation and evolution. Key assumptions include that nature includes six isomers of most elementary particles and that stuff that has bases in five isomers underlies dark-matter effects. Key new modeling uses integer-arithmetic equations; stems from, augments, and does not disturb successful popular modeling; and helps explain aspects and data regarding general physics, elementary-particle physics, astrophysics, and cosmology.

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1. Introduction

This paper suggests explanations for data that physics seems not to otherwise explain. This paper suggests and uses principles and modeling that seem to provide useful advances regarding the following physics challenges.

1. Catalog properties of objects.
2. Catalog elementary particles.
3. Explain data that associate with the two-word term dark matter.
4. Explain data that associate with the two-word term dark energy.

This paper develops SUPP to supplement POST.

- POST is an acronym for POpular physics modeling based on ST. ST associates with notions of principles and modeling that associate with Space-Time coordinates. Much POST modeling associates with continuous coordinates (such as space-time coordinates) and continuous variables (such as position and velocity).
- SUPP is an acronym for SUggested physics modeling based on PP. PP associates with notions of principles and modeling that associate with Particle Properties. SUPP modeling associates with integers and solutions to integer-arithmetic equations.

POST has sought to address each one of the above four challenges for at least the most recent 80 years.

This paper suggests that SUPP complements and does not disturb subsets of POST that comport with data. The notion of combining POST and SUPP suggests some changes to some POST models that currently somewhat comport with data. SUPP suggests limitations regarding the applicability of some POST models that currently seem not to comport with data.

The combination of POST and SUPP provides quantitative explanations for observed ratios of dark-matter effects to ordinary-matter effects. (Table 12 summarizes nine different types of observed ratios for which the combination provides quantitative explanations.) That POST plus SUPP explains the ratios and POST alone seems not to explain the ratios might tend to confirm key aspects of SUPP.

2. Perspective

This unit provides perspective regarding using physics patterns and physics principles and regarding context for SUPP methods and models.

2.1. Physics patterns and physics principles

This unit notes that physics advances can have bases in extrapolating from patterns or in extrapolating based on principles.

Sometimes, physics moves forward based on extrapolations from patterns. The following examples feature advances - based on patterns - that occurred before physics included principles that explained the patterns.

- Galileo Galilei and near-contemporaries to Galileo suggested patterns regarding the motions of free-falling objects and the motions of spherical objects that roll down ramps. For objects that start as having no motion, the distance traveled is proportional to the square of the time that elapses after the time at which motion starts. Publications of text such as the text that Ref. [1] includes occurred decades before Newton proposed - in Ref. [2] - principles that might associate with such motions.
- Mendeleev suggested cataloging the then-known chemical elements based on similarity with respect to chemical interactions and on atomic weight. Mendeleev published the suggestions in Ref. [3], decades before there was enough atomic physics to explain chemical interactions and decades before there was enough nuclear physics to explain atomic weights. (Technically, people continue to pursue modeling regarding phenomena that determine the relative abundances of isotopes. For example, Ref. [4] discusses the so-called cosmological lithium problem.)

Sometimes, physics moves forward based on extrapolations that have bases in mathematical models that associate with principles. Examples include the suggesting and discovering of the Higgs boson and of various antimatter counterparts to matter elementary particles.

2.2. Notions regarding new modeling and discussing new modeling

This unit discusses aspects regarding the approach that this paper takes.

Regarding new modeling, the following notions pertain. New modeling might help to explain observations that popular modeling seems not to explain. New modeling should be compatible with principles and methods that associate with successful popular modeling. New modeling should augment successful popular modeling. New modeling can have bases that might seem to differ markedly from bases for popular modeling. New modeling can introduce new principles or quantum numbers. New modeling can suggest insight about bases for popular modeling. New modeling can suggest insight regarding the range of applicability of popular modeling. New modeling can suggest ways to extend the range of applicability of popular modeling. A combination of new modeling and popular modeling should explain observations that popular modeling alone does not explain or should make verifiable predictions that popular modeling alone does not make. Such explanations or predictions should pertain to significant aspects of physics. Such explanations or predictions do not necessarily need to pertain to all seemingly relevant aspects within an area of physics. Such explanations or predictions do not necessarily need to pertain (at least directly) to seemingly complicated aspects within an area of physics. Such explanations or predictions can be fully quantitative or partly quantitative.

Regarding discussing new modeling, communication might benefit by including discussion elements that might seem - in the context of popular modeling - to include contextual or philosophical notions.

2.3. Context for SUPP methods and models

This unit provides perspective about the context for SUPP methods and models.

2.3.1. Popular (POST) modeling and suggested additional (SUPP) modeling

This paper intertwines the following modeling.

1. POST (as in POpular Space-Time modeling). POST has bases in mathematics related to continuous coordinates (such as coordinates that associate with notions of space-time). POST includes bases for modeling, serves physics branches, and includes hypothesized attributes.
    1. POST includes the following pair of bases for modeling. CM (or, classical mechanics) includes ND (or, Newtonian dynamics), SR (or, special relativity), and GR (or, general relativity). QM (or, quantum mechanics) includes QFT (or, quantum field theory).
    2. Within the physics branch of elementary particles, POST includes the SM (or, the elementary particle Standard Model). The SM has bases in QFT.
    3. Within the physics branch of cosmology and astrophysics, POST includes CC (or, concordance cosmology). CC includes notions about stars, solar systems, black holes, galaxies, galaxy clusters, and so forth. CC has bases in ND, SR, and GR.
    4. POST includes the following trio of hypothesized attributes. OM (or, ordinary matter) associates (approximately) with stuff that associates directly with observations of light. DM (or, dark matter) associates with notions that suggest more gravitational attracting between objects than the gravitational attracting that POST associates with OM. DE (or, dark energy) associates with notions that suggest gravitational repelling between large objects that POST associates with OM plus DM.
    5. The SM evolved - based on physics observations - based on proposals for new elementary particle internal quantum numbers (such as color charge) and proposals for new elementary particles (such as quarks and gluons).
2. SUPP (as in SUggested physics modeling based on PP). SUPP has bases in equations that feature integer arithmetic. (SUPP does not have direct bases in space-time coordinates, POST notions of

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tangent spaces to space-time spaces, or POST notions of phase spaces. SUPP points to properties - such as velocity - that associate with POST tangent spaces and with POST phase spaces.)

1. SUPP suggests - based on CC observations - that nature includes six isomers (or, near copies) of all elementary particles except LRI (or, long-range interaction) elementary bosons. LRI elementary bosons include the (known) photon and the (might-be) graviton. (SUPP uses notation of the form SL to denote LRI elementary bosons. S associates with POST notions of spin. The symbol 1L associates with the word photon. The symbol 2L associates with the word graviton. Each one of the symbols 3L and 4L associates with a might-be LRI elementary particle that SUPP suggests.)
2. SUPP suggests that stuff (such as atoms) that associates with any one of five of the isomers associates with POST notions of DM. SUPP suggests that stuff that associates with the other isomer associates with POST notions of OM.
3. SUPP suggests an elementary-particle internal quantum number - isomer - that POST does not include. SUPP suggests that a notion of isomeric composition (or, the amounts of each of the six isomers) pertains regarding objects (including, for example, galaxies).
4. SUPP suggests a modeling principle - that POST does not include - that links aspects of POST and aspects of SUPP.
5. SUPP proposes specifications for DM and DE. A notion of POST-like multipole expansions regarding gravity and the notion of six isomers (of most elementary particles) provide bases for the specifications for DM and DE.
6. The specifications for DM and DE have inspirations in and seem to explain CC observations.
7. Modeling that associates with multipole expansions and with isomers extrapolates to suggest various catalogs.
8. SUPP outputs a catalog of properties (including charge and mass) of objects.
9. SUPP outputs a catalog of known elementary particles (including the electron, the Z boson, and all other known elementary particles). By extrapolating based on that catalog, SUPP suggests possible new elementary particles.

This paper suggests that the combination of the blending of properties that SUPP suggests with POST properties and the blending of elementary particles that SUPP suggests with POST elementary particles might provide insight about elementary particles and might explain (otherwise seemingly unexplained) cosmology-and-astrophysics data.

2.3.2. Information about POST and topics that this paper discusses

The following references provide information about topics that this paper discusses.

- Electromagnetism, gravity, physics constants, and physics properties.

Ref. [5] explores notions of a coupling between electromagnetism and gravity. Refs. [6][7] discuss Einstein-Maxwell equations that suggest combining electromagnetic stress-energy tensors and the Einstein field equations, which are associated with modeling regarding gravitation. Ref. [8] discusses the Einstein field equations. Refs. [9][10][11] discuss gravitoelectromagnetism, which suggests similarities between gravity and electromagnetism.

Ref. [12] and articles to which Ref. [12] alludes discuss, at least in the context of general relativity, possible relationships between mass and angular momentum.

Ref. [13] discusses notions of repulsive components of gravity.

Refs. [14][15][16][17][18] discuss tests of theories of gravity.

- Elementary particles.

Ref. [19] provides an overview of elementary particles and the elementary particle Standard Model.

Ref. [20] lists some types of modeling that people have considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Ref. [21] provides information about some of these types of modeling. Refs. [22][23][24] provide information about modeling and about experimental results. Ref. [25] (including reviews numbered 86, 87, 88, 89, 90, and 94) provides other information about modeling and about experimental results.

Ref. [26] suggests the notion of an inflaton field.

Ref. [27] discusses the notion of a graviton.

Ref. [28] discusses the notion of neutrino mass mixing.

Ref. [28] discusses notions of sterile neutrinos and heavy neutrinos. Refs. [29][30] discuss lower limits regarding masses of heavy neutrinos.

Based on possibly relevant symmetries and other notions, people suggest candidate elementary particles that might associate with dark-matter effects. Refs. [31][32] discuss notions that might suggest that nature includes an elementary boson for which people now use the term axion. Ref. [22] reviews modeling and experiments regarding axions. Ref. [33] discusses notions that might suggest that nature includes various elementary particles for which people use the two-word term supersymmetric partners. Refs. [34][35] discuss notions that might suggest that nature includes an elementary fermion for which people use the word Elko (as in Elko spinors).

A symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. Ref. [36] discusses theory. Ref. [24] reviews modeling and experiments regarding magnetic monopoles. Ref. [37] discusses a search - for magnetic monopoles - that did not detect magnetic monopoles.

Ref. [23] reviews modeling and experiments regarding leptoquarks.

Ref. [28] discusses modeling and data about neutrino masses and neutrino oscillations.

Ref. [38] notes that quantum field theory suggests that massless elementary particles cannot have spins that exceed two.

- Cosmology and astrophysics.

Ref. [39] provides an overview of concordance cosmology and related topics regarding general physics, dark matter, and elementary particles. Ref. [40] provides an overview of cosmology. Refs. [41][42][43][44] review aspects of concordance cosmology. Ref. [45]
discusses observational tests for cosmological models.

Refs. [46][47] discuss challenges regarding concordance cosmology. Ref. [46] ponders possible future developments regarding concordance cosmology.

Ref. [48] discusses possibilities leading up to a Big Bang.

Refs. [42][49] discuss inflation.

Ref. [50] discusses attempts to explain the rate of expansion of the universe.

Refs. [51][52] discuss so-called tensions between cosmology models and cosmology data.

Refs. [43][53][54][55][56] discuss the notion that concordance cosmology underestimates recent increases in the rate of expansion of the universe. Ref. [43]
suggests that possible resolutions regarding such an underestimate might focus on phenomena early in the history of the universe.

Refs. [57][58][59] discuss possible types of dark matter.

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Ref. [58] notes that physics has yet to determine directly whether nature includes cold dark matter.

Ref. [60] suggests that notions of warm dark matter might reduce discrepancies between data regarding clustering within galaxies and modeling that associates with cold dark matter.

Ref. [61] suggests the following notions regarding dark matter. Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. Some modeling suggests limits on the masses of basic dark matter objects. Observations suggest small-scale challenges to the notion that all dark matter might be cold dark matter. People use laboratory techniques to try to detect dark matter. People use astrophysical techniques to try to infer properties of dark matter. (Ref. [62] discusses astrophysical and cosmological techniques.)

Ref. [63] suggests notions of dark matter charges and dark matter photons. Ref. [64] discusses possible effects of dark matter photons.

Refs. [65][66][67] discuss the notion that dark matter might include atom-like objects.

Ref. [68] suggests that dark matter might include hadron-like particles.

Ref. [69] suggests evidence of non-gravitational interactions - in galaxies and in galactic clusters - between dark matter and ordinary matter.

Ref. [70] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Ref. [70] discusses parameters for classifying and describing galaxies. Ref. [70]

seems not to preclude galaxies that have few ordinary matter stars. Ref. [70] seems not to preclude galaxies that have little ordinary matter.

Ref. [71] suggests that concordance cosmology might not adequately explain gravitational interactions between neighboring galaxies.

- Multipole expansions.

Ref. [72] discusses multipole expansions regarding electrostatics and the property of charge. Ref. [73] discusses a multipole expansion regarding gravitation and the property of mass. Ref. [74] discusses - regarding gravitational radiation - multipole series. Ref. [75] discusses multipole expansions regarding acoustics.

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2.4. Development and discussion of SUPP methods and models

This unit provides perspective about the development of SUPP methods and models and about this paper's discussion of SUPP methods and models.

2.4.1. Development

Development started in the year 2011.

Early development focused on the notion that multipole expansions might provide insight regarding the rate of expansion of the universe and the notion that dark matter and ordinary matter might associate with isomers of a set of elementary particles. Some possible links between properties of elementary particles arose. Refs. [76][77] summarize results as of 2015 and 2016.

Later development benefited from the existence of new data - especially data about ratios of dark-matter effects to ordinary-matter effects, the gravitational constant, and the Higgs boson. Ref. [78] summarizes results as of 2022.

Recent development includes improving the modeling that underlies and links many aspects that SUPP addresses and improving discussions of relationships between SUPP modeling and POST modeling.

2.4.2. SUPP roots in POST principles, seeming happenstance, and deeper principles

Some aspects of the development of SUPP modeling might seem to have adequate roots that associate with links between POST and SUPP. Other aspects of the development of SUPP modeling (including some aspects for which this paper uses terminology such as "SUPP posits ...") might seem (regarding links between POST and SUPP) to be happenstance (or, unexplained useful guesses).

This paper suggests that parallels to Mendeleev's development of the periodic table (for chemical elements) might pertain. The usefulness of the notion of similarity regarding chemical interactions and of the notion of the relevance of atomic weights might associate more with happenstance than with roots in then-contemporary modeling. Notions of deeper principles - principles that now associate (regarding chemical interactions) with atomic physics and (regarding atomic weights) with nuclear physics - arose after Mendeleev's work.

For some perhaps seemingly happenstance aspects of SUPP, this paper does not try to thoroughly explore possibilities for deeper principles.

2.4.3. Results that this paper suggests

This paper suggests advances based on identifying and extrapolating from patterns.

This paper suggests new principles and modeling that underlie some of the patterns.

This paper suggests other advances based on using the new principles and modeling.

Regarding suggestions that this paper makes, the concluding remarks section of this paper summarizes quantitative predictions (that seem to be verifiable), qualitative predictions (that seem to be verifiable), notions regarding how to reduce seeming discrepancies between popular modeling and data, relationships between modeling that this paper suggests and popular modeling, new modeling principles, new quantum numbers, concepts for possible experiments and observations, and notions that might point to new aspects of physics.

3. Methods

This unit suggests new physics modeling (or, SUPP modeling) that includes a new physics modeling principle and that echoes and extrapolates from data, popular physics modeling (or, POST modeling), and patterns that associate with data and popular modeling.

3.1. Objects, trajectories, fields, and multipole expansions

This unit discusses some aspects of POST modeling and introduces some aspects of SUPP modeling that associate with those aspects of POST modeling.

3.1.1. Objects and trajectories

Some POST modeling considers two objects.

- One object is object O (as in observer object). The notion of an object O might include, for example, an array of detectors, systems that process information the detectors produce, and people who interpret information that the systems produce.
- One object is object I (as in inferred object). The notion of an object I reflects aspects such as the following.
  - Models - that object O associates with object I - that include notions of types of properties (of object I) that object O assumes that object O can infer regarding object I.

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- Information - that object O assumes associates with object I - that object O gleans from fields (such as the electromagnetic field or the gravitational field).

POST modeling includes notions of trajectories of objects. For an object I, an object O might infer notions of position (as a function of time), velocity (as a function of time), acceleration (as a function of time), and (perhaps) so forth. POST includes notions of derivatives (with respect to time) that link position, velocity, and so forth. Position associates with zero derivatives. Velocity associates with one derivative.

Regarding the above POST notions of position, velocity, and so forth, the following notions pertain.

- SUPP associates position, velocity, and so forth with aspects of modeling that object O uses.
- SUPP uses the symbol 0d to associate with position, the symbol 1d to associate with velocity, and the symbol 2d to associate with acceleration. SUPP anticipates - and this paper shows - more-general uses for the symbols 0d, 1d, and so forth.

3.1.2. Maxwell's equations and numbers of trajectory-related tetrads

POST includes the notions of electric fields, magnetic fields, and Maxwell's equations.

Some POST models associate with the notion that an object I models as having a point-like distribution of charge. Relative to the notion that the charge of an electron is negative, the value of the charge of object I can be negative, zero, or positive.

The following notions pertain regarding an object I that models as having a point-like distribution of nonzero charge.

- To use Maxwell's equations to describe observed electric-field effects of object I, object O assumes (or infers) a position (relative to object O) of object I at a time at which object I (in effect) emitted the information that object O detects.
- To use Maxwell's equations to describe observed magnetic-field effects of object I, object O assumes (or infers) the position (relative to object O) of object I at a time at which object I (in effect) emitted that information that object O detects, and object O assumes (or infers) a velocity (relative to object O) of object I at a time at which object I (in effect) emitted the information that object O detects.

SUPP suggests a notion of trajectory-related tetrads. For the above discussion, electric-field effects of object I associate with one trajectory-related tetrad - as in one tetrad that consists of a time and a position 3-vector. Here, the word monad associates with time. The word triad associates with the position

3-vector. Magnetic-field effects of object I associate with two trajectory-related tetrads - as in one tetrad that associates with time and position and one tetrad that associates with (the same) time and velocity.

3.1.3. Electromagnetic field modes and tetrads related to trajectories of objects

POST provides the notion that electromagnetism associates with two modes. POST includes the notion of modeling based on two linearly polarized modes. POST includes the notion of modeling based on two circularly polarized modes. (Ref. [79] - volume 1, chapter 33 - discusses circular polarization regarding light.) POST QM includes means to convert representations (regarding amplitudes) between linear-polarization representations and circular-polarization representations. (Ref. [80] discusses relationships between modeling based on photon linear polarization modes and modeling based on photon circular polarization modes.)

SUPP modeling associates most directly with POST representations that have bases in circular-polarization modes. In POST, one mode associates with left-circular polarization. One mode associates with right-circular polarization. Each mode associates with a notion of a spin of one (or, an angular momentum of magnitude $\hbar$).

SUPP associates an integer s with the spin of a mode. SUPP associates the value $s=+1$ with the electromagnetic left-circular polarization mode. SUPP associates the value $s=-1$ with the electromagnetic right-circular polarization mode. (The SUPP choice to associate left-circular with $s=+\cdots$ and not with $s=-\cdots$ is arbitrary, but not consequential.)

SUPP posits the relevance of some equations that involve integers. For the left-circular polarization mode, Eq. (1) associates with electric-field aspects (including one trajectory-related tetrad) and Eq. (2) associates with magnetic-field aspects (including two trajectory-related tetrads). (For the right-circular polarization mode, one reverses the signs of each integer.) SUPP requires that the right-hand side integers sum to s and that the number of right-hand side integers equals the number of trajectory-related tetrads.

$$+1=s=+1 \qquad (1)$$

$$+1=s=-1+2 \qquad (2)$$

SUPP posits - and this paper shows - that generalizing from Eqs. (1) and (2) - might prove useful. SUPP posits that excitations (and de-excitations) of the left-circular mode of the electromagnetic field associate with an overall notion of $+1=s$ and not necessarily with individual sums such as $s=-1+2$. SUPP

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posits that excitations (and de-excitations) of the right-circular mode of the electromagnetic field associate with an overall notion of $-1 = s$ and not necessarily with individual sums such as $s = +1 - 2$.

3.1.4. Gravitational field modes

The gravitational left-circular polarization mode associates with $s = +2$. The gravitational right-circular polarization mode associates with $s = -2$. (Refs. [81][82] discuss the notion of circular polarization regarding gravity.)

3.1.5. Multipole expansions

POST modeling includes uses of the words or one-element terms monopole, dipole, quadrupole, octupole, 16-pole, and so forth.

- One use of the terms associates with systems that include sets of similar objects. For example, a set of two similarly charged objects associates with the word dipole.
- Another use of the terms associates with Taylor series expansions that approximate the potential that associates with a field - such as the electromagnetic field - that associates with a system of similar objects. (Ref. [72] discusses - regarding electromagnetism - such series. Ref. [74] discusses - regarding gravitational radiation - such series. Ref. [73] discusses - regarding gravitation - such a series.) For example, the two-word term monopole potential pertains regarding electrostatics, and the two-word term dipole potential pertains regarding magnetostatics. Generally, such expansions associate with a notion that the accuracy decreases as the distance away from a system decreases.

SUPP uses the terms monopole, dipole, and so forth to describe the number of nonzero integers that appear in the right-hand sides of equations such as Eq. (1) and Eq. (2). For example, monopole associates with exactly one nonzero integer. Dipole associates with exactly two nonzero integers. Quadrupole associates with exactly three nonzero integers. Such SUPP usages can associate with modeling pertaining to single objects. An object can be modeled as having minimal or no structure. For example, for a case of no structure, a notion of point-like distribution of a relevant property can pertain. For such cases, the potential can be adequately accurate, even at small distances from the object.

Such SUPP usages do not necessarily disturb POST modeling regarding systems that are modeled as including more than one object.

3.2. Mathematical bases for SUPP solutions

This unit discusses integer-arithmetic equations that underlie SUPP.

Eq. (3) shows a term in which k is a positive integer and s_k can be one of minus one, zero, or plus one.

$$ks_k \qquad (3)$$

SUPP multipole mathematics has bases in sums of the form that Eq. (4) shows. The symbol Z denotes a set of positive integers. An integer k appears no more than once in each such sum. The symbol $\in$ denotes the set-theory notion of being a member of a set.

$$s = \sum_{k \in Z} ks_k \qquad (4)$$

Regarding sums of the form that Eq. (4) shows, the symbol k_{max} denotes the largest value of k for which $|s_k| = 1$.

Eq. (5) defines Σ.

$$\Sigma \equiv |s| \qquad (5)$$

For each solution that associates with Eq. (4), there is exactly one different solution for which, for each $k \in Z$, the negative of the value s_k replaces s_k. For the second solution, $-s$ replaces s. SUPP uses the one-element term solution-pair to denote such a pair of solutions.

Eq. (6) shows notation that SUPP associates with solution-pairs. The letter g is a convenience regarding notation. (Some applications of SUPP associate $\Sigma = 1$ with electromagnetic properties of objects and $\Sigma = 2$ with gravitational properties of objects. Regarding $\Sigma = 1$ and the letter g, one might think of the two-word term gamma rays. Regarding $\Sigma = 2$ and the letter g, one might think of the word gravity.) The symbol Γ denotes a list - in ascending order - of the positive integers k for which $k \in Z$ and $|s_k| = 1$.

$$\Sigma \mathrm{g} \Gamma \qquad (6)$$

Regarding Eq. (6), SUPP uses the symbol Z_Γ to denote the set of positive integers k for which $k \in Z$ and $|s_k| = 1$. The symbol n_Γ denotes the number of positive integers k for which $k \in Z$ and $|s_k| = 1$.

Table 1 alludes to all $s = \sum_{k \in Z} (ks_k)$ expressions for which $1 \leq k \leq k_{max} \leq 4$.

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k_{max}	Γ	$1 \cdot s_1$	$2 \cdot s_2$	$3 \cdot s_3$	$4 \cdot s_4$	Σ	n_0	n_Γ	n_{sp}	SUPP-pole
1	1	± 1	-	-	-	1	0	1	1	Monopole
2	2	0	± 2	-	-	2	1	1	1	Monopole
2	1‘2	± 1	± 2	-	-	1,3	0	2	2	Dipole
3	3	0	0	± 3	-	3	2	1	1	Monopole
3	1‘3	± 1	0	± 3	-	2,4	1	2	2	Dipole
3	2‘3	0	± 2	± 3	-	1,5	1	2	2	Dipole
3	1‘2‘3	± 1	± 2	± 3	-	0,2,4,6	0	3	4	Quadrupole
4	4	0	0	0	± 4	4	3	1	1	Monopole
4	1‘4	± 1	0	0	± 4	3,5	2	2	2	Dipole
4	2‘4	0	± 2	0	± 4	2,6	2	2	2	Dipole
4	3‘4	0	0	± 3	± 4	1,7	2	2	2	Dipole
4	1‘2‘4	± 1	± 2	0	± 4	1,3,5,7	1	3	4	Quadrupole
4	1‘3‘4	± 1	0	± 3	± 4	0,2,6,8	1	3	4	Quadrupole
4	2‘3‘4	0	± 2	± 3	± 4	1,3,5,9	1	3	4	Quadrupole
4	1‘2‘3‘4	± 1	± 2	± 3	± 4	0,2,2,4,4,6,8,10	0	4	8	Octupole

Table 1. $\Sigma = |s| = |\sum_{k \in Z}(ks_k)|$ solution-pairs for which $1 \leq k \leq k_{max} \leq 4$. The columns labeled $1 \cdot s_1$ through $4 \cdot s_4$ show contributions that associate with terms of the form ks_k. Each entry in the column with the label Σ alludes to a unique solution-pair. The integer n_0 equals the number of k for which $1 \leq k \leq k_{max} \leq 4$ and $s_k = 0$. The integer n_Γ equals the number of k for which k appears in the list Γ. The number n_{sp} equals $2^{n_\Gamma - 1}$ and states the number of solution-pairs. The column for which the one-element label is SUPP-pole associates mathematically with the number of solution-pairs. For a row for which exactly one solution-pair pertains, the column shows the word monopole. For a row for which exactly two solution-pairs pertain, the column shows the word dipole. For a row for which exactly four solution-pairs pertain, the column shows the word quadrupole. For a row for which exactly eight solution-pairs pertain, the column shows the word octupole. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solution-pairs. Regarding $\Sigma = 2$, $|-1+2-3+4| = 2 = |-1-2-3+4|$. Regarding $\Sigma = 4$, $|-1-2+3+4| = 4 = |+1+2-3+4|$.

SUPP includes solution-pairs for which integers k for which $k \geq 5$ pertain. For each of those solution-pairs, $k_{max} \geq 5$ pertains. In general, the following notions pertain.

SUPP suggests that each relevant solution-pair comports with Eq. (7).

$$1 \in Z_\Gamma \text{ or } 2 \in Z_\Gamma \text{ or } 3 \in Z_\Gamma \text{ or } 4 \in Z_\Gamma \qquad (7)$$

For each solution-pair ΣgΓ, Eq. (8) defines k_{n_0}. (That is, k_{n_0} denotes the largest value of k for which k is less than or equal to four and $k \in Z_\Gamma$.)

$$k_{n_0} \equiv \max\{k | 1 \leq k \leq 4 \text{ and } k \in Z_\Gamma\} \qquad (8)$$

For each solution-pair $\Sigma \mathrm{g} \Gamma$, Eq. (9) computes n_0. The symbol $\notin$ denotes the set-theory notion of not being a member of a set.

$$n_0 = \text{ the number of } k \text{ for which } 1 \leq k \leq k_{n_0} \text{ and } k \notin Z_\Gamma \qquad (9)$$

Eq. (7) and Eq. (9) imply that the range $0 \leq n_0 \leq 3$ pertains regarding n_0.

For $n_\Gamma \geq 4$, each one of some combinations of Γ and Σ can associate with more than one solution-pair. For a combination of Γ and Σ that associates with more than one solution-pair, Eq. (10) shows a symbol that SUPP uses.

$$\Sigma \mathrm{g} \Gamma \mathrm{x} \qquad (10)$$

Eq. (11) specifies the values of k that have relevance for this paper. Some $\Sigma = 0$ solution-pairs include values of k for which $k \geq 16$ and k is a power of two.

$$-1 \leq k \leq 8, \text{ or } \log_2(k) - 3 \text{ is a positive integer} \qquad (11)$$

3.3. Some electromagnetic properties of some objects

This unit discusses electromagnetic properties that associate with objects.

Discussion related to Eq. (1) and Eq. (2) associates with modeling for which an object I has a point-like distribution of charge. Regarding Eq. (1), SUPP uses the notation 0d>:1g1 to characterize the relevant solution pair. The notation 0d>: associates with the notion of 0d (as in the series 0d, 1d, and so forth) and notions of symbols of the form $\Sigma \mathrm{g} \Gamma$ for which $\Sigma > 0$. Regarding Eq. (2), SUPP uses the notation 1d>:1g1`2 to characterize the relevant solution pair. (Generally, 0d>: associates with - at least - position and does not associate with linear velocity. Generally, 1d>: associates with - at least - position and linear velocity.)

Some objects I can model as having zero net charge and as making nonzero contributions to the magnetic field. One example is a bar magnet. A second (thought-experiment) example features two equally sized, concentric, uniformly charged shells that rotate around a common axis. One shell features a specific total magnitude of positive charge and rotates in one direction. The other shell features an equal magnitude of negative charge and rotates - with the same magnitude of angular velocity as the positively charged shell - in the other direction.

For the thought-experiment example, the number of trajectory-related triads equals two and pertains to aspects that associate with 0d. One trajectory-related tetrad associates with position. One trajectory-related tetrad associates with angular velocity. Two solution-pairs pertain. Solution-pair 0d>:1g1`2

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associates with position and the relevant angular velocity. Solution-pair 1d>:1g1`2`4 associates with position, relevant angular velocity, and a possible (regarding the object) linear velocity.

SUPP suggests that 0d>:1g1 associates with contributions to the electric field. SUPP suggests that each one of 1d>:1g1`2 and 0d>:1g1`2 associates with contributions to the magnetic field. Such notions might leave two unresolved issues. What about 0d>:3g1`2? What about $\Gamma$=1`2`4?

Regarding 0d>:3g1`2, SUPP suggests the following. If a model ascribes notions of spatial spherical symmetry (or notions of spatially point-like) - with respect to charge - to 0d>:1g1`2, 0d>:3g1`2 associates with naturally occurring (electromagnetic-centric) oblateness - with respect to charge - for CM (and with anomalous magnetic moment for QM). If a model associates all naturally occurring electromagnetic oblateness - with respect to charge - with aspects of 0d>:1g1`2, 0d>:3g1`2 associates with no contributions to the electromagnetic field.

SUPP suggests that Γ=1 associates with contributions to the electric field and that Γ=1`2 associates with contributions to the magnetic field.

Regarding Γ=1`2`4, the following possibilities pertain: 2d>:1g1`2`4, 1d>:1g1`2`4, and 0d>:1g1`2`4.

Regarding 2d>:1g1`2`4, POST SR modeling might associate perceived nonzero acceleration with notions that at least one of object I and object O models as being part of a system and models as not being entirely a distinct object.

Regarding 1d>:1g1`2`4, the notion that object I models as being an object implies that only one linear velocity pertains. One linear velocity pertains for all 1d> solution-pairs. For example, for the present discussion, the velocity associates with both 1d>:1g1`2 and 1d>:1g1`2`4.

Regarding 0d>:1g1`2`4, the Earth provides an example of a possibly relevant property. The magnetic field that the Earth produces associates with an axis and with 1d>:1g1`2. The precession (with a period of one day) of that axis associates with 0d>:1g1`2`4. (This precession does not associate with Larmor precession, which involves influences of externally - with respect to object I - produced aspects of the magnetic field.) POST would consider that 0d>:1g1`2`4 associates with a confluence of electromagnetic aspects (the Earth-produced contributions to the magnetic field) and gravitational (or, possibly inertial) aspects that associate with internal (to the Earth) angular momentum.

Eq. (12) shows a generalization that SUPP posits regarding using SUPP notions within the context of POST modeling. (Discussion related to Eq. (29) suggests other examples of relevance for Eq. (12). Atomic

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energy levels associate with gravitational or inertial aspects, such as masses, as well as with electromagnetic aspects.)

$$\text{For } 4 \in Z_\Gamma,\ 0d >\text{: } \Sigma g\Gamma \text{ disassociates with POST notions of solely electromagnetic effects} \tag{12}$$

3.4. Some sets of SUPP solution-pairs and some relationships between solution-pairs

This unit discusses notation that SUPP uses regarding some sets of solution-pairs.

3.4.1. Some sets of SUPP solution-pairs

SUPP uses symbols of the form Σg′ to denote solution-pairs for which the integer Σ is positive and $\Sigma \in Z_\Gamma$. For example, 1g1\`2 associates with 1g′.

SUPP uses symbols of the form Σg″ to denote solution-pairs for which the integer Σ is positive and $\Sigma \notin Z_\Gamma$. For example, 3g1\`2 associates with 3g″. (Here, one solution associates with $s = +1 + 2 = +3$.)

SUPP uses symbols of the form Σg to denote solution-pairs for which the integer Σ is positive. For example, 1g1\`2 associates with 1g. 3g1\`2 associates with 3g.

SUPP uses the symbol 0g to denote solution-pairs for which the integer Σ is zero. The solution-pair 0g1\`2\`3 provides an example. (Here, one solution associates with $s = -1 - 2 + 3 = 0$.) Arithmetically, 0g associates with $n_\Gamma \geq 3$.

3.4.2. Cascades that interrelate SUPP solution-pairs

SUPP includes the notion of adding - to one Γ - one new positive integer and thereby producing a new Γ. SUPP associates the word cascade with the notion that, for an original solution-pair $\Sigma_1 \mathrm{g} \Gamma_1$ and a resulting solution-pair $\Sigma_2 \mathrm{g} \Gamma_2$, $\Sigma_2 = \Sigma_1$.

For one original solution-pair, more than one cascade solution-pair might pertain.

SUPP also associates the word cascade with a network of solution-pairs that cascade (from each other) based on multiple cascade steps that ensue from one solution-pair. The solution-pair 1g1\`2 associates with a first step in a cascade that starts with the solution-pair 1g1. A next cascade step provides the 1g1\`2\`4 solution-pair. A next cascade step produces two 1g1\`2\`4\`6 solution-pairs and one 1g1\`2\`4\`8 solution-pair.

3.5. Some gravitational properties of some objects

This unit discusses gravitational properties that associate with objects.

Based on doubling values of Σ and the various k (with $k \in Z_{\Gamma}$) that pertain regarding discussion above about electromagnetic properties, SUPP suggests the following notions regarding gravitational properties.

- 0d>:2g2 - energy. (The number of trajectory-related triads equals one. The triad associates with position.)
- 1d>:2g2`4 - momentum. (The number of trajectory-related triads equals two. The triads associate with position and velocity.)
- 0d>:2g2`4 - angular momentum (The number of trajectory-related triads equals two. The triads associate with position and angular velocity.).
- 0d>:6g2`4 - (for some applications) CM oblateness (based on angular momentum) regarding mass (or, energy) or QM "anomalous angular momentum moment". (The number of trajectory-related triads equals two. The triads associate with position and with an oblateness-related moment of inertia.)

Uses of 0d>:2g2`4 associate with uses of 1d>:2g2`4`8. Uses of 0d>:6g2`4 associate with uses of 1d>:6g2`4`8.

SUPP suggests the following notions pertain regarding gravitational properties. (For each case, a correct number of trajectory-related triads pertains.) For each item below, at least one 1d>:ΣgΓ solution-pair exists.

- 0d>:2g1`2`3 - two unique moment-of-inertia axes. (The number of trajectory-related triads equals three. The triads associate with position, one moment of inertia, and the other moment of inertia.)
- 0d>:2g1`2`3`4x - rotation relative to each one of the two unique moment-of-inertia axes. (Table 1 alludes to two 2g1`2`3`4 solution-pairs. The number of trajectory-related triads equals four. The triads associate with position, one moment of inertia, the other moment of inertia, and angular velocity.)
- 0d>:2g1`2`3`4`8x - a possible 0d property.

3.6. Associations between multipole contributions to forces

This unit discusses notions that - for, for example, electromagnetism and gravity - effects that associate with non-scalar properties (such as magnetic moment and intrinsic angular momentum) add to or subtract from effects that associate with scalar properties (such as charge and energy).

3.6.1. Some electromagnetic properties of objects

SUPP associates 0d>:1g1 with the property of charge. SUPP associates 1d>:1g1`2 with the property of charge-current.

POST SR associates charge and charge-current with a 4-vector.

An observer that senses an object as non-moving and non-rotating observes - regarding contributions by the object to the electromagnetic field - that $E_1 \propto |q|$ and $B_1 = 0$. $|q|$ denotes the magnitude of the charge of the object. E_1 denotes the magnitude of the electric field. B_1 denotes the magnitude of the magnetic field. (The subscript $_1$ anticipates notions of E_Σ and B_Σ in which - for example - $\Sigma = 2$ associates with gravitational fields. The case of $\Sigma = 2$ might associate with CM notions of gravitoelectromagnetism. For the case of $\Sigma = 2$, SUPP associates the symbol $E_{2:n_\Gamma}$ with solution-pairs that associate with both E_2 and a particular value of n_Γ. For the case of $\Sigma = 2$, SUPP associates the symbol $B_{2:n_\Gamma}$ with solution-pairs that associate with both B_2 and a particular value of n_Γ.)

For an observer that senses an object for which $|q| > 0$ as moving and not rotating, $B_1 > 0$.

SR associates Eq. (13) with Lorentz invariance. c denotes the speed of light.

$$(E_1)^2 - c^2(B_1)^2 = \text{ a constant } \geq 0 \qquad (13)$$

Per Eq. (13), an observer that senses the object as moving senses a larger value of E_1 than does an observer that senses the object as not moving. An observer that senses the object as moving senses a larger value of $|q|$ than does an observer that senses the object as not moving.

SUPP suggests that - in effect - effects 1d>:1g1`2 subtract from effects of 0d>:1g1.

SUPP associates 0d>:1g1`2 with the property of magnetic moment.

Similarly (to the case of 1d>:1g1`2), SUPP suggests that - in effect - effects of 0d>:1g1`2 subtract from effects of 0d>:1g1. (One might consider that - for a model that considers the magnetic moment to associate with charges that revolve around an axis - the motion associating with such revolving associates with nonzero charge-currents.)

SUPP suggests the notion that B_1 associates with 1d>:1g1`2, with 0d>:1g1`2, and possibly with 0d>:3g1`2. SUPP uses the notation $1_3$g1`2 to associate with the notion that 0d> use of 3g1`2 associates with B_1 and with the magnetic field component of the electromagnetic field.

SUPP suggests that E_1 associates with $\Gamma = 1$. SUPP suggests that B_1 associates with $\Gamma = 1‘2$.

SUPP does not suggest the need to consider a component - of strictly just the electromagnetic field (and not, for example, also of the gravitational field) - that would extend the series monopole E_1 and dipole B_1 to include a quadrupole item (which, presumably, would associate with - for example - $\Gamma = 1‘2‘4$).

3.6.2. Force-related associations between multipole contributions

SUPP extrapolates from discussion related to Eq. (13).

SUPP posits that - for an object that exhibits a nonzero value of the property that associates with (either a 1d> or a 0d>) use of a Σg solution-pair $\Sigma \text{g} \Gamma_2$ that cascades in one step from a solution-pair $\Sigma \text{g} \Gamma_1$ - the nonzero value associates with dilution of the effects (of the object) that associate with 0d> use of the solution-pair $\Sigma \text{g} \Gamma_1$. (For cases in which $\Sigma \text{g} \Gamma_1$ is not a monopole solution-pair, the effects that associate - via solution-pairs from which $\Sigma \text{g} \Gamma_1$ cascades - with $\Sigma \text{g} \Gamma_2$ propagate toward the relevant monopole Σg Γ.)

For any one value of Σ, Eq. (14) and Eq. (15) pertain.

$$\text{dipole, octupole, ... effects subtract from monopole effects} \qquad (14)$$

$$\text{quadrupole, 16-pole, ... effects add to monopole effects} \qquad (15)$$

Regarding gravity, the following notions pertain. (Table 3 associates with the following notions and with other notions.) 0d> use of solution-pairs that associate with $E_{2:1}$, 0d> use of solution-pairs that associate with $E_{2:3}$, and so forth associate with gravitational attraction. 0d> use of solution-pairs that associate with $B_{2:2}$, 0d> use of solution-pairs that associate with $B_{2:4}$, and so forth associate with gravitational repulsion. 0d> uses of solution-pairs that associate with $E_{2:\geq 3}$ or with $B_{2:\geq 4}$ do not associate with POST notions of scalar properties or vector properties.

3.6.3. POST notions that associate with $V(r) \propto r^{n_V}$ potentials for which $n_V \leq -1$

POST ND includes notions of electromagnetic potential energies V for which Eq. (16) pertains. Here, r denotes a radial coordinate that associates with a distance from an object I that is modeled as point-like. n_V is a negative integer. Similar modeling pertains to gravitational potentials.

$$V(r) \propto r^{n_V} \qquad (16)$$

$n_V = -1$ associates with POST ND notions of monopole. $n_V = -2$ associates with POST ND notions of dipole.

For $\Sigma = 1$, SUPP notions regarding (charge and) monopole associate with POST ND notions of $n_V = -1$ and monopole.

For $\Sigma = 2$, SUPP notions regarding (energy - or mass - and) monopole associate with POST ND notions of $n_V = -1$ and monopole.

Eq. (17) links - regarding 0d> uses of $\Sigma \geq 1$ solution-pairs that associate with LRI fields - aspects of POST ND modeling and aspects of SUPP.

$$n_V = -n_\Gamma \qquad (17)$$

For each one of the combinations of POST SR and SUPP and the combination of POST GR and SUPP, the notion of n_Γ pertains, and the notion of n_V does not necessarily pertain.

3.7. Opportunities for and approaches to more methods

This unit provides an overview regarding methods that this paper discusses below.

Discussion above leaves open opportunities that associate with questions such as the following questions.

- Might SUPP associate with aspects of other (than electromagnetic and gravitational) interactions - such as the strong interaction - that POST includes?
- Might SUPP embrace types of other (than charge, mass, and so forth) properties - such as atomic states - that an object O might infer about an object I?
- Might solution-pairs for which $\Sigma = 0$ have significance?
- Might the use of a more general (than just the notions above regarding $\Sigma \geq 1$) notion of tetrads provide insight about POST?
- Might SUPP catalog and predict elementary particles?
- Might SUPP provide insight regarding phenomena that POST associates with the term dark-matter phenomena?

This paper suggests, regarding each of the questions, an answer of yes. This paper discusses methods - beyond methods that units (of this paper) above discuss - that reflect the following two notions.

- Some of the new methods assume that nature includes six isomers of non-LRI elementary particles. One isomer associates with the POST notion of ordinary (or baryonic) matter. Five isomers associate with POST notions of dark matter. (Data, such as data to which Table 12 alludes - and for which discussion related to Table 12
cites references - and modeling that POST and SUPP jointly underlie, seem to support the notion that the six isomer assumption might associate with a useful specification for dark matter. Early such data associate $5^{+}:1$ ratios (of non-ordinary-matter effects to ordinary-matter effects) with the effects pertaining to the observable universe, some galaxy clusters, and some galaxies.)
- Presentation of the new methods can associate with a network approach that features links between POST methods and SUPP methods, extrapolations from (other) SUPP notions, and associations with data. (Each one of some aspects of the new methods links to more than one other aspect. Each one of some aspects of the new methods might seem - at least as far as this paper discusses methods - to associate just with one extrapolation from one other aspect.)

This paper discusses (below) new methods, with the presentation featuring a network approach.

3.8. Solution-pairs that associate with elementary particles

This unit differentiates between solution-pairs that associate with long-range interactions and solution-pairs that associate with elementary particles.

SUPP suggests that some solution-pairs for which $\Sigma \geq 1$ associate with properties - of objects - that associate with interactions between objects and LRI fields.

SUPP suggests that some solution-pairs for which $\Sigma = 0$ associate with elementary particles.

SUPP extends the use of notation - that Eq. (6) shows - to include S$\Phi\Gamma$, in which Φ associates with the notion of a family of elementary particles and S is the spin (as in the POST expression S(S+1)$\hbar^2$) that pertains to each member of the family of elementary particles. (Table 6 and discussion related to Table 6 provide details regarding families of elementary particles.) The symbol g does not directly associate with any family Φ. Each family SΦ associates with 0gΓ solution-pairs. Each S$\Phi\Gamma$ solution-pair associates with 0gΓ solution-pairs.

Each family SΦ for which Φ =L associates with elementary particles that associate with LRI fields. For example, 1L and the photon associate with 1g. 2L and the might-be graviton associate with 2g.

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Each family SΦ for which $\Phi \neq$L associates with elementary particles that do not directly associate with LRI fields.

3.9. Dark matter and the notion of six isomers of most elementary particles

This unit discusses the specification - that SUPP suggests - for dark matter. (This paper elsewhere - for example, in discussion related to Table 12 - motivates the specification and tests the specification against observational data.)

POST does not yet provide an established description of dark matter.

POST associates with exactly one use of the set of all known elementary particles.

SUPP uses the two-word term isomeric set to denote the set of all (known and yet-to-be-found) elementary particles except the elementary particles that do (if known) intermediate LRI or might (if found) intermediate LRI. (Table 6 points to all known elementary particles and to yet-to-be-found elementary particles that SUPP suggests. 1L is known; associates with LRI; and is, thus, not a member of the isomeric set. Each one of 2L, 3L, and 4L is not known, would associate with LRI, and would not be a member of the isomeric set.)

SUPP associates the word isomer with each one of six uses (that SUPP suggests that nature includes) of the isomeric set.

SUPP suggests that most DM associates with five new (compared to POST) uses of the isomeric set (of elementary particles).

SUPP uses the one-element term isomer-zero to denote the isomer that generally associates with OM. (SUPP does not rule out some CC notions - such as notions of primordial black holes - of OM objects that might measure as DM.)

SUPP suggests that - generally (but not always) - effects that associate with the five non-isomer-zero uses of the isomeric set measure - from the perspective of POST - as effects of DM. (The following notions associate with the use - in the previous sentence - of the word generally and with the non-use in the previous sentence of a word such as always. SUPP suggests that OM objects detect light emitted by atoms that associate with one DM isomer. SUPP suggests that OM objects would not detect - at least other than with marginal effectiveness - thermal electromagnetic radiation from DM stars.)

Table 12a, Table 12b, and Table 12c associate with the SUPP suggestion that SUPP notions regarding dark matter might help explain data that POST otherwise does not necessarily explain.

3.10. Isomer-related properties of elementary particles

This unit discusses differences - among the six isomers - regarding properties of elementary particles.

Regarding each LRI field, for each Σg solution-pair, one solution associates with $s > 0$ and with the POST notion of a left-circular polarization mode of the field, and one solution associates with $s < 0$ and with the POST notion of a right-circular polarization mode of the field.

POST notions of left-handedness pertain to (at least) all known elementary particles - possibly except for neutrinos - that have nonzero mass and nonzero spin. (LRI elementary particles associate with zero mass. POST associates the notion of polarization - and not necessarily the notion of handedness - with all known zero-mass elementary particles.)

Table 6 (below) suggests that 0g solution-pairs for which $\{1, 3\} \subset Z_\Gamma$ and $\{5, 7\} \cap Z_\Gamma = \emptyset$ associate with elementary particles. (The symbol $\subset$ denotes the set-theory notion of being a subset of. Here, each one of one and three must be a member of the set Z_Γ. The symbol $\cap$ denotes the set-theory notion of intersection. The symbol $\emptyset$ denotes the empty set - or, a set with no members. Here, neither one of five and seven can be a member of the set Z_Γ.)

For each 0g solution-pair that associates with a non-LRI elementary particle, SUPP suggests that the notion of two solutions associates with a notion of a pair of isomers. SUPP associates the one-element term isomer-pair with such a pair of isomers. SUPP suggests that one isomer associates with POST notions of left-handedness and that one isomer associates with POST notions of right-handedness.

SUPP suggests the notion of three-isomer pairs.

SUPP names the isomers with one-element terms - isomer-zero, isomer-one, ..., and isomer-five. The three isomer-pairs associate, respectively, with isomer-zero and isomer-three, isomer-one and isomer-four, and isomer-two and isomer-five. The notion of left-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-zero, isomer-two, and isomer-four. The notion of right-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-one, isomer-three, and isomer-five.

SUPP suggests that POST associates with isomer-zero. For example, POST non-LRI elementary particles that have nonzero mass and nonzero spin associate with left-handedness. POST does not associate with the other five isomers that SUPP suggests.

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SUPP posits that the masses of counterpart non-LRI elementary particles do not vary between isomers. Per discussion related to Table 9, SUPP posits the following notions about flavours regarding isomeric counterparts of known isomer-zero nonzero-charge fermion elementary particles.

- For each one of isomer-zero and isomer-three, the flavour of the lowest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-one and isomer-four, the flavour of the lowest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-two and isomer-five, the flavour of the intermediate-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the highest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the lowest-mass charged lepton equals the remaining quark flavour.

3.11. Reaches that associate with SUPP multipole solution-pairs

This unit discusses the extent to which various components of LRI forces associate with interactions between isomers.

3.11.1. Notions that associate with solution-pairs that associate with LRI fields

SUPP uses the word instance and the word reach to describe aspects of the extent to which $\Sigma \geq 1$ LRI solution-pairs associate with interactions within and between isomers.

SUPP suggests that Eq. (18) pertains to uses of each $\Sigma \geq 1$ LRI solution-pair. The positive integer n_I denotes a number of instances of a solution-pair. The positive integer R_I denotes the reach - in number of isomers - that associates with one instance of the solution-pair.

$$n_I R_I = 6 \qquad (18)$$

POST suggests that, to a first approximation, DM appears - to OM - to be electromagnetically dark. SUPP suggests that, to a first approximation, each isomer appears - to each other isomer - to be electromagnetically dark. For 0d> use of the solution-pair 1g1, SUPP suggests that $n_I = 6$ and $R_I = 1$.

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(Each instance - of a 0d> use of the solution-pair 1g1 - associates with one isomer. Each isomer associates with its own instance of a 0d> use of the solution-pair 1g1.) Thereby, SUPP points to six instances of the property of charge.

0d> use of 1g1 associates with $n_0 = 0$. (Eq. (9) defines n_0.) SUPP posits that $R_I = 1$ pertains for all cases for which 0d>:$n_0 = 0$. (SUPP extends - from 0d>:ΣgΓ - the use of the notation 0d>: to include 0d>: n_0 and 0d>:Z_Γ.)

POST suggests that gravity associates with interactions between OM and OM, interactions between OM and DM, and interactions between DM and DM. For 0d> use of the solution-pair 2g2, SUPP suggests that $n_I = 1$ and $R_I = 6$. SUPP points to one instance of the property of energy. The one instance of the property of energy associates with all six isomers.

0d> 2g2 associates with $n_0 = 1$. SUPP posits that $R_I = 6$ pertains for all 0d> cases for which $n_0 = 1$.

SUPP posits the following reaches for 0d> uses of LRI solution-pairs - $R_I = 1$ for $n_0 = 0$, $R_I = 6$ for $n_0 = 1$, $R_I = 2$ for $n_0 = 2$, and $R_I = 1$ for $n_0 = 3$. These four pairs - each of one reach and one number of instances - seem to help explain data. (Discussion related to Eq. (91) might link the reaches to aspects of POST modeling.)

SUPP suggests that the R_I for a 1d> use of an LRI solution-pair equals the R_I for the 0d> use of the solution-pair from which the 1d> solution-pair cascades in one step. (Otherwise, the notion of object would not pertain.)

Eq. (19) shows SUPP notation for the notion that a reach of R_I associates with each instance of a $\Sigma \geq 1$ solution-pair ΣgΓ.

$$\Sigma\text{g}\Gamma :: R_I \qquad (19)$$

3.11.2. Notions that associate with elementary-particle solution-pairs

SUPP extends the use of the notation - that Eq. (19) shows - to include S$\Phi\Gamma$ :: R_I, in which Φ associates with the notion of a family of elementary particles and S is the spin (as in the POST expression S(S+1)$\hbar^2$) that pertains for each member of the family of elementary particles. Notation of the forms SΦ::R_I and S $\Phi\Gamma$::R_I pertains.

Other than for Φ =L, SUPP suggests that $n_I = 6$ for each use of SΦ or S$\Phi\Gamma$ and that $R_I = 1$ for each use of SΦ or S$\Phi\Gamma$.

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3.11.3. Reaches for long-range interaction $\Sigma g'$ solution-pairs

Table 2 shows reaches that associate with uses of Σg′ solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5, 7\} \cap Z_\Gamma = \emptyset$.

†	0d>:	1d>:	SL	n_I	R_I
†	1g1	1g1‘2	1L	6	1
-	1g1‘2	1g1‘2‘4	1L	6	1
-	1g1‘2‘4	1g1‘2‘4‘8, 1g1‘2‘4‘6x	1L	1	6
-	1g1‘2‘4‘8	1g1‘2‘4‘6‘8x ‡	1L	1	6
-	1g1‘2‘4‘6‘8x ‡	⃠	-	-	-
-	1g1‘2‘4‘6x	1g1‘2‘4‘6‘8x ‡	-	-	-
†	1g1‘4‘6	1g1‘4‘6‘8	-	-	-
-	1g1‘4‘6‘8	1g1‘2‘4‘6‘8x ‡	-	-	-
†	2g2	2g2‘4	2L	1	6
-	2g2‘4	2g2‘4‘8	2L	3	2
-	2g2‘4‘8	⃠	-	-	-
†	2g1‘2‘3	2g1‘2‘3‘4x, 2g1‘2‘3‘6	2L	6	1
-	2g1‘2‘3‘4x	2g1‘2‘3‘4‘8x	2L	6	1
-	2g1‘2‘3‘4‘8x	2g1‘2‘3‘4‘6‘8x ‡	2L	6	1
-	2g1‘2‘3‘4‘6‘8x ‡	⃠	-	-	-
-	2g1‘2‘3‘6	2g1‘2‘3‘6‘8x	-	-	-
-	2g1‘2‘3‘6‘8x	2g1‘2‘3‘4‘6‘8x ‡	-	-	-
†	3g3	3g3‘6	3L	3	2
-	3g3‘6	⃠	-	-	-
†	3g2‘3‘4	3g2‘3‘4‘8, 3g2‘3‘4‘6	3L	1	6
-	3g2‘3‘4‘8	3g2‘3‘4‘6‘8 ‡	3L	1	6
-	3g2‘3‘4‘6	3g2‘3‘4‘6‘8 ‡	-	-	-
-	3g2‘3‘4‘6‘8 ‡	⃠	-	-	-
†	4g4	4g4‘8	4L	6	1
-	4g4‘8	⃠	-	-	-
†	4g1‘2‘3‘4x	4g1‘2‘3‘4‘6x	4L	6	1
-	4g1‘2‘3‘4‘6x	4g1‘2‘3‘4‘6‘8x ‡	-	-	-
-	4g1‘2‘3‘4‘6‘8x ‡	⃠	-	-	-
†	4g1‘2‘3‘4‘8x	4g1‘2‘3‘4‘6‘8x ‡	4L	6	1

Table 2. Reaches that associate with uses of $\Sigma g'$ solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5, 7\} \cap Z_{\Gamma} = \emptyset$.The symbol † alludes to the notion that the 0d> solution-pairs do not cascade from other 0d> solution-pairs that the table shows. The 1d>: column shows solution-pairs that cascade in one step from the 0d> solution-pairs. The symbol ‡ alludes to the notion that the solution-pairs appear more than once in the column that lists 1d> solution-pairs. The three rightmost columns designate rows that show SUPP-relevant pairs of one 0d> solution-pair and one

1d> solution-pair. (A 0d> solution-pair can associate with more than one 1d> solution-pair.) The symbol SL associates with a known LRI elementary particle or a might-be LRI elementary particle and with the notion that S equals Σ. The symbol ⦸ associates with the notion that - for the 0d> solution-pairs - no 1d> solution-pairs pertain. SUPP assumes that each one of 1d> ⦸ and $6 \in (0d >: Z_\Gamma)$ associates with the notion that 0d> uses of the solution-pairs that the column labeled 0d>: lists are not relevant regarding SUPP. (Per Eq. (23), SUPP suggests that - regarding elementary particles - 0d> use of solution-pairs for which $6 \in Z_\Gamma$ associates with fermion elementary particles and does not associate with boson elementary particles. The notion of SL associates with boson elementary particles and does not associate with fermion elementary particles.) The next-to-rightmost column shows the number of instances for each pair - of one 0d> solution-pair and one 1d> solution-pair - that is relevant regarding SL. The rightmost column shows the reach for each instance - of one 0d> solution-pair and one 1d> solution-pair - that is relevant regarding SL.

The notion that - for 0d> use of the 1g1`2`4 solution-pair - $R_I = 6$ associates with the notion that SUPP suggests that detectors that have only OM physical components can detect electromagnetic radiation that - for $l_I \geq 1$ - isomer-l_I stuff radiates.

3.11.4. Notions that associate with the notion of two solutions per solution-pair

SUPP suggests relevance - regarding the following - for the notion of the existence of two solutions per one solution-pair.

- Two isomers (in the context of isomer-pair).
- Two handednesses (in the context of isomers).
- Two handednesses (in the context of nonzero spin, nonzero mass elementary particles).
- Two circular polarizations (in the context of zero-mass elementary particles).

3.11.5. Properties and reaches for some long-range-interaction solution-pairs

Table 3 shows properties and reaches that associate with 0d> uses of some Σg solution-pairs for which $1 \leq \Sigma \leq 4, 1 \leq k_{max} \leq 8$, and $\{5,7\} \cap Z_\Gamma = \emptyset$.

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0d>	Property	E_Σ or B_Σ	SL	n_I	R_I
1g1	Charge	E_1	1L	6	1
1g1‘2	Magnetic moment, assuming a spherical charge distribution	B_1	1L	6	1
1_3g1‘2	Magnetic moment correction for an oblate charge distribution	B_1	1L	6	1
2g2	Energy	$E_{2:1}$	2L	1	6
2g2‘4	Angular momentum, assuming a spherical energy distribution	$B_{2:2}$	2L	3	2
2_6g2‘4	Angular momentum correction for an oblate energy distribution	$B_{2:2}$	2L	3	2
2g1‘2‘3	Two distinct axes of moments of inertia	$E_{2:3}$	2L	6	1
2g$\cdots$	$\cdots$	$\cdots$	2L	$\cdots$	$\cdots$
3g3	f_{l-r}	E_3	3L	3	2
4g4	f_{B-L}	E_4	4L	6	1

Table 3. Properties and reaches that associate with 0d> uses of some Σg solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5,7\} \cap Z_\Gamma = \emptyset$. The symbol SL associates with a known LRI elementary particle or a might-be LRI elementary particle and with the notion that S equals Σ. Here, Σ associates with the notion (as in the leftmost column in the table) of either one of ΣgΓ or $\Sigma\ldots$gΓ and with one of E_Σ, B_Σ, $E_{\Sigma:n_\Gamma}$, and $B_{\Sigma:n_\Gamma}$. The next-to-rightmost column shows the number of instances for each pair - of one 0d> solution-pair and one (one cascade-step) 1d> solution-pair - that is relevant regarding SL. The rightmost column shows the reach for each instance - of one 0d> solution-pair and one (one cascade-step) 1d> solution-pair - that is relevant regarding SL.

3.12. Gravitational properties of objects

This unit discusses reaches that associate with various gravitational properties of objects.

GR associates a notion of repulsion between objects with a notion of pressure. POST associates a notion of DE with such a pressure.

Per Table 2, Eq. (14), and Eq. (15), SUPP suggests that an excitation of a field (such as the 2L field - or, the gravitational field) encodes knowledge of the isomer-related instances of the properties that associate with the excitation. In effect, the gravitational field carries knowledge of the isomers that associate with the excitation.

Regarding the active gravitational properties of an object A, the following notions can pertain. The number of instances, n_I, of gravitationally attractive 0d> monopole properties is one. The number of instances of gravitationally diluting 0d> dipole properties can be as many as three. The number of instances of gravitationally additive 0d> quadrupole properties can be as many as six. The number of instances of gravitationally diluting 0d> octupole properties can be as many as six. The number of instances of gravitationally additive 0d> 16-pole properties can be as many as six.

Regarding a one-isomer object, there are six possibilities regarding isomer. The following examples pertain regarding interactions between a sun (as a one-isomer object A) and a planet (as a one-isomer object C).

- To the extent that the sun exhibits only gravitationally active $R_I = 6$ aspects (or, monopole 0d> gravitational properties), SUPP suggests that POST modeling is not necessarily inadequate.
- To the extent that the sun exhibits gravitationally active $R_I \neq 6$ aspects, SUPP suggests that POST modeling might be inadequate. Nonzero rotation of an isomer-zero sun provides a basis for an example. 0d>:2g2`4::2 associates with nonzero rotation. A one-isomer planet that associates with isomer-zero or with isomer-three senses the rotation of the sun. The rotation affects the trajectory of the planet. A one-isomer planet that associates with isomer-one, isomer-two, isomer-four, or isomer-five does not sense the rotation of the sun. The rotation does not affect the trajectory of the planet.

In POST, the effective active gravitational properties of object A depend only on aspects of object A.

SUPP suggests that the effective active gravitational properties (of object A) that an object C senses depend on aspects of object A and on aspects of object C.

For a modeling case of a point-like (and possibly multi-isomer) object C interacting with the 2L field that associates with an object A, object C senses all (nonzero value of property) 2gΓ solution-pair components that associate with the 2L field that associates with object A. The weighting that associates with any one 0d> solution-pair associates with the geometric factor of the pole (monopole, dipole, or so forth) that associates with the 0d> solution-pair and (for a non-monopole component) with an orientation factor that associates with a tensor-like notion (vector for dipole, 2-tensor for quadrupole, and so forth) that associates with the 0d> solution-pair. (SUPP uses the word weighting to avoid possibly inappropriate conflation with POST notions such as probability and amplitude. This paper does not operationally define the one-word term weighting.) Per Eqs. (16) and (17), for ND modeling, the geometric factor associates with r^{-n_Γ}. Generally, possibly, effects that associate with one geometric factor or with two geometric factors dominate compared to effects that associate with other geometric factors.

SUPP suggests that the passive gravitational properties of an object equal the active gravitational properties of the object.

3.13. Inertial properties of objects

This unit discusses relationships between inertial properties of objects and gravitational properties of objects.

Discussion as to the extent to which inertial properties of objects equal gravitational properties of objects dates to around the 1680s and Ref. [2].

Based on the notion that $R_I = 6$ for 0d>:2g2, SUPP suggests the following notions. For POST ND, inertial mass equals active gravitational mass and equals passive gravitational mass. For POST SR, inertial energy equals active gravitational energy and equals passive gravitational energy.

Based on the notion that $R_I = 6$ for 1d>:2g2`4, SUPP suggests the following notion. For POST ND and for POST SR, inertial momentum equals active gravitational momentum and equals passive gravitational momentum.

3.14. Tetrads that associate with POST ND r^{n_V} potentials for which $n_V \geq 0$

This unit discusses the notion that some POST ND potentials point to tetrads that associate with aspects - such as three values of color charge - of POST that do not necessarily associate with POST notions of continuous variables. Such tetrads do not necessarily associate with the notion of trajectory-related tetrad.

Aspects of POST ND associate with some nonnegative integer values of n_V. (Discussion related to Eq. (16) defines n_V.)

$n_V = 2$ associates with ND notions of physical harmonic oscillators.

$n_V = 1$ associates with ND notions that might associate with the long-range potential that associates with the strong interaction. POST uses the two-word term asymptotic freedom.

$n_V = 0$ associates with an electromagnetic potential that an observer would attribute to a uniform background of charge, assuming that the background extends adequately broadly with respect to an observation from within the uniform background of charge. $n_V = 0$ associates also with a gravitational potential that an observer would attribute to a uniform background of mass, assuming that the background extends adequately broadly with respect to an observation from within the uniform background of mass.

SUPP posits that for each one of $n_V = 0$, $n_V = 1$, and $n_V = 2$ a notion of tetrad pertains. For this notion of tetrad, each of the four parameters (three triad parameters and one monad parameter) need not necessarily associate with a POST continuous variable or with the SUPP-suggested notion of trajectory-related tetrad.

Regarding $n_V = 0$, SUPP suggests that the triad associates with a notion of three isomer-pairs. Per discussion (in this paper) that leads to Eq. (18), SUPP suggests that nature includes six isomers of elementary particles other than LRI (or SL) elementary particles. (The LRI elementary particles are the 1L elementary particle (or, photon), the 2L possible elementary particle (or, graviton), a possible 3L elementary particle, and a possible 4L elementary particle.) One of the isomers associates with the POST CC notion of ordinary matter. Five of the isomers associate with some POST CC notions of dark matter. Modeling that associates with six isomers associates with the notion that six equals three times two. The factor of three (as in the number of isomer-pairs) associates with the three members of the triad. The factor of two associates with the SUPP notion that each non-SL elementary particle associates with a solution-pair for which $\Sigma = 0$. (For each elementary particle that is not a quark, one solution-pair pertains. For each elementary particle that is a quark, two solution-pairs pertain.) The notion that a solution-pair associates with two solutions associates with the factor of two in the number, six, of isomers. Within each isomer-pair, one isomer associates with the POST notion of left-handedness (as in left-handed matter elementary particles) and one isomer associates with the POST notion of right-handedness (as in right-handed matter elementary particles). Here, the word matter associates with notions of being more prevalent, compared to antimatter. SUPP suggests that isomer is a new (compared to POST) property. SUPP suggests that the monad associates with the notion that some elementary particles (the SL elementary particles) do not necessarily associate with just one isomer.

Regarding $n_V = 1$, SUPP suggests that the triad associates with the three POST QM color-charges (red, blue, and green) that POST QM associates with the strong interaction. The monad associates with the notion that objects other than quarks either (or both, depending on notions regarding modeling) exhibit no color charge or exhibit clear (or white) color charge.

Regarding $n_V = 2$, SUPP suggests (but does not necessarily require) the following notions. The triad associates with the notion of three spatial dimensions. The monad associates with the notion of one temporal dimension.

3.15. Minimal observable nonzero values for some properties of objects

This unit discusses the notion that, for some properties of objects, gaps exist between ranges of observable values.

For each one of charge, energy (or mass), momentum, and angular momentum, CM (including ND, SR, and GR) does not necessarily associate with a minimum magnitude for a nonzero observable value.

The SM associates with minimal nonzero magnitudes $|q_{min}|$ for nonzero values of some 0d> properties, such as charge. For nonzero-charge objects other than quarks, $|q_{min}| = |q_e|$. q_e denotes the charge of an electron. For some quarks, $|q_{min}| = (1/3)|q_e|$. For the other quarks, $|q_{min}| = (2/3)|q_e|$.

Based on the notions that velocity is continuous and charge is one component of a 4-vector, SR associates with the three nonabutting ranges of values for observed charge q that Eq. (20) shows.

$$q \leq -|q_{min}|,\ q = 0,\ \text{and } q \geq |q_{min}| \qquad (20)$$

3.16. Tetrads and elementary-particle aspects other than angular momentum states

This unit discusses tetrads that associate both with monopole components of long-range interactions and with properties of elementary particles.

SUPP posits that tetrad notions associate with elementary particles and with 0d> uses of the solution-pairs 1g1, 2g2, 3g3, and 4g4.

- The following notions pertain regarding 0d> uses of 1g1. For each one of the six isomers, there are three triad aspects - $Q = 1/3$, $Q = 2/3$, and $Q = 1$. Q denotes the magnitude of the charge (of the elementary particle) divided by the magnitude of the charge of the electron. For each one of the six isomers, there is one monad aspect - $Q = 0$.
- The following notions pertain regarding 0d> uses of 2g2. For each one of the six isomers, there are three triad aspects - $m_b = 0$, $m_b > 0$, and $m_f > 0$. m_b denotes the mass of a boson elementary particle. m_f denotes the mass of a fermion elementary particle. LRI elementary particles do not necessarily associate with single isomers. For LRI elementary particles, there is one monad aspect - $m_b = 0$.
- The following notions pertain regarding 0d> uses of 3g3. For each one of the three isomer-pairs, there are three triad aspects - $f_{l-r} = +1$, $f_{l-r} = 0$, and $f_{l-r} = -1$. The symbol f_{l-r} denotes - for fermion elementary particles - whether a matter particle (in the context of matter particle and antimatter

particle) elementary fermion is left-handed ($f_{l-r} = +1$), does not associate with handedness ($f_{l-r} = 0$, in which case the elementary fermion is its own antiparticle), or is right-handed ($f_{l-r} = -1$). For example, the isomer-zero electron associates with $f_{l-r} = +1$ and the isomer-zero positron associates with $f_{l-r} = -1$. POST suggests the possibility that neutrinos are Majorana fermions. For Majorana neutrinos, $f_{l-r} = 0$ would pertain. For non-fermion elementary particles, there is one monad aspect - $f_{l-r} = 0$.

- The following notions pertain regarding 0d> uses of 4g4. For each one of the six isomers, there are three triad aspects - $|f_{B-L}| = 0$, $|f_{B-L}| = 1/3$, and $|f_{B-L}| = 1$. The symbol f_{B-L} denotes - for fermion elementary particles - the POST notion of baryon number minus lepton number. POST suggests the possibility that neutrinos are Majorana fermions. For Majorana neutrinos, $f_{B-L} = 0$ would pertain. Regarding elementary fermions other than Majorana neutrinos, $|f_{B-L}| = 1/3$ for quarks and $|f_{B-L}| = 1$ for leptons. For non-fermion elementary particles, there is one monad aspect - $f_{B-L} = 0$.

Paralleling discussion related to Eq. (20), SUPP suggests that - for SR - f_{l-r} can associate with notions of being the scalar component of a 4-vector. Paralleling discussion related to Eq. (20), SUPP suggests that - for SR - f_{B-L} can associate with notions of being the scalar component of a 4-vector.

Table (4) lists some tetrads that SUPP suggests regarding elementary particles.

Association	Triad notion	n_I	R_I	Monad notion
ND $n_V = 1$	Color charges (red, blue, green)	6	1	The EP is not an EF
ND $n_V = 0$	Three isomer-pairs	3	2	The EP associates with one SL
$\Sigma = 1$ - 0d>:1g1	Charge ($Q = 1/3$, $Q = 2/3$, $Q = 1$)	6	1	The EP associates with $Q = 0$
$\Sigma = 2$ - 0d>:2g2	Mass ($m_b = 0$, $m_b > 0$, $m_f > 0$)	1	6	The EP associates with one SL
$\Sigma = 3$ - 0d>:3g3	EF f_{l-r} ($+1$, $0^\dagger$, -1)	3	2	The EP is not an EF
$\Sigma = 4$ - 0d>:4g4	EF $|f_{B-L}|$ ($0^\dagger$, $1/3$, 1)	6	1	The EP is not an EF
6 $\in$0d>:Z_Γ	Three EF flavours	6	1	The EP is not an EF

Table 4. Some tetrads that SUPP suggests regarding elementary particles. The acronym EP abbreviates the two-word term elementary particle. The acronym EF abbreviates the two-word term elementary fermion. The symbol † associates with the POST notion of Majorana fermion. The symbol 6 $\in$0d>:Z_Γ alludes to the notion that, for 0d>:ΣgΓ, $6 \in Z_\Gamma$.

3.17. A principle that associates with tetrad counts regarding properties of objects

This unit suggests a principle that links POST modeling and SUPP aspects.

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SUPP defines the number n'_Γ as follows. For properties that associate with ND notions of $n_V \geq 0$, $n'_\Gamma = 0$. For properties that associate with ND notions of $n_V \leq -1$ (and, hence, with $\Sigma \geq 1$), $n'_\Gamma = n_\Gamma = -n_V$.

SUPP uses the symbol n_T to denote the number of tetrads that associate with modeling for a property.

Eq. (21) shows a modeling principle that SUPP posits.

$$n_T = n'_\Gamma + 1 \qquad (21)$$

SUPP suggests that, for the combination of POST SR and SUPP, the notion of n'_Γ pertains and the notion of n_V does not necessarily pertain. SUPP suggests that, for the combination of POST GR and SUPP, the notion of n'_Γ pertains and the notion of n_V does not necessarily pertain.

For each instance of modeling that associates with a number n_T, exactly one triad is not a trajectory-related triad.

3.18. Conservation laws

This unit suggests new - compared to POST notions - conserved quantities and discusses numbers of instances of the applicability of various conservation laws.

Eq. (22) shows notation that SUPP uses to describe a reach that includes all six isomers and all appropriately related LRI phenomena.

$$R_I = 6 \uplus \qquad (22)$$

Table 5 lists some conserved quantities that POST includes or that SUPP suggests.

(a) SUPP interpretation.

Association	Conserved triad notion	n_I	R_I	Conserved monad notion
ND $n_V = 2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
ND $n_V = 1$	Color charge	6	1	-
$\Sigma = 1$ - 0d>:1g1	Charge	6	1	-
$\Sigma = 2$ - 0d>:2g2	Energy	1	$6\uplus$	-
$\Sigma = 2$ - 1d>:2g2‘4	Momentum	1	$6\uplus$	-
$\Sigma = 2$ - 0d>:2g2‘4 and 0d>:2_6g2‘4	Angular momentum	1	$6\uplus$	-
$\Sigma = 3$ - 0d>:3g3	EF f_{l-r}	3	2	-
$\Sigma = 4$ - 0d>:4g4	EF f_{B-L} †	6	1	-

(b) POST-like interpretation. POST does not necessarily include notions that associate with (at least) $n_I \geq 3$ and f_{l-r}.

Association	Conserved vector notion	n_I	R_I	Conserved scalar notion
ND $n_V = 2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
ND $n_V = 1$	-	6	1	Color charge
$\Sigma = 1$ - 0d>:1g1	-	6	1	Charge
$\Sigma = 2$ - 0d>:2g2	-	1	$6\uplus$	Energy
$\Sigma = 2$ - 1d>:2g2‘4	Momentum	1	$6\uplus$	-
$\Sigma = 2$ - 0d>:2g2‘4 and 0d>:2_6g2‘4	Angular momentum	1	$6\uplus$	-
$\Sigma = 3$ - 0d>:3g3	-	3	2	EF f_{l-r}
$\Sigma = 4$ - 0d>:4g4	-	6	1	EF f_{B-L} †

Table 5. Some conserved quantities that POST includes or that SUPP suggests. The rows for which ND $n_V = 2$ are speculative and might pertain to notions of space-time dimensions that are relevant for CM modeling and QM modeling. The symbol 2_6g2`4 associates with notions - for rotating objects - of oblateness and with the solution-pair 6g2`4. The acronym EF abbreviates the one-element term elementary-fermion. Conserved notions that SUPP suggests and POST does not necessarily include are numbers of dimensions and EF f_{l-r}. The symbol † associates with the POST notion that conservation of EF f_{B-L} would not pertain to the extent that neutrinos model as Majorana fermions.

For example, for an object and absent interactions that exchange charge with other objects, each of the six instances of charge is conserved.

Each of the rows - in Table 5 - for which the association column notes a value of Σ associates with n_I instances of a quantity that adds across components (including SL aspects) of a multi-component system.

3.19. Methods for cataloging elementary particles and for cataloging spin states

This unit discusses a method for cataloging elementary particles and a method for cataloging spin states of two-component systems.

3.19.1. *Perspective* - $\Sigma \geq 1$ *and* $\Sigma = 0$

Solution-pairs for which $\Sigma \geq 1$ can associate with pinpointing and cataloging some properties (such as electromagnetic properties and gravitational properties, but not strong interaction properties) of objects.

Table 1 alludes to solution-pairs for which $\Sigma = 0$. 0g1`2`3 is an example.

Solution-pairs for which $\Sigma = 0$ associate with $n_\Gamma \geq 3$. (For $1 \leq n_\Gamma \leq 2$, it is not possible arithmetically to have $\Sigma = 0$.)

Regarding POST modeling that treats an object as having components, SUPP associates the word system (or, the two-element phrase multi-component system) with the object.

For objects and systems, the following notions pertain. SUPP notions of $\Sigma \mathrm{g} \Gamma$ solution-pairs for which $\Sigma = 0$ can pertain. Notions of cascades can pertain. Notions of tetrads (and of the associated monads and triads) can pertain. Notions of series 0d, 1d, and so forth that associate with POST notions of temporal derivatives do not pertain. Eq. (21) does not necessarily pertain.

To facilitate discussing $\Sigma = 0$ cascades, SUPP uses the notation 0d0 (and not 0d>) and the notation 1d0 (and not 1d>). The 0d0 and 1d0 notation associates with $\Sigma = 0$ and contrasts with 0d> and 1d>, which associate with $\Sigma > 0$. For each relevant 0d0 use of a solution-pair, there is at least one relevant 1d0 use of a one-step-cascade solution-pair.

SUPP deploys the notation 0d0′ to associate with cases in which a notion pertains for both 0d> cases and 0d0 cases. SUPP deploys the notation 1d0′ to associate with cases in which a notion pertains for both 1d> cases and 1d0 cases.

3.19.2. *Catalogs - elementary particles and spin states*

SUPP posits the following cases.

1. All known elementary particles associate with 0d0 uses of solution-pairs that associate with zero-step cascades or more-than-zero-step cascades from solution-pairs for which $\Sigma = 0$, $n_\Gamma = 3$, $\{1,3\} \subset Z_\Gamma$, and $\{5,7\} \cap Z_\Gamma = \emptyset$. (Table 6 pertains regarding elementary particles.)
2. Spin states of two-component systems can associate with 0d0 uses of solution-pairs that associate with zero-step cascades or more-than-zero-step cascades from solution-pairs for which $\Sigma = 0$, $n_\Gamma = 4$, and $\{5,7\} \cap Z_\Gamma \neq \emptyset$. (Discussion related to Eq. (27) pertains regarding spin states of two-component systems.)

For each one of the above cases, SUPP posits the following notions. (For case 1, Table 6 and discussion related to Table 6 illustrate the notions. For case 2, discussion related to Eq. (27) illustrates the notions.) Here, the symbol 0d0: denotes the notion that the following Z_Γ set or the following n_Γ integer pertains for the 0d0 solution-pair and does not necessarily pertain for the 1d0 solution-pair. (Eq. (23) associates with aspects of Table 2. The use of 0d0′ - and not just 0d0 - associates with the notion that LRI elementary particles are bosons. Eq. (26) associates with aspects of Eq. (12).)

- Boson or fermion? - Eq. (23) pertains. (The symbol $\Leftrightarrow$ stands for the mathematical notion of if and only if.)

$$\text{The object is a fermion} \;\Leftrightarrow\; 6 \in \text{0d0'}{:}Z_\Gamma \qquad (23)$$

- Magnitude of spin? - For boson elementary particles and for boson states of two-component systems, 0d0 use of a solution-pair associates with the spin that Eq. (24) computes. For fermion elementary particles and for fermion states of two-component systems, 0d0 use of a solution-pair associates with the spin that Eq. (25) computes.

$$\text{If the object is a boson, } S = |(\text{0d0}{:}n_\Gamma) - 4| \qquad (24)$$

$$\text{If the object is a fermion, } S = |(\text{0d0}{:}n_\Gamma) - 4.5| \qquad (25)$$

- Charge or no charge? - Eq. (26) pertains. Q denotes the magnitude of the charge (of the object) divided by the magnitude of the charge of the electron.

$$Q > 0 \Leftrightarrow 4 \notin \text{0d0}{:}Z_\Gamma \;\text{ and }\; Q = 0 \Leftrightarrow 4 \in \text{0d0}{:}Z_\Gamma \qquad (26)$$

3.19.3. Cascades and the role of $6 \in \text{0d0}{:}Z_\Gamma$

Per Eq. (23), for boson states, $6 \notin \text{0d0}{:}Z_\Gamma$.

For boson states, per Eq. (24), a cascade - based on inserting an integer other than six - that adds one to n_Γ associates with changing S by minus one or plus one. Regarding POST modeling, the change associates with changing by - respectively, minus two or plus two - the number of states that can associate with QM angular momentum relative to an axis. For example, for an original S and a cascade that increases the spin to $S+1$, the original set of spin states equals $\{-S, -S+1, \ldots, S-1, S\}$ and 0d0 use of the set that associates with $S+1$ includes two additional states that associate, respectively, with $-(S+1)$ and $S+1$.

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A cascade from a boson state to a fermion state associates with inserting the integer six into 0d0:Z_Γ. The insertion associates with changing - by minus one or plus one - the number of states that can associate with QM angular momentum relative to an axis. For example, for an original S and a cascade that increases the spin to $S+0.5$, the original set of spin states equals {$-S$, $-S+1$, . . ., $S-1$, S} and 0d0 use of the set that associates with $S+0.5$ equals {$-(S+0.5), -(S-0.5), \ldots, S-0.5, S+0.5$}.

A cascade from a fermion state to a fermion state associates with inserting an integer other than six into 0d0:Z_Γ. For a cascade from $n_\Gamma = 4$ to $n_\Gamma = 5$, each one of a before-cascade solution-pair and an after-cascade solution-pair associates with $S = 0.5$. Otherwise, for an original S and a cascade that increases the spin to $S+1$, the original set of spin states equals {$-S$, $-S+1$, . . ., $S-1$, S} and 0d0 use of the set that associates with $S+1$ includes two additional states that associate, respectively, with $-(S+1)$ and $S+1$.

3.20. Some spin-related properties of a two-component system and its two components

This unit discusses SUPP modeling regarding two-component systems.

3.20.1. Spins S regarding two-component systems

Here, the notion of 0d0:0g pertains. The notion of an upper limit on k_{max} does not pertain. Regarding spin S, the notion that $2S$ is a nonnegative integer pertains. The range $0 \le S < \infty$ pertains. Eq. (23) pertains.

Per Eq. (24) and Eq. (27), 0d0 use of the solution-pair 0g1\`3\`5\`7 associates with $S = 0$.

$$0 = |+1-3-5+7| \qquad (27)$$

For an integer l, SUPP uses the notation $+l\hat{}$ to denote the series to which Eq. (28) alludes. Each item in the series totals to l.

$$+l\hat{}\ \text{denotes the series}\ +l,\ -l+2l,\ -l-2l+4l,\ -l-2l-4l+8l,\ \cdots \qquad (28)$$

SUPP uses the notation $+l_n\hat{}$ to denote the item in Eq. (28) that includes exactly n terms. For example, $+l_2\hat{}$ denotes , $-l+2l$.

The following three sets of solution-pairs associate with $\Sigma = 0$ and with $4 \notin Z_\Gamma$. Eq. (28) defines $+8\hat{}$ and $+16\hat{}$. Regarding the notion of 0d0:0gΓ, for each solution-pair, the integers shown below (or alluded to by the series just above) appear in Γ and no other integers appear in Γ. For each set, for other than the first solution-pair, each solution-pair cascades from the first solution-pair.

1. $|-1-2-5+8|$, $|-1-2-5-8+16\hat{}|$,$|+1+2-5-6+8|$, and $|+1+2-5-6-8+16\hat{}|$
2. $|+2-3-7+8|$, $|+2-3-7-8+16\hat{}|$, $|+2+3-6-7+8|$, and $|+2+3-6-7-8+16\hat{}|$.
3. $|+1-3-5+7|$, $|-1-3+5-7+8\hat{}|$, $|-1-3+5-6+7|$, and $|-1-3-5+6-7+8\hat{}|$.

For each set, SUPP suggests that Eq. (24) pertains regarding the first two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an even integer. For each set, SUPP suggests that Eq. (25) pertains regarding the second two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an odd integer.

SUPP suggests, regarding a two-component system, that $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$ can associate with one component, $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the other component, and $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the system.

Removal of (just) the criterion that $4 \notin Z_\Gamma$ results in the following notions. Regarding $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, $n_\Gamma = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-2-4+5|$, $|-1-2-5+8|$, $|+1-4-5+8|$, and $|+2-3-4+5|$. Regarding $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, $n_\Gamma = 4$ solution-pairs that might associate with $S = 0$ associate with $|-1-2-4+7|$, $|+2-3-7+8|$, and $|+3-4-7+8|$. Regarding $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$, $n_\Gamma = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-3-5+7|$ and $|+2-4-5+7|$. The numbers of $S = 0$ solution-pairs are four for the case of $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, three for the case of $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, and two for the case of $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$.

The discussion above de-emphasizes the notion that a solution-pair associates with two solutions. SUPP suggests - but this paper does not explore - the following notion. For some modeling, one linear combination of the two solutions might associate with S - as in the POST notion of $S(S+1)\hbar^2$ - and another (perhaps orthogonal to the previous) linear combination of the two solutions might associate with s - as in the POST notion of the $s \in \{-S, -S+1, \cdots, S-1, S\}$ that pertains regarding measuring an angular momentum $s\hbar$ with respect to a spatial axis.

3.20.2. Spins S regarding atoms

SUPP suggests that modeling can treat an atom as a two-component system. One component is the nucleus of the atom. The other component is the electron cloud.

For an atom, the nucleus has a nonzero charge and (unless the atom is fully ionized) the electron cloud has a nonzero charge. Based on Eq. (26), SUPP suggests that the following notions might pertain. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|-1-2-5+8|$ associate with spins S of the electron cloud. The solution-pair that associates with $|+1-2-4+5|$ associates with the spin $S=0$ state of the electron cloud if the atom is an ion that has no electrons. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-3-7+8|$ associate with spins S of the nucleus. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-4-5+7|$ associate with spins S of the atom, if the atom (is not an ion and thus) has a charge of zero. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+1-3-5+7|$ associate with spins S of the atom, if the atom (is an ion and thus) has a nonzero charge.

3.21. Electromagnetic properties and events that associate with atoms or stars

This unit suggests that detectors based on OM stuff can detect electromagnetic atomic radiation (but not necessarily electromagnetic thermal radiation) emitted by stuff that associates with one DM isomer.

3.21.1. Electromagnetic events associating with two-component systems such as atoms

SUPP posits that data that underlie Table 12a suggest Eq. (29). The symbol $(R_I)_{\mathrm{ef}}$ denotes a notion of an effective reach.

$$\text{Regarding electromagnetism: for atomic states and atomic transitions, } (R_I)_{\mathrm{ef}} = 2 \qquad (29)$$

SUPP suggests that, for 0gΓ solution-pairs for which 0d0:n_Γ is greater than or equal to four, $\{5,7\} \subset$ 0d>:Z_Γ associates with states of (at least some) two-component systems.

SUPP posits that, for $\{5,7\} \subset$ 0d>:Z_Γ, 1g solution-pairs can associate with electromagnetic properties of objects that model as being two-component systems. Modeling - for an atom and electromagnetism that is external to the atom - can associate with the position (of the atom), the ionization state (or net charge of the atom), the principal energy levels (for the electron cloud of the atom), the fine-structure state

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(regarding the electron cloud), the hyperfine state (regarding the spin of the electron cloud and the spin of the atomic nucleus), and the atomic weight (or, numbers of protons and neutrons in the atomic nucleus). SUPP posits - based on that list of six properties - that $n_\Gamma = 5$ pertains for relevant 0d>:1g Γ solution-pairs.

Arithmetically, the relevant 0d>:1gΓ solution-pairs can associate with 0d>:1g2`3`5`7`8x and with 0d>:1g1`4`5`7`8x.

The solution-pairs 0d>:1g2`3`5`7`8x associate with the expressions $1 = |-2-3+5-7+8|$ and $1 = |+2+3-5-7+8|$ and with 1d> solution-pairs 1d>:1g2`3`4`5`7`8x and 1d>:1g2`3`5`6`7`8x.

The solution-pairs 0d>:1g1`4`5`7`8x associate with the expressions $1 = |-1-4+5-7+8|$, $1 = |-1-4-5-7+8|$, and $1 = |+1-4-5-7+8|$ and with 1d> solution-pairs 1d>:1g1`4`5`6`7`8 and 1d>:1g1`2`4`5`7`8x.

SUPP posits the following notions.

- 0d>:1g2`3`5`7`8x solution-pairs associate with the charge of the electron cloud and with the charge of the atomic nucleus. Here, $R_I = 6$. However, SUPP suggests (per discussion below regarding the evolution of stuff that associates with dark matter isomers) that isomer-one stuff, isomer-two stuff, isomer-four stuff, and isomer-five stuff do not form counterparts to isomer-zero atoms and isomer-three atoms. Regarding atomic physics, SUPP suggests that 0d>:1g2`3`5`7`8x solution-pairs associate with $(R_I)_{\text{ef}} = 2$.
- 0d>:1g1`4`5`7`8x associates, via the expression $1 = |-1-4+5-7+8|$, with electron-cloud principal energy levels and with electromagnetic interactions via which the atom transits to new principal energy-level states.
- 0d>:1g1`4`5`7`8x associates, via the expressions $1 = |-1-4-5-7+8|$ and $1 = |+1-4-5-7+8|$, with fine-structure energy levels and hyperfine energy levels and with electromagnetic interactions via which the atom transits to new fine-structure and hyperfine energy-level states.

For each one of the solution-pairs that associates with 0d>:1g2`3`5`7`8x or with 0d>:1g1`4`5`7`8x, $(R_I)_{\text{ef}} = 2$.

SUPP suggests that the notions above associate with the notion that detectors based on OM stuff can detect electromagnetic atomic radiation emitted by stuff that associates with one DM isomer, isomer-three.

3.21.2. Electromagnetic events that associate with stars

SUPP suggests that stars tend to associate with single isomers. For a one-isomer star, SUPP suggests the following contributions to the electromagnetic field.

If the star has a net nonzero charge, some contributions to the electromagnetic field associate with the 0d>:1g1 and 1d>:1g1`2 properties of the star. The relevant reach, R_I, for electromagnetic effects is one isomer.

Regarding modeling for stellar radiation (such as thermal radiation) from the star, SUPP suggests that 5 $\in$0d>:Z_Γ might associate with the bulk of the star and 7 $\in$0d>:Z_Γ might associate with the corona of the star. Thermal radiation might associate with, for example, 0d>:1g4`5 or 0d>:1g1`2`3`4`5x. For each one of those two cases, the relevant reach, R_I, for the emitted electromagnetic radiation is one isomer. $R_I = 1$ comports with the notion that ordinary matter seems not to detect thermal radiation emitted by stars that (if they exist) contain essentially no ordinary matter.

3.21.3. Implications regarding cosmic background radiation and sensing dark matter

Per discussion related to Eq. (29), components of cosmic (electromagnetic) background radiation that associate with creation (of electromagnetic radiation) via atomic phenomena can associate with a reach R_I of two isomers. Components of cosmic (electromagnetic) background radiation that associate with creation via other phenomena can associate with a reach R_I of one isomer.

SUPP suggests that electromagnetic radiation that associates with creation via single-isomer atomic phenomena can associate with a reach R_I of two. SUPP suggests that detectors that have bases in OM stuff can detect such radiation created by isomer-zero atomic phenomena or by isomer-three atomic phenomena. Discussion related to Table 12a
suggests that detectors that have bases in OM stuff may have detected electromagnetic radiation created by isomer-three atomic phenomena.

4. Results

This unit suggests (based on POST modeling and SUPP modeling) explanations for data that POST modeling alone does not necessarily explain. This unit suggests possible future data.

4.1. A catalog of elementary particles

This unit shows a catalog of all elementary particles of which people know or that SUPP suggests.

This unit associates with 0g, $\{1, 3\} \subset Z_\Gamma$, and $\{5, 7\} \cap Z_\Gamma = \emptyset$. This unit associates with perspective and terminology centric to OM. (Generally, regarding the other five isomers, similar notions pertain.)

SUPP uses the following notions to catalog elementary particles. A symbol of the form SΦ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S associates with the POST expression S(S+1)$\hbar^2$ that associates with angular momentum. Regarding POST, known values of S include 0, 0.5, and 1. The symbol Φ associates with a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of the charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, SUPP omits - from the symbols for families - the symbol Q. Regarding quarks, SUPP uses the symbol $\mathrm{Q}_{>0}$ to associate with cases for which either one of $\mathrm{Q}_{1/3}$ or $\mathrm{Q}_{2/3}$ pertains.

Table 6 catalogs all known elementary particles and some elementary particles that SUPP suggests nature might include.

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0d0	Names	Families	n_{EP}	Q	m	1d0
$\|-1-3+4\|$	Z	1Z	1	0	>0	$\|+1-2-3+4\|$; $\|-1-3-4+8\|$; $\|+1-3-4+6\|$.
$\|-1-2+3\|$	W	$1\mathrm{W}_1$	1	1	>0	$\|+1-2-3+4\|$; $\|-1-2-3+6\|$.
$\|+1-2-3+4\|$	Higgs boson	0H	1	0	>0	$\|+1-2-3-4+8\|$; $\|-1+2-3-4+6\|$.
$\|-1-3-4+8\|$	Inflaton	0I	1 †	0	=0	$\|+1-2-3-4+8\|$; $\|-1-3-4-8+16\|$; $\|-1+3-4-6+8\|$.
$\|+1-3-4+6\|$	Neutrinos	0.5N	3	0	>0	$\|-1+3-4-6+8\|$; $\|-1+2-3-4+6\|$.
$\|-1-2-3+6\|$	Charged leptons	$0.5\mathrm{C}_1$	3	1	>0	$\|-1+3-4-6+8\|$; $\|-1+2-3-4+6\|$.
$\|-1+2-3-6+8\|$ and $\|-1+2-3-4+6\|$	Quarks	$0.5\mathrm{Q}_{2/3}$; $0.5\mathrm{Q}_{1/3}$	6	1, 0	>0	$\|-1-2-3+4-6+8\|$, $\|+1-2+3-4-6+8\|$.
$\|+1-2-3-4+8\|$	Gluons	1G	8	0	=0	$\|+1-2+3-4-6+8\|$, $\|-1-2-3+4-6+8\|$.
$\|+1-2-3-4+8\|$	Jay	1J	1 †	0	=0	$\|+1-2+3-4-6+8\|$, $\|-1-2-3+4-6+8\|$.
$\|-1-3-4-8+16_1\hat{}\|$	Photon	1L	1	0	=0	$\|-1-3-4-8+16_2\hat{}\|$, $\|+1-2-3-4-8+16_1\hat{}\|$, $\|-1+3-4-6-8+16_1\hat{}\|$.
$\|-1-3-4-8+16_S\hat{}\|$, with S being the S in SL	Graviton, TBD, ...	2L, 3L, ...	1 †, 1 †, ...	0, 0, ...	=0, =0, ...	$\|-1-3-4-8+16_{(S+1)}\hat{}\|$, $\|+1-2-3-4-8+16_S\hat{}\|$, $\|-1+3-4-6-8+16_S\hat{}\|$

Table 6. All known elementary particles and some elementary particles that SUPP suggests nature might include. For each one of the 0d0 column and the 1d0 column, the table alludes to solution-pairs by using sums that echo Eq. (4) and Eq. (5), rather than by directly deploying 0gΓ notation and listing the elements of Γ. The leftmost three columns provide information about elementary particles. The three charged leptons are the electron, the muon, and the tau. n_{EP} denotes the number of elementary particles. Q is a magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron). m denotes mass. Regarding the $0.5\mathrm{Q}_{2/3}$ family of three quarks, SUPP posits that a notion of two-thirds times the $Q=1$ 0d0 solution-pair plus one-third times the $Q=0$ 0d0 solution-pair pertains. Regarding the $0.5\mathrm{Q}_{1/3}$ family of three quarks, SUPP posits that a notion of one-third times the $Q=1$ 0d0 solution-pair plus two-thirds times the $Q=0$ 0d0 solution-pair pertains. The symbol † denotes that the elementary particles are as-yet unfound. The word inflaton associates with POST notions of a possible inflaton elementary particle. 2L cascades from 1L, 3L cascades from 2L, and so forth. The acronym TBD abbreviates the three-word phrase to be determined. Eq. (28) and related remarks define notation of the form $+16_n\hat{}$. The table de-emphasizes (but SUPP does not necessarily rule out) the possibilities that - for each one of some S ≥ 1 - 0d0 use of the solution-pair $|+1-2-3-4-8+16_{\mathrm{S}}\hat{}|$ associates with an elementary boson that has spin S+1. Such elementary bosons might associate with notions of (S $+$ 1)G families or (S $+$ 1)J families. The table de-emphasizes (but SUPP does not necessarily rule out) the possibilities that - for each one of some S ≥ 1 - 0d0 uses of combinations of the solution-pair $|-1+3-4-6-8+16_{\mathrm{S}}\hat{}|$ and the solution-pair

$|-1-2-3-4-6+16_{(\mathrm{S}-1)}\hat{}\,|$ associate with elementary fermions that have spins of $\mathrm{S}+0.5$. Such elementary fermions might associate with notions of $(\mathrm{S}+0.5)\mathrm{Q}_{>0}$ families. In this table, each 0d0 solution-pair for which $n_\Gamma \geq 4$ cascades (in $n_\Gamma - 3$ steps) from (at least) one of solution-pair $|-1-3+4|$ and solution-pair $|-1-2+3|$.

SUPP suggests the following notions regarding properties of elementary particles that associate with 0d0 solution-pairs that Table 6
shows. (These notions supplement notions that discussion related to Eq. (23) suggests.)

- Mass or no mass? - Eq. (30) pertains. The symbol m associates with the property of mass.

$$m = 0 \Leftrightarrow (6 \notin 0\mathrm{d}0{:}Z_\Gamma \text{ and } 8 \in 0\mathrm{d}0{:}Z_\Gamma) \qquad (30)$$

- Number of fermion flavours? - For fermion elementary particles, each 0d0 use of a solution-pair associates with $6 \in Z_\Gamma$ and with three flavours. (Per Eq. (23), $6 \in 0d0 : Z_\Gamma$ associates with the notion of fermion.)
- The magnitude of charge (that associates with a relevant solution-pair)? Eq. (31) pertains (and is more specific than Eq. (26)).

$$Q = 1 \Leftrightarrow 4 \notin 0\mathrm{d}0{:}Z_\Gamma \;\text{ and }\; Q = 0 \Leftrightarrow 4 \in 0\mathrm{d}0{:}Z_\Gamma \qquad (31)$$

- LRI elementary particle or not an LRI elementary particle? For elementary particles, Eq. (32) pertains if and only if the elementary particle is an LRI elementary particle.

$$(\{1,3,4,8,16\} \subset 0\mathrm{d}0{:}Z_\Gamma) \text{ and } (\{2,6\} \cap 0\mathrm{d}0{:}Z_\Gamma = \emptyset) \qquad (32)$$

SUPP suggests the following notion regarding properties of elementary particles that associate with 1d0 (or, one-step cascade) solution-pairs that Table 6 shows.

- Can model (in POST) as existing independently of other elementary particles? - Eq. (33) pertains.

$$\text{The object cannot model as independent } \Leftrightarrow \{1,2,3,4,6,8\} \subset 1\mathrm{d}0{:}Z_\Gamma \qquad (33)$$

Each one of the quarks, the gluons, and the might-be jay boson associates with the same pair of one-step-cascade solution-pairs and associates with the strong interaction. Regarding the gluons and the jay boson, the two one-step-cascade solution-pairs associate with eight gluons and one might-be jay boson. (Here, each one-step-cascade solution-pair might associate with three - as in triad - tetrad-related aspects. Two instances of three tetrad-related aspects might associate with nine - as in three times three - elementary particles. Discussion that includes Eq. (92) suggests that a - perhaps new to POST - use of an aspect of group theory might pertain.) SUPP suggests that the jay boson might associate with repulsive aspects of the residual strong force. SUPP suggests that the jay boson might associate with Pauli repulsion between like fermions, whether the like fermions are elementary fermions or are not elementary fermions.

SUPP suggests the following statements regarding interactions that involve elementary particles.

- Any elementary boson that associates with a 1d0 solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ can transform into a pair of elementary bosons that are similar to each other and are not similar to the original elementary boson. For example, a Z boson can transform into two photons. For the W boson, there is no 1d0 solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. The W boson cannot transform into two (hypothetical) elementary particles for which each of the two produced elementary particles would associate with $Q = 0.5$.
- Any elementary boson that does not associate with a 1d0 solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ cannot directly transform into a pair of elementary bosons that are similar to each other and are not similar to the original elementary boson. For the gluons, there is no 1d0 solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. Gluons do not transform directly into, for example, pairs of photons.
- Any elementary boson that associates with a 1d0 solution-pair for which $6 \in Z_\Gamma$ can transform into a pair of elementary fermions. For example, a Z boson can transform into two elementary fermions that are antiparticles to each other. The W boson can transform into a pair of fermions (for example, an electron and a neutrino). The W boson is the only elementary boson that cannot transform into two elementary fermions that are antiparticles to each other.

SUPP associates the symbol that Eq. (34) shows with a possible maximum spin S for LRI elementary particles.

$$\mathrm{S}_{max,\mathrm{L}} \qquad (34)$$

Each one of the following two notions might suggest that $S_{max,\mathrm{L}}$ is no greater than four. The first notion associates with the following sentence. Discussion related to Eq. (91) suggests the limit $n_0 \leq 3$ and hence a limit of $\Sigma \leq 4$ regarding the relevance of 0d>:ΣgΓ solution-pairs for which Σ is the only element in the list Γ. The second notion associates with the following sentences. Eq. (41) suggests that the solution-pair 1g1 associates with an interaction strength that includes a factor of four and that the solution-pair 2g2 associates with an interaction strength that includes a factor of three. Extrapolation suggests that the solution-pair 3g3 associates with an interaction strength that includes a factor of two, that the solution-pair 4g4 associates with an interaction strength that includes a factor of one, and that the solution-pair 5g5 would associate with an interaction strength that includes a factor of zero.

Ref. [38] notes that QFT suggests that zero-mass elementary particles might not have spins that exceed two. SUPP suggests that each phenomenon for which SUPP suggests an explanation has at least one SUPP-based explanation that can comport with the possibility that $\mathrm{S}_{max,\mathrm{L}} \leq 2$. For example, a basis for baryon asymmetry can be associated with Eq. (60) and at least one 1g′ solution-pair (such as 0d> use of 1g1`2`4). (The notion that 3g3 might not be associated with an elementary particle does not disturb the possibility that Eq. (60) is associated with a mechanism that produced baryon asymmetry. The notion that 3g3 and 4g4 might not be associated with elementary particles does not necessarily disturb notions that Table 4 and Table 5 discuss.) This paper does not explore the extent to which multi-isomer analogs to QFT might suggest limits other than $\mathrm{S}_{max,\mathrm{L}} \leq 2$ regarding the spins of zero-mass elementary particles.

4.2. Relationships among properties of boson elementary particles

This unit discusses interrelationships that SUPP suggests pertain regarding properties of boson elementary particles and points to a notion - that associates with integers N' - that might portend new physics that this paper does not fully develop.

This unit is associated with perspective and terminology centric to OM. (Generally, regarding the other five isomers, similar notions pertain.)

4.2.1. Relationships among the masses of nonzero-mass elementary bosons

SUPP suggests that Eq. (35) pertains regarding the masses of the nonzero-mass elementary bosons.

$$(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{Higgs}})^2 :: 7 : 9 : 17 \qquad (35)$$

Eq. (35) is not inconsistent with data. Based on information that Ref. [83] provides, the following notions pertain. The most accurately known of the three masses is m_{Z}. Based on the nominal value of m_{Z}, the nominal value (that Eq. (35) suggests) for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}. Based on the nominal value of m_{Z}, the nominal value (that Eq. (35) suggests) for m_{W} is within 3.6 experimental standard deviations of m_{W}. Based on information that Ref. [84] provides, the following notions pertain. Based on the nominal value of m_{Z}, the nominal value (that Eq. (35) suggests) for m_{W} is within 1.1 experimental standard deviations of m_{W}. (Ref. [84] does not provide new information about m_{Higgs}.) Based on the nominal value of m_{Z} that Ref. [84] suggests and on information that Ref. [83] provides about m_{Higgs}, the nominal value that Eq. (35) suggests for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}.

SUPP suggests that Eq. (35) might point to possible insight regarding - and a possible extension to - the POST notion of the weak mixing angle.

4.2.2. Links between properties of all known and SUPP-suggested elementary bosons

Table 7 discusses SUPP-suggested relationships between properties of all known elementary bosons and some elementary bosons that SUPP suggests.

Elementary boson(s)	Family	Status	M'	S'	Q'	μ'	$\sum(X')^2$	T'	N'
Higgs boson	0H	Known	$\sqrt{17}$	0	0	0	17	1	4
Z	1Z	Known	$\sqrt{9}$	1	0	0	10	1	3
W	$1\mathrm{W}_1$	Known	$\sqrt{7}$	1	1	1	10	1	3
Photon	1L	Known	0	1	0	0	1	0	1
Gluons	1G	Known	0	1	0	0	1	0	1
Inflaton	0I	SUPP-suggested	0	0	0	0	0	0	0
Jay	1J	SUPP-suggested	0	1	0	0	1	0	1
Graviton	2L	SUPP-suggested	0	2	0	0	4	0	2
TBD	3L	SUPP-suggested	0	3	0	0	9	0	3
TBD	4L	SUPP-suggested	0	4	0	0	16	0	4

Table 7. SUPP-suggested relationships between properties of all known elementary bosons and some elementary bosons that SUPP suggests. The symbol M' denotes the mass (in units of $m_{\mathrm{Z}}/3$) of the elementary boson. The symbol S' denotes the spin that POST associates with the expression $S'(S'+1)\hbar^2$. The symbol Q' denotes the magnitude (in units of the magnitude $|q_e|$ of the charge - q_e - of the electron) of the charge of the elementary boson. The symbol μ' denotes the magnitude of the magnetic moment divided by the magnetic moment of the W boson. The column with the label $\sum(X')^2$ shows the sum of the squares of the numbers to the left of the column. The symbol T' denotes one, if the mass is nonzero. (Here, POST suggests that the notion of longitudinal polarization pertains). The symbol T' denotes zero, if the mass is or would be zero. (Here, POST suggests or presumably would suggest that the notion of longitudinal polarization does not pertain.) The symbol N' denotes a positive integer for which $\sum(X')^2$ equals the sum of the squares of the numbers to the right of the $\sum(X')^2$ column. The table omits possible SG and SJ elementary bosons for which S exceeds one. The table omits possibilities for SL elementary bosons for which S exceeds four.

SUPP suggests that Eq. (36) pertains for each elementary boson to which Table 6 alludes.

$$(N')^2 \equiv (M')^2 + (S')^2 + (Q')^2 + (\mu')^2 - (T')^2 \qquad (36)$$

For each known or SUPP-suggested elementary boson, the notion that N' is an integer is not inconsistent with data.

SUPP suggests that Eq. (37) and Eq. (38) pertain for boson elementary particles. (SUPP does not suggest any elementary bosons for which $0\text{d}0{:}n_\Gamma \leq 2$.)

$$M' > 0 \;\Leftrightarrow\; N' = (0\text{d}0{:}n_\Gamma)\;; \quad M' = 0 \;\Leftrightarrow\; N' = (0\text{d}0{:}n_\Gamma) - 4 \qquad (37)$$

$$N' \in \{0, 1, 2, 3, 4\} \qquad (38)$$

For each elementary boson to which Table 6 alludes, SUPP suggests that Eq. (39) pertains. (SUPP does not suggest any elementary bosons for which $M' > 0$ and $N' \leq 2$.)

$$M' > 0 \;\Leftrightarrow\; N' = 4 - S' \geq 3\;; \quad M' = 0 \;\Leftrightarrow\; N' = S' \qquad (39)$$

4.3. Relationships among properties of fermion elementary particles

This unit discusses relationships that SUPP suggests pertain regarding properties of fermion elementary particles. This unit points to a notion - that associates with integers N' - that might portend new physics that this paper does not fully develop.

This unit associates with perspective and terminology centric to OM. (Generally, regarding the other five isomers, similar notions pertain. However, SUPP suggests that - for DM counterparts to OM charged leptons - different associations between flavour and mass can pertain.)

4.3.1. A relationship between the tau mass, electron mass, and strengths of two forces

Regarding charged leptons, SUPP suggests a link between the strength of electromagnetism and the strength of gravity.

Eq. (40) and Eq. (41) define, respectively, β' and β. m_τ denotes the mass of the tau particle (which is a charged lepton). m_e denotes the mass of the electron (which is a charged lepton). The right-hand side of Eq. (41) is the ratio of the electrostatic repulsion between two electrons to the gravitational attraction between the two electrons. The ratio does not depend on the distance between the two electrons.

$$\beta' \equiv m_\tau / m_e \qquad (40)$$

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$$(4/3) \cdot (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \qquad (41)$$

Based on data, $\beta \approx 3477.1891 \pm 0.0226$. (Ref. [83]

provides the relevant underlying data.) The standard deviation is associated almost entirely with the standard deviation for G_N, the gravitational constant.

Eq. (42) shows an equality that SUPP suggests.

$$\beta' = \beta \qquad (42)$$

Eq. (43) results from Eq. (42). The standard deviation is associated almost entirely with the standard deviation for G_N.

$$m_{\tau,\,\text{calculated}} \approx 1776.8400 \pm 0.0115 \text{ MeV}/c^2 \qquad (43)$$

Eq. (43) comports with data. More than eight standard deviations fit within one standard deviation from the data-based nominal value for m_τ.

Eq. (44) suggests (based on Eq. (42)) a restatement - that reflects (just) strengths of (electromagnetic and gravitational) interactions between like particles - of Eq. (41). One might also note that the charge q_τ of the tau equals q_e.

$$(4/3) \cdot ((G_N(m_\tau)^2)/(G_N(m_e)^2))^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \qquad (44)$$

4.3.2. A formula that might approximately link the masses of all elementary fermions

SUPP suggests a formula that might approximately link the masses of all elementary fermions.

For each charged elementary fermion, Table 8 provides a value of an integer l_m. For each charged elementary fermion, Table 8 provides a value of Q. Each value of Q is one of $Q = 1$, $Q = 2/3$, and $Q = 1/3$. The formula $l_q = 3Q$ defines l_q.

Eq. (45) defines $m(l_m, l_q)$ and has bases in the equations that immediately follow Eq. (45). Eq. (46) defines the fine-structure constant. m_μ denotes the mass of the muon, which is a charged lepton. Eq. (51) has bases in trying to fit data.

$$m(l_m, l_q) \equiv m_e \cdot (\beta^{1/3})^{l_m + j_m w_m(2)} \cdot (\alpha^{-1/4})^{(1+l_m)n_q + j_q w_q(l_m)} \qquad (45)$$

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c) \qquad (46)$$

$$j_m = 0, +1, 0, -1 \text{ for, respectively, } l_m \bmod 3 = 0, 1, 3/2, 2\text{; with } 3/2 \bmod 3 \equiv 3/2 \qquad (47)$$

$$w_m(2) = 2 - (\log(m_\mu/m_e)/\log(\beta^{1/3})) \approx 3.840613 \times 10^{-2} \qquad (48)$$

$$n_q = 0, 3/2, 3/2, 3/2, 3/2, \text{for, respectively, } l_q = 3, 2, 3/2, 1, 0 \qquad (49)$$

$$j_q = 0, -1, 0, +1, 0 \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \qquad (50)$$

$$w_q(0) \sim 0.324,\ w_q(1) \sim -1.062,\ w_q(2) \sim -1.509 \qquad (51)$$

$$w_q(l_m) = 0 \text{ for } l_m \leq -1 \text{ and for } l_m \geq 3 \qquad (52)$$

For each pair - of one l_m and one l_q - that associates with a known charged elementary fermion, discussion related to Table

8 suggests that $m(l_m, l_q)$ approximates the mass of the respective charged elementary fermion.

SUPP suggests that the number of seemingly independent not-necessarily-rational numbers input into the above equations is seven. For example, the list consisting of m_e, m_μ, β, α, $w_q(0)$, $w_q(1)$, and $w_q(2)$ includes seven not-necessarily-rational numbers.

4.3.3. Nominal properties of known charged elementary fermions

Table 8 shows information about properties of all known charged fermion elementary particles. (Ref. [83] provides the data that underlie Table 8.) Regarding similar tables for each one of isomer-one, isomer-two, isomer-four, and isomer-five, SUPP suggests (per Table 9) that the values of l_f that Table 8 shows for the charged leptons are not appropriate. For example, for isomer-two, the l_f values in the leftmost column would be 3 (for the row for which - for quarks - $l_f = 1$), blank (for the row for which - for quarks - $l_f = 2$), 1 (for the row for which - for quarks - $l_f = 3$), and 2 (for the remaining row).

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l_f (0.5C$_1$)	l_f (0.5Q$_{>0}$)	l_m	$Q=1$	$Q=2/3$	$Q=1/2$	$Q=1/3$
1 (Electron)	1 (Up, Down)	0	0.00 Electron	0.63 Up	0.80 †	0.97 Down
-	2 (Charm, Strange)	1	1.23 †	3.40 Charm	2.83 †	2.26 Strange
2 (Mu)	3 (Top, Bottom)	2	2.32 Muon	5.53 Top	4.72 †	3.91 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

Table 8. Approximate values of $\log_{10}(m(l_m, l_q)/m_e)$ for all known charged fermion elementary particles. Regarding flavour, this table generalizes, based on POST terminology that associates with charged leptons and with neutrinos. (For example, POST uses the term electron-neutrino.) In this table, the symbol l_f numbers the three flavours. The "l_f (0.5C$_1$)" terms pertain to fermions in the 0.5C$_1$ family. The symbol 0.5Q$_{>0}$ denotes the pair 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$. The "l_f (0.5Q$_{>0}$)" terms pertain to quarks (or, elementary particles in the two families 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$). l_m is an integer parameter. The domain $-6 \leq l_m \leq 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. The family 0.5C$_1$ associates with $Q=1$. The family 0.5Q$_{2/3}$ associates with $Q=2/3$. The family 0.5Q$_{1/3}$ associates with $Q=1/3$. Regarding this table, $l_q = 3Q$ pertains. Regarding the rightmost four columns, items show $\log_{10}(m(l_m, l_q)/m_e)$ and - for particles that nature includes - the name of an elementary fermion. For each † case, no particle pertains. Each number in the column with the label $Q=1/2$ equals the average of the number in the $Q=2/3$ column and the number in the $Q=1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q=2/3$ particle and the mass of the $Q=1/3$ particle. Regarding each † case, Eq. (45) provides the number $m(l_m, l_q)$.

For each charged elementary fermion except the top quark, Eq. (45) suggests a mass that is within one experimental standard deviation of the nominal mass that Ref. [83] reports. Ref. [83] alludes to three estimates for the mass of the top quark. Eq. (45) provides a mass (for the top quark) that is within 4.4 standard deviations below the nominal mass that associates with direct measurements, within 4.3 upward standard deviations above the nominal mass that associates with cross-section measurements, and within 1.6 standard deviations below the nominal mass that associates with the four-element phrase pole from cross-section measurements.

The caption for Table 8 suggests that the domain $-6 \leq l_m \leq 18$ might have relevance regarding modeling. Below, discussion related to Eq. (58) associates possible neutrino masses with values of l_m for which $-6 \leq l_m \leq -4$. Also, Table 9 notes possibly relevant (for some modeling purposes) notions of the domain $0 \leq l_m \leq 18$. This paper notes - without further comment - that (based on Eq. (45) and paralleling Eqs. (41) and (44)) Eqs. (53) and (54) pertain. The rightmost aspect of Eq. (54) assumes that, for $l_q = 3$, the magnitude of hypothetical charge $q(l_m = 18, l_q = 3)$ associates with $Q = 1$.

$$(4/3) \cdot (m(18,3)/m_e)^2 = (4/3) \cdot (m(18,3)/m(0,3))^2 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \qquad (53)$$

$$(4/3) \cdot (m(18,3))^2 G_N = (q_e)^2/(4\pi\varepsilon_0) = (q(18,3))^2/(4\pi\varepsilon_0) \qquad (54)$$

4.3.4. Anomalous magnetic moments of charged leptons

QFT associates with two complementary aspects of magnetic moment - nominal magnetic moment and anomalous magnetic moment. QFT calculates anomalous magnetic moments that match data regarding the electron and the muon. The calculations feature notions of virtual particles, including virtual photons.

SUPP suggests that POST notions of nominal magnetic moment can associate with SUPP notions of a 0d>:1g1`2 component of magnetic moment. SUPP suggests that POST notions of anomalous magnetic moment can associate with SUPP notions of a 0d>:3g1`2 component of magnetic moment.

QFT suggests Eq. (55). The subscript *cl* abbreviates the two-word phrase charged lepton. The term μ_{cl} associates with the notion of (total) magnetic moment. The factor $|q_{cl}|$ associates with the property of charge. The factor m_{cl} associates with the property of mass. The term g_{cl} associates with the property of (total) magnetic moment. The term that associates with the number two associates with the notion of nominal magnetic moment. The term that associates with a_{cl} associates with the notion of anomalous magnetic moment.

$$\mu_{cl} \propto |q_{cl}| m_{cl} \cdot g_{cl} = |q_{cl}| m_{cl} \cdot (2 + a_{cl}) \qquad (55)$$

SUPP makes the following notational associations. For the electron, *cl* can be either e or 0 (as in $l_m = 0$). For the muon, *cl* can be either μ or 2 (as in $l_m = 2$). For the tau, *cl* can be either τ or 3 (as in $l_m = 3$).

Ref. [83] provides the data $a_0 \approx 0.00115965$ and $a_2 \approx 0.00116592$.

SUPP explores two cases, each based on Eq. (56).

$$m'_{l_m} g_{l_m} = (m'_0 g_0) \cdot ((m'_2 g_2)/(m'_0 g_0))^{l_m/2} \qquad (56)$$

For the first case, $m'_{l_m} = m(l_m, 3)$, per Eq. (45) and Eq. (47). For the second case, $m'_{l_m} = m(l_m, 3)$, per Eq. (45) and the assumption (regarding Eq. (47)) that $j_1 = 0$. The second case differs from the first case only in the notion that - for the second case - $m'_2 c^2 \approx 117.284$MeV, whereas for the first case $m'_2 c^2 \approx 105.658$ MeV, which is the rest energy of the muon.

By the construction of Eq. (56), both cases comport with data regarding a_0 and a_2.

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The first case suggests that $a_3 \approx -0.143938$. The second case suggests that $a_3 \approx 0.00116905$.

Ref. [83] alludes to data that suggest $-0.052 \lesssim a_3 \lesssim 0.013$. SUPP de-emphasizes the first case and assumes that the second case pertains.

SUPP suggests the result for the tauon anomalous magnetic moment that Eq. (57) shows.

$$a_{\tau,\mathrm{SUPP}} = a_3 \approx 0.00116905 \qquad (57)$$

Ref. [85] provides, based on the SM, a first-order result - which SUPP calls $a_{\tau,\mathrm{SM}}$ - for a_τ. Here, SM denotes the two-word term Standard Model. The result is $a_{\tau,\mathrm{SM}} \approx 1.177 \times 10^{-3}$ and leads to a value of $(a_{\tau,\mathrm{SUPP}} - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ of approximately -0.00675. Each one of $a_{\tau,\mathrm{SUPP}}$ and $a_{\tau,\mathrm{SM}}$ comports with data ($-0.052 \lesssim a_\tau \lesssim 0.013$) that Ref. [83] provides.

4.3.5. Notions that associate with neutrino oscillations

The SM suggests that neutrino oscillations associate with a notion that flavour eigenstates do not necessarily match mass eigenstates. Flavour eigenstates associate with the creation of neutrinos via the weak interaction. Mass eigenstates associate with the motion - after the creation of a neutrino - of a neutrino.

SUPP associates the weak interaction bosons (and the weak interaction) with 0g solution-pairs. Per Table 6, neutrino flavour eigenstates associate with 0d0 use of 0g1`3`4`6. SUPP associates mass - and mass eigenstates - with 0d> use of 2g2.

SUPP suggests the possibility that 0d> use of the 6g2`4 solution-pair (and 1d> use of 6g2`4`8) associates with a notion of anomalous angular momentum (including for zero-charge elementary fermions). Paralleling discussion regarding Eq. (55) and anomalous magnetic moments for charged leptons, there might be differences - among the three neutrino mass eigenstates - regarding anomalous angular momentum. SUPP suggests that such differences might associate with SM notions of differences - regarding masses - between neutrinos.

Ref. [28] notes that observations explore the extent to which neutrino oscillations associate with interactions - between neutrinos and the environments through which neutrinos pass. SUPP suggests that such interactions might associate with 2g (and notions that associate with mass), with 3g (and notions that associate with flavour), or with 4g (and notions that associate with flavour). SUPP suggests that properties and events that associate with interactions between neutrinos and their environments might associate (for example) with 0d> use of 2g2`3`7::6 (for which 2g2`3`6`7 is a one-step cascade), with 0d> use of solution-pair 3g4`7::1 (for which 3g4`6`7 is a one-step cascade), or with 0d> use of solution-pair 4g3`7::2 (for which 4g3`6`7 is a one-step cascade).

4.3.6. Neutrino masses

SUPP suggests neutrino masses.

Ref. [25] suggests that data point to the notion that the sum of the three neutrino rest energies is at least approximately 0.06 eV and not more than approximately 0.12 eV. Ref. [86] discusses data and modeling regarding upper bounds for the sum of the masses of the three neutrinos. Ref. [87] discusses a lower bound of 0.06 eV, an upper bound of 0.15 eV, and a possible upper bound of 0.12 eV. Ref. [83] suggests that an upper bound might be approximately 0.10 eV.

Neutrinos associate with $Q = 0$. SUPP suggests that some $m(l_m, 0)$ solutions associate with neutrino masses. Eq. (49) suggests that, for $l_q = 0$, $n_q = 3/2$. For $l_m \leq -1$ and for $l_m \geq 3$, no quarks pertain, and SUPP suggests that $w_q(l_m) = 0$.

Eq. (58) shows a result from Eq. (45).

$$mc^2 = m(-4, 0)c^2 \approx 3.448 \times 10^{-2} \text{ eV} \qquad (58)$$

SUPP suggests the following two possibilities, either of which might comport with bounds regarding the sum of the three neutrino rest energies.

1. $mc^2 = m(-4, 0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of the three neutrinos.
2. $mc^2 = m(-4, 0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of two neutrinos. For one neutrino, one of $m(-6, 0)c^2 \approx 4.2 \times 10^{-6}$ eV and $m(-5, 0)c^2 \approx 4.4 \times 10^{-4}$ eV might pertain.

This paper does not try to explore the extent to which SUPP notions - such as notions regarding anomalous angular momentum and 0d> use of the 6g2`4 solution-pair, or such as notions regarding interactions that associate with 0d>:ΣgΓ properties for which $\{5, 7\} \cap (0d >: Z_\Gamma) \neq \emptyset$ - might suffice to explain neutrino oscillations, including for the case in which just one mass pertains for all three neutrinos.

4.4. Differences - between isomers - regarding properties of fermion elementary particles

This unit discusses differences - regarding the flavours of charged leptons - between isomers.

This unit associates with perspective and terminology centric to OM and with perspective and terminology centric to the five DM counterparts to OM.

If the stuff that associates with each of the five all-DM isomers evolved similarly to the stuff that associates with isomer-zero, SUPP suggestions regarding DM might not adequately comport with the following observations.

- Observations of ratios of $5^+ : 1$ (and not of $5 : 1$) regarding large-scale ratios of dark-matter effects to ordinary-matter effects. (Discussions related to Table 12b and discussions related to Table 12c provide more information.)
- Observations regarding the Bullet Cluster collision of two galaxy clusters. (Discussion - below - that cites Ref. [88] provides more information.)

SUPP uses the symbol l_I to number the isomers. The notion of isomer-l_I pertains.

Per discussion (including discussion regarding Table 8) above, regarding each l_I that is at least one, SUPP suggests that the instances (of elementary particles) that associate with isomer-l_I match - with respect to mass - the instances (of the counterpart elementary particles) that associate with isomer-zero.

For modeling regarding flavours (and not - for $0 \leq l_I \leq 5$ - for modeling regarding masses), SUPP associates the quarks in isomer-l_I with three values of l_m. The values are $3l_I + 0$, $3l_I + 1$, and $3l_I + 2$. (Table 8 shows the associations for $l_I = 0$.) Across the six isomers, quarks associate with each value of l_m that is in the range $0 \leq l_m \leq 17$. Regarding quarks and flavours, SUPP suggests that - within isomer-l_I - flavour 1 associates with $l_m = 3l_I$, flavour 2 associates with $l_m = 3l_I + 1$, and flavour 3 associates with $l_m = 3l_I + 2$.

Aspects of Table 8 point to the notion that means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer-zero does not have a charged lepton that associates with $l_m = 1$ and does have a charged lepton that associates with $l_m = 3$. SUPP suggests that - for each l_I - a charged lepton associates with each of $l_m = 3l_I + 0$, $l_m = 3l_I + 2$, and $l_m = 3l_I + 3$.

SUPP posits that - for each isomer-l_I such that $1 \leq l_I \leq 5$ - the charged-lepton flavour that associates with $l_m = 3(l_I) + 0$ equals the flavour that associates with the isomer-$(l_I - 1)$ charged lepton that associates with the same value of l_m and - thus - with $l_m = 3(l_I - 1) + 3$. SUPP suggests that, across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table 9 shows, for isomers of charged elementary fermions, matches between masses and flavours.

l_I	f_{l-r}	l_m $(0.5\mathrm{Q}_{>0})$	Respective l_f $(0.5\mathrm{Q}_{>0})$	l_m $(0.5\mathrm{C}_1)$	Respective l_f $(0.5\mathrm{C}_1)$
0	$+1$	$0, 1, 2$	1,2,3	$0, 2, 3$	1,2,3
1	-1	$3, 4, 5$	1,2,3	$3, 5, 6$	3,1,2
2	$+1$	$6, 7, 8$	1,2,3	$6, 8, 9$	2,3,1
3	-1	$9, 10, 11$	1,2,3	$9, 11, 12$	1,2,3
4	$+1$	$12, 13, 14$	1,2,3	$12, 14, 15$	3,1,2
5	-1	$15, 16, 17$	1,2,3	$15, 17, 18$	2,3,1

Table 9. Matches between masses and flavours, for isomers of charged elementary fermions. The symbol l_I denotes the isomer number. The symbol f_{l-r} denotes - for fermion elementary particles - whether a matter particle (in the context of matter particle and antimatter particle) elementary fermion is left-handed ($f_{l-r} = +1$), does not associate with handedness ($f_{l-r} = 0$, in which case the elementary fermion is its own antiparticle), or is right-handed ($f_{l-r} = -1$). The symbol 0.5Q$_{>0}$ denotes the pair 0.5Q$_{1/3}$ and 0.5Q$_{2/3}$. The symbol l_f numbers the three flavours.

SUPP posits that - for each isomer - the flavour of each of the three neutrinos matches the flavour of the respective one of the three charged leptons.

4.5. Some possibilities for transfers of energy between isomers

This unit discusses a mechanism that might transfer energy between isomers and a mechanism that might transfer net elementary fermion handedness between the two isomers that associate with an isomer-pair.

Eq. (59) symbolizes an interaction in which an isomer-l_I object transits from a state X_{l_I} to a state Y_{l_I} and the interaction produces a pair of isomer-l'_I elementary fermions. The symbol FLH denotes a left-handed fermion elementary particle, and the symbol FRH denotes a right-handed fermion elementary particle. The left-handed fermion elementary particle and the right-handed fermion elementary particle are antiparticles regarding each other.

$$X_{l_I} \to Y_{l_I} + FLH_{l'_I} + FRH_{l'_I} \qquad (59)$$

For each one of 1L, 2L, and 3L, there is at least one 0d>:Σg″ solution-pair for which $R_I = 6$ pertains, a 1d> one-step-cascade solution-pair for which $6 \in Z_\Gamma$ exists, and a 1d> one-step-cascade solution-pair for which $8 \in Z_\Gamma$ exists. Examples of such 0d>:Σg″ solution-pairs include 1g2\`3\`4\`5\`7x, 2g1\`3\`4\`5\`7x, and 3g1\`2\`4\`5\`7x.

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SUPP suggests that interactions that associate with Eq. (59) can result in any isomer-l_1 creating stuff that associates with any isomer-l_1'. The notion that 16-pole pertains regarding the relevant solution-pairs might suggest that such interactions occur mainly in situations in which stuff is dense (or, mainly, in the early history of the universe).

Eq. (60) symbolizes an interaction in which an isomer-l_I matter-and-antimatter pair of similar elementary fermions produces an isomer-l_I left-handed elementary particle and an isomer-l_I' right-handed similar elementary particle. (Similar interactions could produce $FLH_{l_I'} + FRH_{l_I}$.) Here, based on the notion - in Table 5 - of conservation (for each one of the three isomer-pairs) of elementary fermion f_{l-r}, SUPP suggests that $|l_I - l_I'| = 3$.

$$FLH_{l_I} + FRH_{l_I} \rightarrow FLH_{l_I} + FRH_{l_I'} \qquad (60)$$

For each one of 1L and 3L, Table 2 lists at least one 0d>:Σg′ solution-pair for which both $R_I \geq 2$ and there is a one-step-cascade solution-pair for which $6 \in Z_\Gamma$. Examples of such 0d>:Σg′ solution-pairs include 1g1\`2\`4 and 3g3. For 1g1\`2\`4, there is also a one-step-cascade solution-pair for which $8 \in Z_\Gamma$.

For l_I=0 and $l_I' = 3$, Eq. (60) associates with a decrease in the number of isomer-zero right-handed elementary fermions and an increase in the number of isomer-three right-handed elementary fermions.

4.6. Eras in the evolution of the universe

This unit discusses SUPP-suggested notions regarding eras - including possible eras before inflation and known eras after inflation - in the evolution of the universe.

Ref. [89] discusses CC notions regarding cyclic cosmology. SUPP includes the possibility that the present universe arose from an implosion of energy. SUPP does not yet consider either aspects that may have created the energy that would have imploded or whether the present universe might eventually implode.

Ref. [48] discusses possibilities that might lead to a Big Bang.

CC suggests three eras in the rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

SUPP suggests using the notion of eras regarding the separating from each other of clumps - that, today, POST would consider to be large - of stuff. Examples of such clumps might include galaxy clusters and even larger clumps. SUPP suggests (based on Eq. (14) and Eq. (15)) that, for a pair of similar objects that

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always move away from each other, the dominating gravitational effects transit (over time) all or a portion of the following sequence: 16-pole attracting, octupole repelling, quadrupole attracting, dipole repelling, and monopole attracting.

Table 10 discusses six possible eras regarding the rate of separating of large clumps. (Refs. [26][49][90][91] discuss the possible inflationary epoch. Refs. [92][93][94][95] provide data and discussion about the two multi-billion-year eras. Ref. [50] discusses attempts to explain the rate of expansion of the universe.)

Force	0d> s-p	SUPP-pole	R_I	$\rightarrow$	Rate of separating	Duration	Notes
Attractive	2g1‘3‘4‘8‘16	16-pole	6	$\rightarrow$	Is negative	TBD	3
Repulsive	0g1‘2‘3‘4‘8	16-pole	1	$\rightarrow$	Turns positive †	TBD	3
Repulsive	2g1‘2‘3‘4x	Octupole	1	$\rightarrow$	Increases rapidly	Less than a second	2, 3
Attractive	2g1‘2‘3	Quadrupole	1	$\rightarrow$	Decreases	Billions of years	1
Repulsive	2g2‘4	Dipole	2	$\rightarrow$	Increases	Billions of years	1
Attractive	2g2	Monopole	6	$\rightarrow$	Would decrease	-	3

Table 10. Six possible eras regarding the rate of separating of large clumps. The rightmost three columns suggest eras. The leftmost four columns describe phenomena that SUPP suggests as noteworthy causes for the eras. Generally, a noteworthy cause associates with dominant forces and with notions of accelerations. Generally, an era associates with notions of velocities. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before an era starts. The two-element term 0d> s-p abbreviates the two-element phrase 0d> solution-pairs. Subsequent rows associate with later eras. CC suggests notions of a Big Bang (or - at least - of a time that CC associates with the two-word term Big Bang). The symbol † denotes a possible association between the relevant era and some CC notions of a Big Bang. SUPP points to the possibility for the first two eras that the table discusses. 0d> use of the solution-pair 0g1`2`3`4`8 associates with the Pauli exclusion principle (and with the might-be jay boson). The other 0d> solution-pairs to which the table alludes associate with gravitation. CC uses the word inflation (or, the two-word term inflationary epoch) to name the era that associates with the third row in the table. CC suggests that the inflationary epoch started about 10^{-36} seconds after the Big Bang. CC suggests that the inflationary epoch ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for the inflationary epoch. TBD denotes to be determined. The following notions pertain regarding the column with the one-word label notes. The symbol 1 denotes the notion that CC interpretations of data support the notions of each one of the two billions-of-years eras. The symbol 2 denotes the notion that CC suggests the era. The symbol 3 denotes the notion that SUPP suggestions regarding resolving CC tensions (between data and modeling) that associate with the fifth row do not necessarily depend on the existence of the era.

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SUPP suggests that some SUPP notions regarding eras that follow the inflationary epoch might not necessarily depend significantly on SUPP notions regarding the inflationary epoch or on SUPP notions regarding eras that might precede the inflationary epoch.

This paper does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repelling based on 2g2`4 to attracting based on 2g2 might occur.

4.7. Baryon asymmetry

This unit discusses a SUPP-suggested mechanism that might have led to POST notions of baryon asymmetry.

The two-word term baryon asymmetry is associated with the POST notion that - regarding known stuff - there are many more left-handed (or matter) fermion elementary particles than right-handed (or antimatter) fermion elementary particles. CC suggests that baryon asymmetry arose early in the history of the universe. From the perspective of SUPP, such known stuff is associated with isomer-zero.

Discussion related to Eq. (60) points to a means that may have produced baryon asymmetry. Possibly, POST notions of lasing pertained regarding relevant excitations of LRI fields. SUPP suggests that processes leading to baryon asymmetry led to isomer-three stuff having fewer left-handed (or antimatter, from the perspective of isomer-three) fermion elementary particles than right-handed (or matter, from the perspective of isomer-three) fermion elementary particles.

This paper does not address the topic of the extent to which steps leading to a predominance in isomer-zero stuff of left-handed elementary particles (and not to a predominance of right-handed elementary particles) have a basis other than random chance.

4.8. Evolution of stuff that is associated with dark matter isomers

This unit discusses SUPP-suggested notions about the evolution of stuff that is associated with the five isomers that SUPP associates with dark matter.

SUPP uses the two-element term isomer-l_I stuff to denote objects (including hadron-like particles, atom-like objects, and stars) that are associated with isomer-l_I elementary particles.

4.8.1 Evolution of isomer-1, isomer-2, isomer-4, and isomer-5 stuff

Here, SUPP uses the one-element term alt-isomer to designate an isomer other than isomer-zero and isomer-three.

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For each one of the six isomers, a charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, the hadron that includes just two tops and one bottom has a larger total mass than does the hadron that includes just one top and two bottoms.)

Per Table 8 and Table 9, alt-isomer flavour 3 charged leptons are less massive than isomer-zero flavour 3 charged leptons. When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the stuff that is associated with an alt-isomer converts more charged baryons to zero-charge baryons than does the stuff that is associated with isomer-zero. Eventually, regarding the stuff that is associated with the alt-isomer, interactions that entangle multiple W bosons result in the stuff that is associated with the alt-isomer having more neutrons and fewer protons than does the stuff that is associated with isomer-zero. The sum of the mass of a proton and the mass of an alt-isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer-zero neutrons, alt-isomer neutrons scarcely decay.

SUPP posits that each alt-isomer does not associate with significant numbers of analogs to protons, other charged hadrons, or electrons and does not associate with significant numbers of analogs to isomer-zero atoms.

The IGM (or, intergalactic medium) that is associated with the alt-isomer scarcely interacts with itself via electromagnetism.

4.8.2. Evolution of isomer-3 stuff

The following two possibilities pertain. In one possibility, the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff. In the second possibility, the evolution of isomer-three stuff does not parallel the evolution of isomer-zero stuff. The second possibility might be associated with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer-three and isomer-zero.

This paper nominally assumes that the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff.

4.9. Explanations for tensions - between data and models - regarding large-scale phenomena

This unit discusses SUPP-suggested notions that might help resolve so-called tensions - between data and POST CC - regarding the rate of expansion of the universe, regarding large-scale clumping of matter, and regarding gravitational interactions between neighboring galaxies.

4.9.1. The rate of expansion of the universe

Table 10 lists two known eras in the history of the universe.

CC underestimates - for the second multi-billion-years era - increases in the rate of expansion of the universe. (Refs. [53][54][55][56][96][97][98][99] provide further information. Ref. [100] suggests that the notion that DM is similar to OM might help resolve the relevant tension. Ref. [101] discusses various possible resolutions. Ref. [102] provides data about the Hubble constant.)

SUPP suggests the following explanation for such underestimates.

When using modeling based on GR, CC might try to extend - to modeling regarding more recent times - the use of an equation of state (or the use of a cosmological constant) that works well regarding early in the first multi-billion-years era. Regarding the first multi-billion-years era, SUPP suggests dominance by attractive effects that associate with 0d> use of the 2g1`2`3 component of gravity. The notion of a reach of one pertains. The symbol 0d>:2g1`2`3::1 pertains. SUPP suggests that - later in the first multi-billion-years era - repulsive effects that associate with 0d> use of 2g2`4::2 and with 1d> use of 2g2`4::6 become significant. Dominance by 0d>:2g2`4::2 and 1d>:2g2`4::6 pertains by the time the second multi-billion-years era starts. However, use of an equation of state that has roots in the time period in which 0d>:2g1`2`3::1 dominates might - at best - extrapolate based on a notion of 0d>:2g2`4::1 and 1d>:2g2`4::1 (and not based on a notion of 0d>:2g2`4::2 and 1d>:2g2`4::6) and would underestimate the strength of the key gravitational driver - of expansion - by a (time-varying) factor of at least two.

SUPP points - conceptually - to the following possible remedy.

CC might change (regarding the stress-energy tensor or the cosmological constant) the aspects that would associate with repelling and the 2g2`4 component of gravity. The contribution - to the pressure - that associates with 0d>:2g2`4 might need to double (compared to the contribution that would associate with 0d>:2g2`4::1). The contribution - to the momentum - that associates with 1d>:2g2`4 might need multiplication by a factor of 6 (compared to the contribution that would associate with 1d>:2g2`4::1)

4.9.2. Large-scale clumping of matter

CC overestimates large-scale clumping of matter - OM and DM. (Refs. [56][103][104][105] provide data and discussion.)

SUPP suggests that CC modeling associates with repulsive components - 0d>:2g2`4::1 and 1d>:2g2`4::1 - of gravity. SUPP suggests that 0d>:2g2`4::2 and 1d>:2g2`4::6 pertain. The additional (compared to CC modeling) repelling might explain the overestimation that CC suggests.

4.9.3. Effects - within galaxies - of the gravity associated with nearby galaxies

CC might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (Ref. [71] provides further information.)

SUPP suggests that CC modeling associates primarily with an attractive component - 0d>:2g2::6 - of gravity and a repulsive component - 1d>:2`4::6 - of gravity. SUPP suggests, regarding a repulsive component - 0d>:2g2`4 - of gravity, that POST CC modeling associates with SUPP notions of 0d>:2g2`4::6. SUPP suggests that 0d>:2g2`4::2 pertains. The reduced (compared to CC modeling) repelling might explain at least some aspects of the data that Ref. [71] discusses.

4.10. Formation, evolution, and composition of galaxies

This unit discusses SUPP-suggested notions regarding the formation, evolution, and composition of galaxies.

4.10.1. Mechanisms regarding the formation and evolution of galaxies

Ref. [58] suggests that galaxies form around early clumps of stuff. Ref. [58] associates the word halo with such clumps.

SUPP suggests that each one of many early halos associates with one isomer. SUPP associates with such early halos the three-element term one-isomer original clump. Clumping occurs based on gravitational effects. Differences - between the evolution of stuff associating with any one of isomer-zero and isomer-three and the evolution of stuff associating with any one of isomer-one, isomer-two, isomer-four, and isomer-five - are not necessarily significant regarding this gravitationally based clumping. The six isomers might form such clumps approximately equally.

Table 11 discusses SUPP suggestions regarding the formation and evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Force	0d> s-p	R_I	$\rightarrow$	Aspects of the stage	Stage	Era
Attractive	2g1‘2‘3	1	$\rightarrow$	A one-isomer original clump forms.	1	First
Repulsive	2g2‘4	2 †	$\rightarrow$	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.	2	First
Attractive	2g2	6	$\rightarrow$	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.	3	Second
Attractive	2g2	6	$\rightarrow$	Another galaxy subsumes the original clump and might subsequently merge with yet other galaxies.	4	Third

Table 11. Stages and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests stages, with subsequent rows associating with later stages. The column labeled with the four-word term aspects of the stage describes events. The leftmost three columns in the table describe a component of 2L that is a noteworthy cause for the stage. Generally, a noteworthy cause associates with dominant forces and with notions of accelerations. The two-element term 0d> s-p abbreviates the two-element phrase 0d> solution-pairs. The symbol † denotes the notion that the 1d>:2g2`4::6 also pertains. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before a stage starts. This table associates with a scenario in which a galaxy forms based on one original one-isomer clump and initially does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer. The rightmost column in this table suggests terminology regarding the evolution of galaxies. (A galaxy can include stuff from more than one earlier galaxy.)

4.10.2. Aspects regarding the evolution of galaxies

Table 11 suggests three eras regarding the evolution of galaxies. The first era associates with the first two rows in Table 11. The second era associates with the 2g2 attractive force that associates with the third row in Table 11. The third era associates with collisions between and mergers of galaxies.

SUPP suggests the possibility that some galaxies do not exit the first stage and do not significantly collide with other galaxies.

SUPP suggests the possibility that some galaxies do not exit the second stage and do not significantly collide with other galaxies.

SUPP suggests that some galaxies result from aspects associating with the 2g2 attractive force that associates with the third row in Table 11. Here, this paper discusses three cases. (Mixed cases and other cases might pertain.)

- Each one of some era-one galaxies does not collide with other galaxies. Such a galaxy accumulates (via 2g2 attracting) stuff associating with various isomers that have representation in nearby IGM. The galaxy becomes an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.
- Each one of some era-two galaxies merges (via 2g2 attracting) mainly just with galaxies that feature the same five isomers. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.
- Each one of some era-one or era-two galaxies merges (via 2g2 attracting) with other galaxies. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era-three galaxy. The galaxy might include stuff that significantly associates with as many as six isomers.

Presumably, some galaxies form based on two or more clumps, for which all the clumps associate with just one isomer. Possibly, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

4.10.3. Amounts of dark matter in galaxies

POST CC suggests the possibility that DM associates with an explanation for phenomena that associate with the three-element term galaxy rotation curves. Absent an explanation, CC modeling would suggest that stuff rotating around the center of a galaxy would associate with lower velocities than the velocities that observations suggest.

SUPP is not incompatible with the possibility that DM associates with such an explanation. However, SUPP suggests that models based on POST CC might overestimate the amount of DM in some galaxies.

From the perspective of SUPP, POST CC modeling (that associates with galaxy rotation curves) associates with $R_I = 6$ for all (2g′) components that contribute to gravitational phenomena. However, SUPP suggests that - for 0d>:2g2`4 - $R_I = 2$. SUPP associates 0d>:2g2`4 with rotation and with dilution of the attraction that associates with 0d>:2g2. $R_I = 2$ would associate with less effect than would $R_I = 6$. Here, less effect associates with more gravitational attraction.

SUPP suggests that POST might consider the greater gravitational attraction to be a MOND (as in MOdified Newtonian Dynamics) adjustment (regarding gravity) that might account for some aspects that associate with galaxy rotation curves.

SUPP suggests - for each one of some galaxies - that the galaxy might include - within almost any (not adequately small) radial distance from the center of the galaxy - less DM than POST would currently estimate based on the observed rotation speed that associates with the distance.

4.11. Explanations for ratios of dark-matter effects to ordinary-matter effects

This unit suggests that SUPP notions comport with and explain various observed ratios of dark-matter effects to ordinary-matter effects. POST CC seems not to explain the ratios.

Table 12 lists observed ratios of dark-matter effects to ordinary-matter effects and alludes to SUPP notions that seem to explain the ratios. The discussion below defines terms such as dissimilar evolution regarding isomeric stuff, isomer-zero dark matter, and misinterpreted data.

(a) Ratios - that pertain to light that dates to about 380,000 years after the Big Bang - of observed effects to effects that POST estimates. The three-word phrase cosmic optical background associates with radiation that - recently - measures as optical radiation or measures as close (with respect to wavelengths) to optical radiation. The acronym CMB associates with radiation that - recently - measures as cosmic microwave background radiation. DM:OM denotes a ratio of DM effects to OM effects that this paper suggests.

Aspect	Observed : POST-CC-expected	SUPP-suggested DM:OM
Amount of cosmic optical background	2 : 1	1 : 1
Some absorption of CMB	2 : 1	1 : 1

(b) Suggested explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that associate with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z. Here, z denotes redshift. The word later associates with the notion that observations pertain to objects later in the history of the universe. The two-element term DM galaxy denotes a galaxy that contains much less OM than DM. Possibly, people have yet to directly detect early DM galaxies. Table 11 provides information about the explanations.

Objects	DM:OM	Examples	Explanation
Some early galaxies	$0^+ : 1$	Reported	OM original clump. Stage-1 or stage-2.
Some later galaxies	$0^+ : 1$	Reported	OM original clump. Stage-1 or stage-2.
Some early galaxies	$1 : 0^+$	No known reports	DM-isomer(s) original clump. Stage-1 or stage-2.
Some later galaxies	$1 : 0^+$	Reported	DM-isomer(s) original clump. Stage-1 or stage-2.
Some later galaxies	$\sim 4 : 1$	Reported	Non-isomer-three original clump. Stage-3.
Many later galaxies	$5^+ : 1$	Reported	Any-isomer(s) original clump(s). Stage-4.

(c) Suggested explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. More than one explanation might pertain. If none of these or other explanations pertained, SUPP would suggest DM:OM ratios of 5 : 1.

Aspect	DM:OM	Comment
Densities of the universe	$5^+ : 1$	Each one of dissimilar evolution regarding isomeric stuff, isomer-zero dark matter, and misinterpreted data might pertain.
Some galaxy clusters	$5^+ : 1$	SUPP suggests that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe. (Similarity to DM:OM ratios for many stage-4 galaxies also pertains.)

Table 12. Observed ratios of dark-matter effects to ordinary-matter effects and SUPP notions that seem to explain the ratios.

4.11.1. Ratios that might pertain regarding the cosmic electromagnetic background

Table 12a lists ratios - that pertain to light that dates to about 380,000 years after the Big Bang - of observed effects to effects that POST modeling estimates. The acronym CMB abbreviates the three-word term cosmic microwave background (or, the four-word term cosmic microwave background radiation). (Refs. [106][107][108][109] provide data and discussion regarding the amount of cosmic optical background. Refs. [110][111][112] provide data and discussion regarding absorption of CMB.)

The following two paragraphs provide SUPP-suggested explanations for the observations to which Table 12a alludes.

The three-word phrase cosmic optical background associates with now nearly-optical light remaining from early in the universe. CC suggests that atomic transitions produced radiation that today measures as cosmic (optical and microwave) background radiation. SUPP associates POST atomic transitions with isomer-zero. Observations found twice as much light as CC expected. SUPP suggests that isomer-one, isomer-two, isomer-four, and isomer-five stuff did not result in much stuff that is similar to isomer-zero atoms. SUPP suggests that isomer-three stuff evolved similarly to isomer-zero stuff. For four types of changes in atomic energy levels, discussion related to Eq. (29) alludes to electromagnetic-radiation-producing events that associate with $R_I = 2$. SUPP suggests that such events explain the two-to-one observed-to-expected ratios regarding the cosmic optical background. Isomer-zero (or, OM) stuff produced half of the observed light. Isomer-three (or, DM) stuff produced half of the observed light.

The four-element phrase some absorption of CMB associates with the notion that measurements of some specific depletion of CMB indicate twice as much depletion as CC expected based solely on hyperfine interactions with (isomer-zero) hydrogen atoms. SUPP suggests (per discussion related to Eq. (29)) that isomer-three (or, DM) hydrogen-like atoms account for the half of the absorption for which isomer-zero (or, OM) hydrogen atoms do not account.

4.11.2. Ratios that pertain to some galaxies

Table 12b suggests explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. (Refs. [113][114] provide data and discussion. Ref. [113] influenced the choice - that this paper reflects - of a time range to associate with the word early. Regarding the combination of 0^+:1 and later, Refs. [115][116][117][118][119][120][121] provide data and discussion. Ref. [122] discusses a galaxy that might have started as containing mostly OM. Ref. [123] discusses a DM-deficient galaxy. Regarding observed DM galaxies, Refs.

[58][124][125][126] provide data and discussion. Current techniques might not be capable of observing early DM galaxies. Refs. [127][128] suggest, regarding galaxy clusters, the existence of clumps of DM that might be individual galaxies. Extrapolating from results that Refs. [58][129] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM $1 : 0^+$ later galaxies. Ref. [130] discusses a trail of galaxies for which at least two galaxies have little DM. Ref. [130] suggests that the little-dark-matter galaxies result from a collision that would have some similarities to the Bullet Cluster collision. Regarding galaxies for which DM:OM ratios of $\sim$4:1 pertain,

Refs. [131][132] provide data and discussion. Regarding later galaxies for which DM:OM ratios of 5^{+}:1 pertain, Ref. [58] provides data and discussion. Refs. [133][134] provide data about collisions of galaxies.)

Table 12b does not rule out the notion that galaxies somewhat fully populate DM:OM ranges within the interval of $0:1$ to, say, $6:1$. For DM:OM ratios of less than (say) ten, Table 11 suggests that each range of DM:OM ratios to which Table 12b alludes might stand out statistically (in terms of numbers of galaxies) from ranges (of positive-number ratios) near to the range to which Table 12b alludes.

Table 12b does not rule out the notion that galaxies somewhat fully populate a DM:OM range of $10^{p_1}:1$ to $10^{p_2}:1$ for which SUPP does not suggest a value of p_1; p_1 exceeds, say, three; p_2 exceeds p_1; and SUPP does not suggest a value of p_2.

4.11.3. Ratios that pertain regarding phenomena that are bigger than galaxies

SUPP suggests the following aspects that might contribute toward the notion that measurements of large-scale presences of DM might exceed five times measurements of large-scale presences of OM.

- Dissimilar evolution regarding isomeric stuff. Here, the term alt-isomer refers to isomer-one, isomer-two, isomer-four, or isomer-five. The evolution of alt-isomer stuff might deviate - compared to the evolution of isomer-zero stuff - early enough that (nominally) isomer-zero high-energy excitations of the electromagnetic field produce alt-isomer stuff significantly more copiously than (nominally) alt-isomer excitations of the electromagnetic field produce isomer-zero stuff. (Discussion related to Eq. (59) pertains.)
- Isomer-zero dark matter. POST CC suggests notions - such as notions of primordial black holes or yet-to-be-found elementary particles - of stuff that might measure as DM and (in the context of SUPP) associate mainly with isomer-zero stuff. (SUPP does not necessarily suggest isomer-zero elementary particles that would associate with notions of DM.)
- Misinterpreted measurements. Interpretations of measurements might - based on notions that, for example, the $R_I=2$ for 0d>:2g2`4 differs from the $R_I=6$ for 0d>:2g2 and for 1d>:2g2`4 - might lead to inferred ratios of DM effects to OM effects that do not associate exactly with actual ratios of DM stuff to OM stuff.

Table 12c suggests explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. (Ref. [83] provides data and discussion regarding densities of the universe. Refs. [135][136][137][138] provide data and discussion regarding galaxy clusters.)

4.11.4. Aspects related to collisions of pairs of galaxy clusters

Ref. [88] discusses the Bullet Cluster collision of two galaxy clusters.

CC suggests two general types of trajectories for stuff. Most DM - from either one of the clusters - exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Also, OM stars - from either cluster - exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, OM IGM - from either cluster - lags behind the cluster's OM stars and DM. CC suggests that the OM IGM interacted electromagnetically with the other cluster's OM IGM, as well as gravitationally with the other cluster.

SUPP suggests that SUPP might comport (regarding each cluster) with the interpretations of data, with one possible exception. The possible exception is associated with the notion that SUPP suggests that isomer-three IGM interacts electromagnetically and follows trajectories that are consistent with OM IGM trajectories.

Regarding the possible exception, at least three possibilities arise.

For one possibility, per discussion related to Eq. (29), the light that CC associates with OM IGM might include light that SUPP associates with OM IGM and light that SUPP associates with isomer-three IGM.

For one possibility, isomer-three IGM measures as DM, and CC does not adequately report (or otherwise account for) lagging isomer-three IGM.

For one possibility, isomer-three IGM follows trajectories that are consistent with other DM trajectories.

SUPP suggests that interpretations of data may not be sufficient to rule out each one of the first two possibilities or to rule out a combination of the first two possibilities.

SUPP notions of DM are not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - regarding DM.

5. Discussion

This unit suggests possible additional (compared to previous units) associations between possible future data and SUPP modeling. This unit suggests possible additional (compared to previous units) associations between POST modeling and SUPP modeling. This unit suggests (based on POST modeling and SUPP modeling) bases that might point toward additional (compared to previous units) principles or other notions that -- in the future -- might underlie physics or physics modeling.

5.1. Possible elementary particles that POST has yet to include

This unit suggests possible insight regarding the existence and properties of some as-yet-unfound elementary particles that POST hypothesizes or that SUPP might suggest.

5.1.1. Elementary particles that SUPP suggests

Table 6 catalogs as-yet-unfound elementary particles that SUPP suggests.

5.1.2. Right-handed W boson

Ref. [139] discusses a fraction of decays - of OM top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+, is 3.6×10^{-4}. Ref. [25] provides a confidence level of 90 percent that the rest energy of a might-be W_R (or, right-handed W boson) exceeds 715 GeV. Ref. [140] provides other information.

SUPP suggests that W_R bosons associate only with isomers one, three, and five. SUPP suggests possibilities for inter-isomer interactions.

Aspects of SUPP might approximately reproduce the above result that SM modeling suggests.

Aspects related to Eq. (45) suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, SUPP does not suggest that $m(5,3)$ associates with the inertial mass of an isomer-one charged lepton. However, perhaps such mass-like quantities associate with some measurable aspects of nature. For charged leptons and $0 \leq l_I \leq 4$ and $0 \leq l'_f \leq 3$, $m(3(l_I+1)+l'_f,3) = \beta m(3(l_I+0)+l'_f,3)$. One might conjecture that isomer-zero observations of some aspects of isomer-one phenomena associate with notions of non-inertial mass-like quantities that are β times the inertial masses for isomer-zero elementary particles (and that are β times inertial masses for the counterpart isomer-one elementary particles).

Based on notions of scaling that might calculate non-inertial mass-like quantities, SUPP might suggest that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that Ref. [139] discusses. A notion of $m_{\text{non-inertial,W}_R\text{isomer-one}}c^2 = \beta m_{\text{W}}c^2 \approx 2.8 \times 10^5$ GeV might pertain. The notion of a non-inertial mass-like quantity might associate with interactions that associate with photons.

5.1.3. Magnetic monopole

Table 1 seems not to suggest an electromagnetic interaction with a monopole other than an electric monopole. SUPP does not suggest a property (of objects) that would associate with a magnetic monopole.

5.2. Phenomena that might involve the SUPP-suggested jay boson elementary particle

This unit discusses phenomena that might associate with the SUPP-suggested jay (or, 1J) boson (that Table 6 lists).

5.2.1. Pauli repulsion

POST includes the notion that two identical fermions cannot occupy the same state. Regarding QM, one notion is that repelling between identical fermions associates with overlaps of wave functions. Another QM notion features wave functions that are antisymmetric with respect to the exchange of two identical fermions.

SUPP might be compatible with such aspects of POST and, yet, not necessitate - regarding POST dynamics modeling - the use of wave functions. QM based on jay bosons might suffice. CM based on potentials that would associate with effects of jay bosons might suffice.

SUPP suggests that QM or CM based on jay bosons might suggest that the prevention of two identical fermions from occupying the same state might associate with, in effect, interactions - mediated by jay bosons - that try to change aspects related to the fermions. Notions of changing a spin orientation might pertain. For elementary fermions, notions of changing a flavour might pertain.

5.2.2. Energy levels in positronium

Ref. [141] discusses the transition - between two states of positronium - characterized by the expression that Eq. (61) shows.

$$2^3S_1 \rightarrow 2^3P_0 \qquad (61)$$

Four standard deviations below the nominal observed value of the energy that associates with the transition approximately equals four standard deviations above the nominal value of the energy that

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POST suggests.

SUPP notions regarding jay bosons might explain the might-be discrepancy regarding positronium. Compared to QFT, a new notion of virtual charge exchange or a new notion of virtual flavour change might pertain.

To the extent that QFT does not suffice to explain positronium energy levels, SUPP notions related to the jay boson might help to close the gap between observations and modeling.

5.2.3. Pauli crystals

Ref. [142] reports the detection of Pauli crystals. SUPP suggests that modeling based on the notion of jay bosons might help explain relevant phenomena.

5.3. Some relationships between tetrads and modeling for elementary particles

This unit suggests possible relationships between tetrads that associate with elementary particles and modeling that catalogs elementary particles.

Table 4 suggests tetrads that associate with elementary particles. Table 6 suggests a catalog of elementary particles.

The following notions seem to interrelate aspects of Table 4
and aspects of Table 6. The notions feature two paths that start at a common point.

- The starting point.
  - $n_\Gamma = 3$ (which associates with the W and Z bosons) in Table
    6 might associate with three tetrads - isomer-pairs, charge, and mass - in Table 4.
- A path regarding boson elementary particles. (Each step obviates notions in one tetrad.)
  - For $n_\Gamma \geq 4$, the charge is zero.
  - For $n_\Gamma \geq 5$, the mass is zero.
- A path regarding fermion elementary particles. (Each step suggests relevance for one - or, possibly two - new tetrads.)
  - For $n_\Gamma \geq 4$, three elementary-fermion flavours pertain and (possibly conservation of) f_{l-r} pertains.
  - For $n_\Gamma = 5$, three elementary-fermion color charges pertain and (possibly conservation of) f_{B-L} pertains.

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The starting point and two paths touch each row in Table 4.

Table 13 reorganizes information that Table 6 shows.

Legend	$n_\Gamma = 3$	$n_\Gamma = 4$	$n_\Gamma = 5$	$n_\Gamma = 6$
Fermions:				
Families		$0.5\mathrm{C}_1$	$0.5\mathrm{Q}_{>0}$ $(Q = 1)$	
0d0		$\lvert -1-2-3+6 \rvert$	$\lvert -1+2-3-6+8 \rvert$	
Families		0.5N	$0.5\mathrm{Q}_{>0}$ $(Q = 0)$	
0d0		$\lvert +1-3-4+6 \rvert$	$\lvert -1+2-3-4+6 \rvert$	
Bosons:				
Families	$1\mathrm{W}_1$	0H	1G	
0d0	$\lvert -1-2+3 \rvert$	$\lvert +1-2-3+4 \rvert$	$\lvert +1-2-3-4+8 \rvert$	
Families	1Z	0I †	1J †	
0d0	$\lvert -1-3+4 \rvert$	$\lvert -1-3-4+8 \rvert$	$\lvert +1-2-3-4+8 \rvert$	
Families			1L	2L †
0d0			$\lvert -1-3-4-8+16 \rvert$	$\lvert -1-3-4-8-16+32 \rvert$

Table 13. 0d0 solution-pairs that associate with all known elementary particles and with some elementary particles that SUPP suggests nature might include. The symbol † denotes that the elementary particles are as-yet unfound. For the 1G family, the number of elementary particles is eight. For each one of the four fermion families ($0.5\mathrm{C}_1$, 0.5N, $0.5\mathrm{Q}_{2/3}$, and $0.5\mathrm{Q}_{1/3}$), the number of elementary particles is three. For each one of the other families, the number of elementary particles is one.

5.4. Some phenomena that associate with galaxies

This unit discusses SUPP-suggested notions that might help explain some phenomena that associate with some galaxies.

5.4.1. Some stopping of the accrual of matter

Ref. [143] discusses a galaxy that seems to have stopped accruing both OM and DM about four billion years after the Big Bang.

The galaxy that Ref. [143] discusses might (or might not) associate with the notion of a significant presence early on of one of isomer-zero and isomer-three, one of isomer-one and isomer-four, and one of isomer-two and isomer-five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

5.4.2. Some quenching of star formation

Some galaxies seem to stop forming stars. The word quenching can pertain. (Refs. [144][145] discuss examples of quenching.) Such quenching might take place within three billion years after the Big Bang, might be associated with a lack of hydrogen atoms, and might (per Ref. [145]) pertain to half of the galaxies that are associated with the notion of a certain type of galaxy.

SUPP suggests that some such quenching might be associated with repelling that is associated with 2g2`4. Some quenching might be associated with galaxies for which original clumps featured isomer-zero stuff or isomer-three stuff.

5.4.3. Aspects regarding stellar stream GD-1 in the Milky Way galaxy

Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-OM clump - in the Milky Way galaxy - with a mass of 10^6 to 10^8 solar masses. (Refs. [146][147] provide data and discussion regarding the undetected object. Ref.
[147] cites Refs. [148][149].)

SUPP suggests that the undetected object might be a clump of DM.

5.5. Some possibilities for directly detecting non-isomer-zero dark matter

This unit discusses some possibilities for directly detecting dark matter that does not associate with isomer-zero stuff.

Discussion related to Eq. (29) points to electromagnetic phenomena that are associated with reaches of two and, thereby, suggests that OM equipment might be able to catalyze or detect transitions within isomer-three atoms. Discussion related to Table 12a
suggests that data point to detection, by OM equipment, of light emitted by transition events that are associated with isomer-three atoms. Presumably, some isomer-three atoms pass (essentially unimpeded by isomer-zero stuff) through isomer-zero stuff that is near to and includes the Earth. SUPP suggests that experiments - based on light produced by OM stuff and light detected by OM stuff - might be able to detect (via transition events that are associated with isomer-three atoms) isomer-three atomic stuff. This paper does not discuss notions regarding whether techniques are now - or when techniques might become - sufficiently sensitive that such experiments would be feasible.

5.6. Some information that gravitational waves might convey

This unit suggests that adequately detailed analyses of the gravitational signatures that are associated with collisions of objects - such as black holes - might enable the development of data that are associated with the extents to which the colliding objects include stuff that is associated with more than one isomer or more than one isomer-pair.

Ref. [150] discusses opportunities for research regarding gravitational waves.

SUPP posits that, for $\{5, 7\} \subset 0\mathrm{d}{>}{:}Z_\Gamma$, 2g solution-pairs can be associated with gravitational properties relevant to collisions of two objects. For 2d0:2g1`3`4`5`7, $R_I = 6$. For 2d0:2g1`2`3`5`7, $R_I = 1$.

SUPP suggests that adequately detailed analyses of the gravitational signatures that associate with collisions of objects - such as black holes - might enable the development of data that associate with the extents to which the colliding objects include stuff that associates with more than one isomer or more than one isomer-pair.

5.7. Modeling regarding gravity

This unit discusses some notions regarding the accuracy of using modeling based on GR.

Present GR is not necessarily adequately compatible with some data.

Present GR is not necessarily adequately compatible with SUPP notions of DM. SUPP suggests that POST modeling based on GR can be less than adequately accurate.

Tests of GR have generally featured phenomena that associate with the isomer-pair that includes isomer-zero and isomer-three. Each one of the Sun, the planet Mercury, and the Earth associates with isomer-zero. Relevant radiation from distant stars and galaxies associates essentially just with isomer-zero stuff and isomer-three stuff.

For cases in which POST suggests that uses of general relativity adequately (or nearly adequately) comport with data, SUPP suggests that the following notions - about uses of SUPP solution-pairs and about the GR stress-energy tensor - might help bridge from SUPP to GR or from GR to SUPP. (The GR stress-energy tensor is symmetric.) 0d>:2g2::6 associates with the one stress-energy-tensor component (T^{00}) that associates with energy density. 1d>:2g2`4::6 associates with the three components ($T^{01}, T^{02}$, and $T^{03}$) that associate with momentum density and also associates with the three components ($T^{10}, T^{20}$, and $T^{30}$) that associate with energy flux. 0d>:2g2`4::2 associates with the three components (T^{11}, T^{22}, and T^{33}) that associate with pressure. Each one of 1d>:2g2`4`8::2 and

0d>:2g1`2`3::1 associates with the three components ($T^{21}, T^{31},$ and T^{32}) that associate with momentum flux and also associates with the three components ($T^{12}, T^{13},$ and T^{23}) that associate with shear stress.

To the extent that octupole components (such as 1d>:2g1`2`3`4x::1 and 0d>:2g1`2`3`4x::1) or 16-pole components (such as 0d>:2g1`3`4`8`16::6) have significant roles, GR might be less than adequately accurate.

This paper does not further explore the usefulness of such notions.

5.8. Applications of a series of formulas for lengths

This unit suggests a series of formulas for lengths (including the Schwarzschild radius and the Planck length) and discusses phenomena that might associate with lengths that the formulas suggest.

Eqs. (62), (63), and (64) define a set of lengths r_l. r_0 has dimensions of length. POST associates the two-word term Planck length with r_0. m denotes a mass. b is a dimensionless number.

$$r_0 = (G_N)^{1/2} m^0 \hbar^{1/2} c^{-3/2} 2^0 \qquad (62)$$

$$b = (G_N)^{-1/2} m^{-1} \hbar^{1/2} c^{1/2} 2^{-1} \qquad (63)$$

$$r_l = r_0 b^l \qquad (64)$$

POST associates the two-word term Schwarzschild radius with r_1. Eq. (65) pertains.

$$r_1 = (G_N)^1 m^1 \hbar^0 c^{-2} 2^1 \qquad (65)$$

For a charged pion, Eq. (64) yields $r_{-1} \sim 0.7 \times 10^{-15}$ meters. Ref. [151] states a value of $(0.640 \pm 0.007) \times 10^{-15}$ meters for a measured charge radius of a charged pion.

For a Z boson, Eq. (64) yields $r_{-1} \sim 2 \times 10^{-18}$ meters. (The r_{-1} for a W boson is about 1.1 times the r_{-1} for a Z boson.) Ref. [152] suggests that, for a separation of approximately 10^{-18} meters between two interacting particles, the weak interaction and the electromagnetic interaction have similar magnitudes. For a separation of approximately 3×10^{-17} meters, the magnitude of the weak interaction is less than the magnitude of the electromagnetic interaction by approximately a factor of 10^4.

SUPP suggests that the notion of r_{-1} might have significance regarding some sizes of objects and some ranges of interactions.

5.9. Some notions that might associate with elementary bosons and fermions

This unit discusses mathematics for which applications might provide insight about the notions of boson and fermion and about relationships between properties of elementary particles.

5.9.1. Notions regarding integers N' for elementary bosons

SUPP suggests that each term on the right side of Eq. (36) might associate with an expression of the form that Eq. (66) shows. Here, $d‘= 1$ pertains. Eq. (36) pertains for each isomer and for the LRI elementary bosons (which can associate with multiple isomers).

$$(X')^2 \propto \int_0^{X'} x^{d‘} dx \qquad (66)$$

5.9.2. Notions regarding raising and lowering operators for elementary bosons

Regarding QM raising and lowering operators for elementary boson states, the following notions pertain. States associate with the range $0 \leq n_b$ for integers n_b. For the raising operator, a factor that Eq. (67) shows pertains. For the lowering operator, a factor of $(n_b)^{1/2}$ pertains.

$$(1 + d‘n_b)^{1/2} = (1 + n_b)^{½} \qquad (67)$$

5.9.3. Notions regarding raising and lowering operators for elementary fermions

Eq. (36) interrelates the properties of elementary bosons.

Compared to Eq. (36), equations relating mass, charge, and other properties for elementary fermions would include two new (compared to elementary bosons) aspects - flavour and fractional charge. Flavour associates with one new (compared to elementary bosons) tetrad. (Notions related to the $k = 6$ row in Table 4 pertain.) Fractional charge associates with one new (compared to elementary bosons) tetrad. (Notions related to the $\Sigma = 1$ row in Table 4 pertain.)

SUPP suggests that the change of two regarding a relevant (per Table 4) number of tetrads might associate with notions that associate with Eq. (68). Here, the symbol EF denotes elementary fermions. The symbol EB denotes elementary bosons.

$$d‘(EF) = d‘(EB) - 2 = -1 \qquad (68)$$

Regarding QM raising and lowering operators for elementary fermion states, the following notions might pertain. States associate with the range $0 \leq n_f \leq 1$ for integers n_f. For the raising operator, a factor that

Eq. (69) shows pertains. For the lowering operator, a factor of $(n_f)^{1/2}$ pertains.

$$(1 + d' n_f)^{1/2} = (1 - n_f)^{½} \qquad (69)$$

Notions above might point to possibilities for future physics principles. This paper does not explore these notions further.

5.9.4. Notions regarding possible integers N' for elementary fermions

Eqs. (36), (66), and (68) might point toward relationships among the properties of elementary fermions. $d' = -1$, Eq. (70), and Eq. (71) might pertain regarding elementary fermions.

$$log(X'/X_{ref}) = \int_{X_{ref}}^{X'} x^{d''} dx, \text{ for a property for which the } X' \text{value is nonzero} \qquad (70)$$

$$N' = \sum_{\{X'\}} log(X'/X_{ref}) \qquad (71)$$

Discussion related to Eq. (95) explores this possibility and points to at least one possible member of the set $\{X'\}$.

5.10. Harmonic oscillator mathematics, gauge symmetries, and the Higgs mechanism

This unit discusses possible similarities between gauge symmetries that POST SM features and some symmetries that SUPP suggests. This unit discusses possible associations between the POST SM notion of the Higgs mechanism and notions that SUPP suggests.

5.10.1. Isotropic harmonic oscillator math - PDE (partial differential equation) solutions

Modeling for a j-dimensional isotropic harmonic oscillator can feature j linear coordinates $x_{k'}$ - each with a domain $-\infty < x_{k'} < \infty$ - and an operator that is the sum - over k' - of j operators of the form that Eq. (72) shows. The number C is positive and is common to all j uses of Eq. (72). The word isotropic associates with the commonality - across all j uses of Eq. (72) - of the number C.

$$-\frac{\partial^2}{\partial (x_{k'})^2} + C \cdot (x_{k'})^2 \qquad (72)$$

For $j \geq 2$, one can split the overall operator into pieces. Eq. (73) associates with a split into two pieces. Here, each of j_1 and j_2 is a positive integer.

$$j = j_1 + j_2 \qquad (73)$$

In the discussion below, the symbol D might be any one of j, j_1, and j_2.

For $D \geq 2$, mathematics related to isotropic harmonic oscillators can feature partial differential equations, a radial coordinate, and $D-1$ angular coordinates. Eq. (74) defines a radial coordinate.

$$x = (\sum_{k'} (x_{k'})^2)^{½} \qquad (74)$$

SUPP suggests replacing x via the expression that Eq. (75) shows. Here, r_{HO} denotes the radial coordinate and has dimensions of length. The parameter η has dimensions of length. The parameter η is a nonzero real number. The magnitude $|\eta|$ associates with a scale length. (Here, r_{HO} associates with mathematics for HO - or, harmonic oscillators - and does not necessarily associate with uses of r elsewhere - for example, in Eq. (16) - in this paper.)

$$x = r_{HO}/\eta \qquad (75)$$

In POST applications, the following HO notions can pertain. Solutions - that can associate with wave functions - to the pair of Eqs. (76) and (77) can have the form $\Psi = \phi_R(r_{HO})Y$, in which Y is a function of $D-1$ angular coordinates and is not a function of r_{HO}. Ω associates with operators that associate with angular coordinates. (For $D=3$, Ref. [153] shows a representation for Ω in terms of an operator that is a function of spherical coordinates.) D is a nonnegative integer. The domain for r_{HO} is $0 \leq r_{HO} < \infty$. Each one of ξ and ξ' is an as-yet unspecified constant. For $D=1$, Eqs. (76) and (77) might not be appropriate.

$$\xi\Psi = (\xi'/2)(-\eta^2\nabla^2 + (\eta)^{-2}(r_{HO})^2)\Psi \qquad (76)$$

$$\nabla^2 = (r_{HO})^{-(D-1)}(\partial/\partial r_{HO})(r_{HO})^{D-1}(\partial/\partial r_{HO}) - \Omega(r_{HO})^{-2} \qquad (77)$$

This paper considers solutions that comport with Eqs. (78), (79), (80), (81), (82), (83), and (84). With respect to the domain $0 \leq r_{HO} < \infty$, ϕ_R associates with the mathematical notion of having a definition almost everywhere. In POST, solutions that associate with Eq. (72) and with $D=1$ have the form $H(x)\exp(-x^2)$, in which $H(x)$ is a Hermite polynomial. In this paper, for each relevant D, each solution that is relevant associates with - in effect - a one-term polynomial. In this paper, $D=1$ is a relevant D. Eqs. (82) and (83) echo Eqs. (76) and (77). (Per Eq. (87), the fact that the function $\phi_R(r_{HO})$ normalizes will be significant. Per the equal-sign symbol in Eq. (84), normalization to a value of one is not necessarily relevant in this paper.)

$$D \text{ is a real number} \qquad (78)$$

$$\Omega \text{ is a constant} \qquad (79)$$

$$\phi_R(r_{HO}) \text{ is a function of just } r_{HO}, \eta, \text{ and a number } \nu \qquad (80)$$

$$0 < r_{HO} < \infty \qquad (81)$$

$$\xi\phi_R(r_{HO}) = (\xi'/2)(-\eta^2\nabla^2 + (\eta)^{-2}(r_{HO})^2)\phi_R(r_{HO}) \qquad (82)$$

$$\nabla^2 = (r_{HO})^{-(D-1)}(\partial/\partial r_{HO})(r_{HO})^{D-1}(\partial/\partial r_{HO}) - \Omega(r_{HO})^{-2} \qquad (83)$$

$$\phi_R(r_{HO}) \ = \ (r_{HO}/\eta)^\nu \exp(-(r_{HO})^2/(2\eta^2)), \text{ with } \eta^2 > 0 \qquad (84)$$

Eqs. (85) and (86) characterize solutions of the form that Eq. (84) shows. The parameter η does not appear in Eqs. (85) and (86).

$$\xi = (D + 2\nu)(\xi'/2) \qquad (85)$$

$$\Omega = \nu(\nu + D - 2) \qquad (86)$$

$\phi_R(r_{HO})$ normalizes if and only if Eq. (87) pertains. The symbol $(\phi_R(r_{HO}))^*$ denotes the complex conjugate of $\phi_R(r_{HO})$.

$$\int_0^\infty (\phi_R(r_{HO}))^*\phi_R(r_{HO}) \cdot (r_{HO})^{D-1} dr_{HO} < \infty \qquad (87)$$

Eq. (88) associates with the domains of D and ν for which normalization pertains for $\phi_R(r_{HO})$. For $D + 2\nu = 0$, normalization pertains in the limit $\eta^2 \to 0^+$. Regarding mathematics relevant to normalization for $D + 2\nu = 0$, the delta function that Eq. (89) shows pertains. Here, $(x')^2$ associates with $(r_{HO})^2$ and 4ϵ associates with η^2. (Ref. [154] provides Eq. (89).) The difference in domains, between $-\infty < x' < \infty$ and Eq. (81), is not material here.

$$D + 2\nu \geq 0 \qquad (88)$$

$$\delta(x') = \lim_{\epsilon\to 0^+}(1/(2\sqrt{\pi\epsilon}))e^{-(x')^2/(4\epsilon)} \qquad (89)$$

5.10.2. Isotropic harmonic oscillator math - ground-state symmetries

Per Ref. [155], for $n \geq 2$, $SU(n)$ symmetry associates with the ground state of an isotropic n-dimensional harmonic oscillator. Eq. (90) pertains. Here, $\text{gen}(GX)$ denotes the number of generators of the group GX.

$$\text{gen}(SU(l)) = l^2 - 1, \ \text{for } l \geq 2 \qquad (90)$$

5.10.3. SUPP, QM excitations, SM gauge symmetries, and the Higgs mechanism

POST makes the following associations between some interactions and gauge groups. The electroweak interaction associates with $SU(2) \times U(1)$ symmetry. The electromagnetic interaction associates with $U(1)$ symmetry. The strong interaction associates with $SU(3)$ symmetry. POST points to difficulties regarding developing a so-called Grand Unified Theory, which would unite - at high energies - into one force the electromagnetic interaction, the weak interaction, and the strong interaction. (Ref. [156] provides an overview of grand unification.)

Regarding elementary particles, SUPP suggests that 1d0 solution-pairs that Table 6 shows associate with aspects of interactions in which elementary particles participate.

Notions related to Table 6 suggest that the following SUPP notions pertain regarding relationships between 0d0 solution-pairs and the related 1d0 (or, one-step) cascades.

- For each case, one 0d0 solution-pair pertains. For some cases, a second 0d0 solution-pair pertains. SUPP suggests that the monad that associates with one solution-pair associates with whether the relevant elementary particles have nonzero mass or zero mass. SUPP suggests that, if a second solution-pair pertains, the monad associates with associating the second 0d0 solution-pair with the first 0d0 solution-pair.
- Each one-step cascade associates with a set 1d0:Z_Γ.
  - For each case that does not associate with the strong interaction, each one-step cascade associates with a set 1d0:Z_Γ that differs from the set 1d0:Z_Γ that associates with each other one-step cascade. SUPP suggests that three triad-related aspects pertain.
  - For each case that associates with the strong interaction, there are exactly two one-step cascades, and the two one-step cascades share one set 1d0:Z_Γ. SUPP suggests that two sets of three triad-related aspects pertain.
- For each case that associates with nonzero mass and nonzero spin, the following notions pertain. (Here, for bosons, SUPP suggests that the word aspect associates - from a perspective of modeling and mathematics - with a one-dimensional harmonic oscillator.)
  - For each case that does not associate with the strong interaction, SUPP suggests that one triad-related aspect associates with excitation and the other two triad-related aspects associate with $SU(2)$ symmetry. (Regarding excitation for fermions, discussion related to Eq. (69) pertains. Regarding excitation for bosons, discussion related to Eq. (67) pertains.) For a case that associates

with fermion elementary particles, SUPP suggests that the three generators that associate with the $SU(2)$ symmetry associate with three flavours. For a case that associates with boson elementary particles, SUPP suggests that the $SU(2)$ symmetry might associate with the $SU(2)$ symmetry that the SM considers to be part of the $SU(2) \times U(1)$ gauge symmetry that the SM associates with the electroweak interaction.

- For each case that associates with the strong interaction and with fermions, quark elementary particles pertain. This paper does not discuss these cases.

- For each case that associates with zero mass (and, thus, with the notion of boson) and nonzero spin, the following notions pertain. (Here, SUPP suggests that the word aspect associates - from a perspective of modeling and mathematics - with a one-dimensional harmonic oscillator.)
  - For each case that does not associate with the strong interaction, SUPP suggests that two triad-related aspects associate with excitation and that the other one triad-related aspect associates with $U(1)$ symmetry. (Regarding excitation for bosons, discussion related to Eq. (67) pertains.) One of the first two aspects associates with excitation for the left-circular polarization mode. The other one of the first two aspects associates with excitation for the right-circular polarization mode. For the case that associates with photon elementary particles, SUPP suggests that the $U(1)$ symmetry might associate with the $U(1)$ symmetry that the SM associates with the electromagnetic interaction.
  - For each case that associates with the strong interaction, SUPP suggests that notions - regarding excitations - discussed just above pertain regarding one set of three aspects and that three new (compared to notions discussed just above) aspects pertain. SUPP suggests that one triad-related aspect that above associates with $U(1)$ associates with aligning the three new aspects with the three previous aspects. SUPP suggests that the three new aspects associate with $SU(3)$ symmetry. SUPP suggests that the $SU(3)$ symmetry might associate with the $SU(3)$ symmetry that the SM associates with the strong interaction.

- For the two cases that associate with zero spin (and, thus, with the notion of boson), the following notions pertain. (Here, SUPP suggests that the word aspect associates - from a perspective of modeling and mathematics - with a one-dimensional harmonic oscillator.)
  - For the case that associates with nonzero mass, the following notions pertain. POST associates the ground state for the Higgs field with a notion of three spatial dimensions. SUPP suggests that the three triad-related aspects associate with modeling that associates excitations with one three-dimensional harmonic oscillator. With respect to three spatial dimensions and per Eq. (85), SUPP

suggests that the ground state of the Higgs field associates with $D = 3$ and $\nu = 0$. Per Eq. (88), SUPP suggests that the state $D = 3$ and $\nu = -1$ can have relevance. SUPP suggests that the ground state of the Higgs boson associates with $D = 3$ and $\nu = -1$. Per Eq. (85), the ground state ($D = 3$ and $\nu = 0$) for the Higgs field would associate with one more unit of energy than does the three-dimensional ground state ($D = 3$ and $\nu = -1$) for the Higgs boson. (SUPP suggests that, regarding POST notions of excitations of the Higgs boson, modeling associates with a ground state that associates with $D = 1$ and $\nu = 0$. SUPP suggests that discussion related to Eq. (67) pertains.) SUPP suggests that these notions might associate with SM notions of the Higgs mechanism.

- This paper does not discuss the case that associates with zero mass (and the inflaton).

Thus, SUPP might include notions that associate with the SM gauge symmetries and the Higgs mechanism. This paper does not discuss such notions further. (The discussion above suggests possibilities for symmetries that might pertain regarding other interactions, including interactions that would associate with gravitons. This paper does not discuss such notions further.)

This paper does not discuss further the notion that SUPP might provide further insight regarding the POST SM notion of grand unification.

5.11. Group theory mathematics, reaches, and the SUPP-suggested jay boson

This unit discusses possibly useful notions that might link group-theoretic expressions to modeling regarding reaches of components of LRI fields and to modeling regarding the SUPP-suggested jay boson.

5.11.1. Reaches that pertain regarding components of LRI fields

Discussion related to Eq. (18) posits reaches R_I that associate with 0d> uses of LRI solution-pairs.

Per Eq. (8) and Eq. (9), $0 \leq n_0 \leq 3$.

SUPP posits that Eq. (91) pertains for $1 \leq n_0 \leq 3$.

$$R_I = \mathrm{gen}(SU(7))/\mathrm{gen}(SU((2 \times (0\mathrm{d}{>}{:}n_0)) + 1)) \qquad (91)$$

SUPP suggests the possibility that modeling might link the factor (in Eq. (91)) of two to the notion that - for a k that is not a member of Z_Γ - the two notions ($s_k = +1$ and $s_k = -1$) of nonzero s_k do not play a role in Eq. (4).

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5.11.2. The jay boson

POST associates the eight gluons with modeling that features the group $SU(3)$. Mathematically, Eq. (92) pertains. (This use of the symbols $GX \supset GY$ associates with the notion that the group GX includes the group GY.)

$$U(3) \supset SU(3) \qquad (92)$$

Mathematically, Eq. (93) pertains. Here, I denotes a representation for the identity (or, trivial) group.

$$U(3) \supset I \qquad (93)$$

SUPP - including notions that associate with Table 6 - suggests the possibility that modeling that associates with Eq. (93) might prove useful. In Table 6, gluons and the jay boson share one 0d0 solution-pair and two 1d0 solution-pairs.

5.12. Possible bases for insight regarding the three-body problem

This unit suggests a possible extrapolation - from SUPP notions regarding the spin states of two-component systems - that might lead to insight regarding the spin states of three-component systems.

SUPP associates internal states of elementary particles with 0g solution-pairs for which $\{1, 3\} \subset Z_\Gamma$ and - for each positive odd number k for which $k \geq 5$ - $k \notin Z_\Gamma$. SUPP associates internal states of some two-component systems with 0g solution-pairs for which $\{5, 7\} \cap Z_\Gamma \neq \emptyset$ and - for each positive odd number k for which $k \geq 9$ - $k \notin Z_\Gamma$.

SUPP suggests - and this paper does not further discuss - the following notions. Internal states of some three-component systems might associate with 0g solution-pairs for which $\{9, 11\} \cap Z_\Gamma \neq \emptyset$. Some properties of some three-component systems might associate with Σg solution-pairs for which $\Sigma \geq 1$ and $\{9, 11\} \cap Z_\Gamma \neq \emptyset$.

5.13. Connections between classical modeling and quantum modeling

This unit suggests the possibility that SUPP might provide bases for insight about connections between continuous CM modeling and discrete QM modeling.

POST explores connections between continuous CM modeling and discrete QM modeling. (Ref. [157] discusses aspects of this exploration and provides further references. Ref. [158] discusses aspects

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regarding electromagnetism.) Such explorations tend to explore notions of developing discrete modeling from bases that feature continuous modeling.

SUPP features discrete notions. SUPP notions, such as the n_T-related modeling principle that associates with Eq. (21), suggest possible relationships between POST continuous and SUPP discrete.

Eq. (20) includes aspects that associate with continuous and aspects that associate with discrete.

Steps from notions that associate with Table 4 to notions that associate with Table 5a to notions that associate with Table 5b might provide insight regarding transitions - regarding modeling - between discrete (including QM) and continuous (including CM).

This paper does not further address the extent to which SUPP might provide bases for insight about connections between continuous CM modeling and discrete QM modeling.

5.14. Isomer, as a new internal quantum number for elementary particles

This unit discusses the notion that l_I (as in isomer number) associates with the notion of a new (compared to POST QM) internal quantum number for elementary particles.

In Table 2, each row that associates with $n_I > 1$ associates with the SUPP notion that isomer is a property of objects.

Regarding elementary particles for which $\Phi \neq$L, SUPP suggests that $n_I = 6$.

Table 9 suggests associations between some internal quantum numbers (for some elementary fermions) and l_I (as in isomer number).

SUPP suggests that the SUPP notion of l_I associates with a new (compared to POST QM) internal elementary-particle quantum number.

5.15. Some notions that might interrelate properties of elementary fermions

This unit discusses the possibility that an analog for fermion elementary particles exists to an equation that suggests integer-based linkage between properties of boson elementary particles.

Eq. (36) suggests integer-based linkage between properties of elementary bosons. Discussion related to Eq. (71) indicates possibilities for an analog - to Eq. (36) - for elementary fermions.

This paper notes two cases that might suggest possibilities for an analog - to Eq. (36) - for elementary fermions. This paper does not provide a completed analog - to Eq. (36) - for elementary fermions.

Case one considers just the three charged leptons.

For $n_q = 3$, Eq. (94) restates Eq. (45).

$$\log(m(l_m, l_q)/m_e) = (l_m + j_m w_m(2)) \log(\beta^{1/3}) \qquad (94)$$

Per discussion related to Eq. (56), the notion of m'_{l_m} might have relevance (at least regarding modeling), even though - for the muon (for which $l_m = 2$) - $m'_{l_m} \neq m(l_m, 3)$. Regarding Eq. (71), l_m might be a member of $\{X'\}$. Here, the relevant X_{ref} can associate with m'_e (which equals m_e). Per Eq. (94), a change of plus one regarding l_m associates with a multiplicative factor of $\beta^{1/3}$.

To include j_m in $\{X'\}$, one might associate (per Eq. (47)) a change of plus one regarding j_m with a multiplicative factor of $\beta^{(1/3)w_m(2)}$. Per Eq. (47), j_m and l_m have some independence from each other. One might consider that, regarding a choice of X_{ref} that pertains to j_m, X_{ref} can associate with $j_m = 2$ (and with the muon).

Case two considers all known elementary fermions, except the three charged leptons.

Per Eq. (49) and discussion related to Eq. (49), $n_q = 3/2$ associates with nine known elementary fermions (that is, with all known elementary fermions, except the three charged leptons). (The three charged leptons associate with $l_q = 3$).

For $n_q = 3/2$, Eq. (95) restates Eq. (45). Here, either $l_q = 3/2$ or $l_q = 0$ pertains.

$$\log(m(l_m, l_q)/m_e) = l_m \log(\beta^{1/3}\alpha^{-3/8}) + j_m w_m(2) \log(\beta^{1/3}) + \log(\alpha^{-3/8}) + 0 \qquad (95)$$

Eqs. (96) and (97) restate Eq. (95).

$$l_m = \log([\{(m(l_m, l_q)\}^{1/\log(\beta^{1/3}\alpha^{-3/8})}]/[\{m_e\}^{1/\log(\beta^{1/3}\alpha^{-3/8})}]) - Y \qquad (96)$$

$$Y = (\log(\beta^{1/3}\alpha^{-3/8}))^{-1}(j_m w_m(2) \log(\beta^{1/3}) + \log(\alpha^{-3/8})) \qquad (97)$$

SUPP suggests that Eqs. (96) and (97) point to the possibility that a version of Eq. (71) might pertain regarding (at least nine and possibly all) elementary fermions. For such a version of Eq. (71), one X' might associate with the integer l_m and with the property $\{(m(l_m, l_q)\}^{1/\log(\beta^{1/3}\alpha^{-3/8})}$. Another X' might associate with the integer j_m and with a property for which a statement of the value includes the constant $w_m(2)$. (Eq. (48) defines $w_m(2)$.)

Beyond the work above regarding case one and case two, this paper does not further explore developing or using a form of Eq. (71) for elementary fermions.

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5.16. Possibilities regarding sub-elementary-particle physics and yet more quantum numbers

This unit suggests that SUPP might hint at aspects of nature that POST might associate with notions of extending - beyond elementary particle physics - a series that might include chemistry, chemical elements, atomic physics, nuclear physics, intra-hadron physics, elementary particle physics, and so forth.

SUPP notions regarding either one (or both) of non-point-like magnetic moment and non-point-like intrinsic angular momentum might suggest that - regarding (at least) elementary fermions - at least one of magnetic moment or intrinsic angular momentum associates with non-point-like spatial distributions of properties. That notion hints at possibilities for modeling that associates with aspects of nature that people might consider as associating with notions of components of elementary particles.

SUPP notions regarding integers N' (as in Eqs. (36) and (71)) - and regarding, in effect, components that add up to yield the integers N' - might suggest that modeling for something that people might consider to be smaller than elementary particles might prove useful.

Work related to Eqs. (36) and (71) might suggest a notion that - in some as yet possibly not fully specified sense - N' functions as a quantum number. That notion hints at possibilities for finding more relationships regarding properties, for considering a new modeling space that associates with such relationships, and for developing principles that associate with the new modeling space and inter-property relationships.

6. Concluding remarks

This unit lists specific predictions that this paper makes. This unit lists specific aspects - that this paper discusses - regarding modeling. This unit suggests a general perspective about notions that this paper discusses.

6.1. Specific predictions and specific aspects regarding modeling

This unit lists specific predictions that this paper makes. This unit lists specific aspects - that this paper discusses - regarding modeling.

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6.1.1. Quantitative predictions

This paper makes the following quantitative predictions. It might be possible to verify or refute most of the predictions soon or within not very many years.

- New elementary particles. (Table 6 lists suggested new elementary particles and all known elementary particles.) Popular modeling anticipates some of the suggested new elementary particles, such as an inflaton and a graviton. Popular modeling seems not to anticipate some of the suggested new elementary particles, such as a so-called jay boson.
- The following properties of elementary particles, plus the following relationships between properties of specific elementary particles.
  - More (than currently known) accurate masses for the W boson and the Higgs boson, plus integer ratios regarding the squares of the masses of the W boson, the Z boson, and the Higgs boson. (Eq. (35) pertains.)
  - Relationships between the properties of all known and some suggested elementary bosons. (Table 7 pertains.)
  - A more (than currently known) accurate mass for the tau elementary fermion. (Eq. (43) shows the predicted mass.)
  - Various ratios of masses of charged elementary fermions. (Eq. (45), Table 8, and Eq. (71) pertain.)
  - A more (than currently known) accurate anomalous magnetic moment for the tau elementary fermion. (Eq. (43) shows the predicted anomalous magnetic moment.)
  - Well-specified masses for two neutrinos and a choice among three masses for the other neutrino. (Discussion related to Eq. (58) shows the suggested masses.)
  - Relationships between properties of all known charged elementary fermions. (Discussion related to Eqs. (45), (71), and (95) pertains.)
- A well-specified description of dark matter. (Discussion that leads to and includes Eq. (18) pertains. Table 9

  pertains.)
- Seemingly preferred ratios of dark-matter effects to ordinary-matter effects. (Table 12 pertains. Observations seem to suggest some of these ratios.)
  - Ratios regarding the production and absorption of cosmic optical background and CMB (as in cosmic microwave background radiation). (Table 12a

    pertains.)

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    - Ratios regarding the composition of some galaxies. (Table 12b pertains.)
    - Ratios regarding the composition of galaxy clusters (that have not collided with other galaxy clusters) and ratios regarding the composition of the universe. (Table 12c pertains.)

6.1.2. Qualitative predictions

This paper makes the following predictions. It might be possible to verify or refute most of the predictions soon or within not very many years.

- Specific aspects of galaxy formation and evolution. (Table 11 pertains.)

6.1.3. Suggestions for reducing seeming gaps between popular modeling and data

This paper makes the following suggestions regarding how to reduce seeming discrepancies between popular modeling and data. It might be possible to test and use most of the suggestions now.

- The new modeling points (at least qualitatively) to how to reduce so-called tensions - between data and popular modeling - regarding the following.
    - The rate of expansion of the universe. (Discussion that cites Ref. [53] pertains.)
    - Large-scale clumping of matter. (Discussion that cites Ref. [103] pertains.)
    - Effects on galaxies of gravity associated with nearby galaxies. (Discussion that cites Ref. [71] pertains.)
- The new modeling points - regarding gravity and the Einstein field equations - to the following notions. (Discussion that mentions specific components - including T^{21} - of the stress-energy tensor pertains.)
    - The new modeling comports with the notion that - regarding circumstances for which physics has tested the Einstein field equations - using the Einstein field equations is appropriate.
    - The new modeling suggests notions that might improve the effectiveness of using the Einstein field equations regarding some circumstances for which notions of tensions - between modeling and data - pertain.
    - The new modeling comports with the notion that - regarding some circumstances - using the Einstein field equations would not be adequately accurate.

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6.1.4. Relationships between suggested new modeling and popular modeling

This paper suggests the following notions regarding the new modeling that this paper discusses.

- The new modeling aligns itself with popular modeling, including the following notions.
  - The new modeling associates with (and extends from) a notion - that uses integer-arithmetic equations - that associates with Newtonian modeling regarding forces. (Discussion that includes Table 1 pertains. Discussion that leads to Eq. (12) pertains. Discussion that leads to Eq. (21) pertains. Discussion that includes Eq. (16) pertains.)
  - The new modeling associates - via a new modeling principle - with aspects of popular modeling. (Discussion that leads to and includes Eq. (21) pertains.)
  - The new modeling suggests reuses for aspects that associate with popular modeling. (The notion of isomers of a set that includes all known elementary particles, except the photon, provides an example. Discussion related to Eq. (18) and discussion related to Table 9 pertain.)
  - The new modeling shows possible associations (regarding the new modeling) with elementary particle Standard Model gauge symmetries and with the Standard Model notion of the Higgs mechanism. (Discussion that cites Ref. [156] pertains.)
- The new modeling augments popular modeling by doing the following.
  - Showing and using modeling that organizes a set of physics properties (including charge, magnetic moment, mass or energy, momentum, and angular momentum). Suggesting new (compared to popular modeling) aspects regarding properties. (Table 3 pertains.)
  - Including modeling that outputs a list of all known and some suggested new elementary particles and that suggests organizing principles for cataloging elementary particles. (Table 6 and related discussion pertain.)
  - Suggesting a new modeling principle. (Discussion that leads to and includes Eq. (21) pertains.)
  - Suggesting a new elementary particle internal quantum number - isomer. (Discussion related to Eq. (18) and Table 9
  - Including modeling that outputs a list of properties that associate with elementary particles. (Table 4 pertains.)
  - Including new modeling for spin-states of two-component objects. (Discussion related to Eq. (27) pertains.)
  - Including new modeling regarding electromagnetic atomic transitions. (Discussion related to Eq. (29) pertains.)

- Including modeling that outputs a list of conservation laws. (Table 5 pertains.)
- Suggesting possibly new insight about relationships among observations of properties, aspects of fields (such as the electromagnetic field) that convey information about the properties of objects, and Lorentz invariance. (Discussion regarding Eq. (13) pertains. Table 3 pertains.)
- Identifying components - of gravitational fields - that associate with attraction and repulsion between objects. (Discussion related to Eqs. (14) and (15) pertains.)

  Suggesting a mechanism that led to baryon asymmetry. (Discussion related to Eq. (60) pertains.)
- Suggesting a mechanism that might convert some energy associated with one of ordinary matter and dark matter to energy that would associate with the other one of ordinary matter and dark matter. (Discussion related to Eq. (59) pertains.)
- Suggesting eras (regarding increasing and decreasing rates of expansion of the universe) and mechanisms that drive those eras. The eras include two known eras, the possible inflationary epoch, and some possible earlier eras. (Table 10 pertains.)
- Suggesting a new way to estimate - given data about two charged leptons - the anomalous magnetic moment of the third charged lepton. (Discussion related to Eq. (57) pertains and yields a result for the tau - that comports with data and seems to comport with results from the Standard Model. Eq. (43) pertains.)

6.1.5. Suggestions regarding possible experiments and observations

This paper makes the following suggestions regarding possible experiments and observations. It does not make suggestions as to when the experiments or observations might be feasible.

- Possible means for directly detecting dark matter objects that have similarities to ordinary matter atoms. (Discussion related to Eq. (29) and Table 12a pertains.)
- Possible new types of information that gravity waves might convey. (Discussion that cites Ref. [150] pertains.)

6.1.6. Suggestions regarding possible new aspects of physics

This paper makes the following suggestions regarding possible new aspects of physics.

- New aspects of physics might arise from notions that (at least some) elementary particles might be modeled as having structure. (Discussion related to Eq. (13), Table 3, and Eq. (55) suggests that the nonzero values of anomalous magnetic moments for charged leptons are associated with modeling that suggests oblate distributions of properties.)

- New aspects of physics might arise from relationships between electromagnetic properties and gravitational properties (Eq. (42) suggests a relationship.), between strong-interaction properties and gravitational properties (Discussion regarding Eq. (64) suggests a relationship.), or between weak-interaction properties and gravitational properties (Discussion regarding Eq. (64) suggests a relationship.).
- New aspects of physics might arise from aspects that new modeling suggests regarding new quantum numbers that associate with elementary particles. (Discussions related to Eqs. (36) and (71) pertain.)

6.2. General perspective

This unit suggests a general perspective about notions that this paper discusses.

Each one of the following sentences describes a physics challenge that has persisted for the most recent eighty or more years. Interrelate physics properties, properties of objects, and physics constants. Provide, for elementary particles, an analog to the periodic table for chemical elements. Describe bases for phenomena that POST (as in modeling that has bases in POpular modeling notions of Space-Time coordinates) associates with the two-word term dark matter. Describe bases for phenomena that POST associates with the two-word term dark energy. Explain the overall evolution of the universe. Interrelate physics models. Develop a list of principles that underlie physics or physics modeling.

Physics amasses data that people can use as bases for developing and evaluating modeling aimed at addressing the challenges.

SUPP (as in SUggested physics modeling based on notions of principles and modeling that associate with Particle Properties) addresses those physics challenges and has bases in the following mathematics - multipole expansions, integer-arithmetic equations, and multidimensional harmonic oscillators.

SUPP suggests a new principle - that associates with notions of tetrads - that links POST and SUPP.

SUPP suggests a new elementary-particle internal quantum number - isomer - that associates with notions that dark matter has similarities to ordinary matter.

SUPP unites and decomposes aspects of electromagnetism and gravity. For each of those two long-range interactions, the decomposition associates with properties - of objects - that people can measure and that POST features. For electromagnetism, the properties include charge and magnetic moment. For gravity, the properties include energy and intrinsic angular momentum.

SUPP points to all known elementary particles and to some might-be elementary particles. POST suggests some, but not all, of the might-be elementary particles. SUPP suggests relationships between properties of elementary particles. SUPP suggests more (compared to data or to POST modeling) accurate values for some properties of some elementary particles.

SUPP includes a notion of isomers of elementary particles that do not mediate long-range interactions. SUPP features a notion of instances of components of long-range interactions.

SUPP suggests a description of dark matter.

SUPP suggests explanations for data regarding dark matter. SUPP points to possible resolutions for tensions - between data and POST - regarding dark-energy phenomena. SUPP suggests insight regarding galaxy formation and evolution.

SUPP suggests explanations for data that POST seems not to explain, suggests results regarding data that people have yet to gather, and points to possible opportunities to develop models that unite aspects of physics and physics modeling. SUPP seems not to disturb aspects of POST that comport with data.

In summary, SUPP suggests augmentations - to POST - that might achieve the following results. Extend the list of elementary particles. Predict masses for at least two neutrinos. Predict masses - that would be more accurate than known masses - for some other elementary particles. Describe dark matter. Explain ratios of dark-matter effects to ordinary-matter effects. Provide insight regarding galaxy formation. Describe bases for phenomena that associate with the two-word term dark energy. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models. Point to new principles regarding physics modeling. Point to physics phenomena that might underlie elementary particle phenomena.

Statements and Declarations

Data Availability

This study created no new data. All data underpinning this publication are cited by the article.

Author Contributions

T.J.B. conceived and designed the analysis, performed the analysis, and wrote the paper.

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References

1. ^*Galilei G (1954). Dialogues Concerning Two New Sciences. Dover Publications. https://www.abebooks.com/9780486600994/DialoguesConcerning-Two-New-Sciences-0486600998/plp.*
2. a, b*Newton I (1687). Philosophiae Naturalis Principia Mathematica. Jussu Societatis Regiae ac Typis Josephi Streater. doi:10.3931/E-RARA-440.*
3. ^*Mendeleev D (1869). "On the Relationship of the Properties to the Atomic Weights of the Elements." Journal of Chemistry. **12**:405–406. https://babel.hathitrust.org/cgi/pt?id=uc1.b3481652.*
4. ^*Hou SQ, He JJ, Parikh A, Kahl D, Bertulani CA, Kajino T, Mathews GJ, Zhao G (2017). "NON-EXTENSIVE STATISTICS TO THE COSMOLOGICAL LITHIUM PROBLEM." The Astrophysical Journal. **834**(2):165. doi:10.3847/1538-4357/834/2/165.*
5. ^*Becker M, Caprez A, Batelaan H (2015). "On the Classical Coupling between Gravity and Electromagnetism." Atoms. **3**(3):320–338. doi:10.3390/atoms3030320.*
6. ^*Banados M, Barnich G, Compere G, Gomberoff A (2006). "Three-dimensional origin of Gödel spacetimes and black holes." Phys. Rev. D. **73**:044006. doi:10.1103/PhysRevD.73.044006.*
7. ^*Barnich G, Gomberoff A (2008). "Dyons with potentials: Duality and black hole thermodynamics." Phys. Rev. D. **78**:025025. doi:10.1103/PhysRevD.78.025025.*
8. ^*Einstein A (1997). The Collected Papers of Albert Einstein, Volume 6: The Berlin Years. Princeton University Press.*

9. ^Medina JR (2006). "Gravitoelectromagnetism (GEM): A Group Theoretical Approach." Drexel University. https://core.ac.uk/download/pdf/190333514.pdf.
10. ^Delphenich D (2015). "Pre-Metric Electromagnetism as a Path to Unification." In: Unified Field Mechanics. World Scientific. https://arxiv.org/ftp/arxiv/papers/1512/1512.05183.pdf.
11. ^Papini G (2020). "Some Classical and Quantum Aspects of Gravitoelectromagnetism." Entropy. **22**(10):1089. doi:10.3390/e22101089.
12. ^a, b Nadis S (2022). "Mass and Angular Momentum, Left Ambiguous by Einstein, Get Defined." Quanta Magazine. https://www.quantamagazine.org/mass-and-angular-momentum-leftambiguous-by-einstein-get-defined-20220713.
13. ^Gorkavyi N, Vasilkov A (2016). "A repulsive force in the Einstein theory." Monthly Notices of the Royal Astronomical Society. **461**(3):2929–2933. doi:10.1093/mnras/stw1517.
14. ^Damour T (2019). "21: Experimental Tests of Gravitational Theory." In: Zla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.
15. ^Kramer M, Stairs IH, Manchester RN, Wex N, Deller AT, Coles WA, Ali M, Burgay M, Camilo F, Cognard I, et al. (2021). "Strong-Field Gravity Tests with the Double Pulsar." Phys. Rev. X. **11**(4):041050. doi:10.1103/physrevx.11.041050.
16. ^Everitt CWF, DeBra DB, Parkinson BW, Turneaure JP, Conklin JW, Heifetz MI, Keiser GM, Silbergleit AS, Holmes T, Kolodziejczak J, et al. (2011). "Gravity Probe B: Final Results of a Space Experiment to Test General Relativity." Phys. Rev. Lett. **106**:221101. doi:10.1103/PhysRevLett.106.221101.
17. ^Touboul P, Metris G, Rodrigues M, Berge J, Robert A, Baghi Q, Andre Y, Bedouet J, Boulanger D, et al. (2022). "MICROSCOPE Mission: Final Results of the Test of the Equivalence Principle." Phys. Rev. Lett. **129**:121102. doi:10.1103/PhysRevLett.129.121102.
18. ^Chae KH (2023). "Breakdown of the Newton–Einstein Standard Gravity at Low Acceleration inInternal Dynamics of Wide Binary Stars." The Astrophysical Journal. **952**(2):128. doi:10.3847/1538-4357/ace101.
19. ^Mann R (2009). An Introduction to Particle Physics and the Standard Model. CRC Press. doi:10.1201/9781420083002.
20. ^Gasiorowicz S, Langacker P. "Elementary Particles in Physics." University of Pennsylvania. https://www.physics.upenn.edu/~pgl/e27/E27.pdf.
21. ^Hebecker A, Hisano J (2019). "94: Grand Unified Theories." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.

22. ^a, b^ *Ringwald A, Rosenberg LJ, Rybka G (2019). "91: Axions and Other Similar Particles." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.*
23. ^a, b^ *Rolli S, Tanabashi M (2019). "95: Leptoquarks." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.*
24. ^a, b^ *Milstead D, Weinberg EJ (2019). "96: Magnetic Monopoles." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.*
25. ^a, b, c^ *Zyla PA, et al. (2020). "Review of Particle Physics." PTEP.* ***2020****(8):083C01. doi:10.1093/ptep/ptaa104.*
26. ^a, b^ *Green B (2020). Until the End of Time: Mind, Matter, and Our Search for Meaning in an Evolving Universe. Alfred A. Knopf. https://www.penguinrandomhouse.com/books/549600/until-the-end-of-time-by-brian-greene/.*
27. ^ *Misner CW, Thorne KS, Wheeler JA (2017). Gravitation. University of Princeton Press. https://press.princeton.edu/books/hardcover/9780691177793/gravitation.*
28. ^a, b, c, d^ *Gonzalez-Garcia MC, Yokoyama M (2019). "14: Neutrino Masses, Mixing, and Oscillations." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.*
29. ^ *Vogel P, Piepke A (2019). "Neutrino Properties." In: Zyla PA et al. Prog.Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.*
30. ^ *Elfgren E, Fredriksson S (2007). "Mass limits for heavy neutrinos." Astronomy and Astrophysics.* ***479****(2):347–353. doi:10.1051/0004-6361:20078898.*
31. ^ *Peccei RD, Quinn HR (1977). "CP Conservation in the Presence of Pseudoparticles." Physical Review Letters.* ***38****(25):1440–1443. doi:10.1103/PhysRevLett.38.1440.*
32. ^ *Peccei RD, Quinn HR (1977). "Constraints imposed by CP conservation in the presence of pseudoparticles." Physical Review D.* ***16****(6):1791–1797. doi:10.1103/physrevd.16.1791.*
33. ^ *Peskin ME (2008). "Supersymmetry in Elementary Particle Physics." In: Colliders and Neutrinos. WORLD SCIENTIFIC. p. 609–704. doi:10.1142/9789812819260_0011.*
34. ^ *Ahluwalia-Khalilova DV, Grumiller D (2005). "Dark matter: A spin one-half fermion field with mass dimension one?" Physical Review D.* ***72****(6):067701. doi:10.1103/PhysRevD.72.067701.*
35. ^ *Alves A, de Campos F, Dias M, da Silva JMH (2015). "Searching for Elko dark matter spinors at the CERN LHC." International Journal of Modern Physics A.* ***30****(01):1550006. doi:10.1142/S0217751X15500062.*
36. ^ *Dirac PAM (1948). "The Theory of Magnetic Poles." Phys. Rev.* ***74****:817–830. doi:10.1103/PhysRev.74.817.*

37. ^Abbasi R, Ackermann M, Adams J, Aguilar JA, Ahlers M, Ahrens M, Alispach C, Alves AA, Amin NM, An R, et al. (2022). "Search for Relativistic Magnetic Monopoles with Eight Years ofIceCube Data." Phys. Rev. Lett. **128**:051101. doi:10.1103/PhysRevLett.128.051101.
38. a, bSchwartz MD (2013). Quantum Field Theory and the Standard Model. Cambridge University Press. doi:10.1017/9781139540940.
39. ^Bonometto S, Gorini V, Moschella U, editors (2002). Modern Cosmology. Institute of Physics Publishing. https://www.routledge.com/Modern-Cosmology/Bonometto-GoriniMoschella/p/book/9780750308106.
40. ^Thorne KS, Blandford RD (2021). Relativity and Cosmology. Princeton University Press.
41. ^Olive KA, Peacock JA (2019). "22: Big-Bang Cosmology." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.
42. a, bEllis J, Wands D (2019). "23: Inflation." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.
43. a, b, cWeinberg DH, White M (2019). "28: Dark Energy." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.
44. ^Freedman WL, Madore BF (2010). "The Hubble Constant." Annu Rev Astron Astrophys. **48**(1):673–710. doi:10.1146/annurev-astro-082708-101829.
45. ^Olivares-Salaverri I, Ribeiro MB (2021). "Testing cosmological models with the brightness profile of distant galaxies." Astrophysics and Space Science. **366**(11). doi:10.1007/s10509021-04016-3.
46. a, bTurner MS (2022). "The Road to Precision Cosmology." Annual Review of Nuclear and Particle Science. **72**(1):1–35. doi:10.1146/annurev-nucl-111119-041046.
47. ^Perivolaropoulos L, Skara F (2022). "Challenges for ΛCDM: An update." New Astronomy Reviews. **95**. https://doi.org/10.1016/j.newar.2022.101659.
48. a, bKhoury J, Ovrut BA, Seiberg N, Steinhardt PJ, Turok N (2002). "From big crunch to big bang." Phys. Rev. D. **65**:086007. doi:10.1103/PhysRevD.65.086007.
49. a, bZhu T, Wang A, Cleaver G, Kirsten K, Sheng Q (2017). "Pre-inflationary universe in loop quantum cosmology." Phys. Rev. D. **96**:083520. doi:10.1103/PhysRevD.96.083520.
50. a, bSilvestri A, Trodden M (2009). "Approaches to understanding cosmic acceleration." Rep. Prog. Phys. **72**(9):096901. doi:10.1088/0034-4885/72/9/096901.
51. ^Di Valentino E (2022). "Challenges of the Standard Cosmological Model." Universe. **8**(8). doi:10.3390/universe8080399.

52. ^Kamionkowski M, Riess AG (2022). "The Hubble Tension and Early Dark Energy." doi:10.48550/ARXIV.2211.04492.

53. a, b, cVerde L, Treu T, Riess AG (2019). "Tensions between the early and late Universe." Nature Astronomy. **3**(10):891–895. doi:10.1038/s41550-019-0902-0.

54. a, bMiller JL (2020). "Gravitational-lensing measurements push Hubble-constant discrepancy past 5σ." Physics Today. **2020**(1):0210a. doi:10.1063/pt.6.1.20200210a.

55. a, bLewton T (2020). "What Might Be Speeding Up the Universe's Expansion?" Quanta Magazizne. https://www.quantamagazine.org/why-is-the-universe-expanding-so-fast-20200427/.

56. a, b, cWanjek C (2020). "Dark Matter Appears to be a Smooth Operator." Mercury. **49**(3):10–11. https://astrosociety.org/news-publications/mercury-online/mercuryonline.html/article/2020/12/10/dark-matter-appears-to-be-a-smooth-operator.

57. ^Del Popolo A (2007). "Dark matter, density perturbations, and structure formation." Astronomy Reports. **51**(3):169–196. doi:10.1134/s1063772907030018.

58. a, b, c, d, e, f, gSimon JD, Geha M (2021). "Illuminating the darkest galaxies." Physics Today. **74**(11):30–36. doi:10.1063/pt.3.4879.

59. ^Oks E (2023). "Review of latest advances on dark matter from the viewpoint of the Occam razor principle." New Astronomy Reviews. **96**:101673. doi:10.1016/j.newar.2023.101673.

60. ^Bode P, Ostriker JP, Turok N (2001). "Halo Formation in Warm Dark Matter Models." The Astrophysical Journal. **556**(1):93–107. doi:10.1086/321541.

61. ^Baudis L, Profumo S (2019). "27: Dark Matter." In: Zyla PA et al. Prog. Theor. Exp. Phys. 083C01 (2020) and 2021 update. doi:10.1093/ptep/ptaa104.

62. ^Boddy KK, Lisanti M, McDermott SD, Rodd NL, Weniger C, Ali-Haimoud Y, Buschmann M, Cholis I, Croon D, et al. (2022). "Snowmass2021 theory frontier white paper: Astrophysical and cosmological probes of dark matter." Journal of High Energy Astrophysics. **35**:112–138. doi:10.1016/j.jheap.2022.06.005.

63. ^Ackerman L, Buckley MR, Carroll SM, Kamionkowski M (2009). "Dark matter and dark radiation." Physical Review D. **79**:023519. doi:10.1103/PhysRevD.79.023519.

64. ^Bolton JS, Caputo A, Liu H, Viel M (2022). "Comparison of Low-Redshift Lyman-α Forest Observations to Hydrodynamical Simulations with Dark Photon Dark Matter." Phys. Rev. Lett. **129**:211102. doi:10.1103/PhysRevLett.129.211102.

65. ^Kaplan DE, Krnjaic GZ, Rehermann KR, Wells CM (2010). "Atomic dark matter." Journal of Cosmology and Astroparticle Physics. **2010**(05):021–021. doi:10.1088/1475-7516/2010/05/021.

66. ^Cyr-Racine F, Sigurdson K (2013). "Cosmology of atomic dark matter." Physical Review D. **87**(10):103515. doi:https://doi.org/10.1103/PhysRevD.87.103515.

67. ^Gurian J, Ryan M, Schon S, Jeong D, Shandera S (2022). "A Lower Bound on the Mass of Compact Objects from Dissipative Dark Matter." The Astrophysical Journal Letters. **939**(1):L12. doi:10.3847/2041-8213/ac997c.

68. ^Tsai YD, McGehee R, Murayama H (2022). "Resonant Self-Interacting Dark Matter from Dark QCD." Physical Review Letters. **128**(17):172001. doi:10.1103/physrevlett.128.172001.

69. ^Chan MH (2022). "Two mysterious universal dark matter–baryon relations in galaxies and galaxy clusters." Physics of the Dark Universe. **38**:101142. doi:10.1016/j.dark.2022.101142.

70. a, b, c, dMo H, van den Bosch F, White S (2010). Galaxy Formation and Evolution. Cambridge University Press. https://www.cambridge.org/us/academic/subjects/physics/astrophysics/galaxy-formation-andevolution-1.

71. a, b, c, dChae KH, Lelli F, Desmond H, McGaugh SS, Li P, Schombert JM (2020). "Testing the Strong Equivalence Principle: Detection of the External Field Effect inRotationally Supported Galaxies." The Astrophysical Journal. **904**(1):51. doi:10.3847/1538-4357/abbb96.

72. a, bJackson JD (1998). Classical Electrodynamics. third edition. WILEY. https://www.wiley.com/en-us/Classical Electrodynamics, 3rd Edition-p-9780471309321.

73. a, bHaranas I, Harney M (2008). "Detection of the Relativistic Corrections to the GravitationalPotential Using a Sagnac Interferometer." Progress in Physics. **3**:3. http://www.ptep-online.com/complete/PiP-2008-03.pdf.

74. a, bThorne KS (1980). "Multipole expansions of gravitational radiation." Reviews of Modern Physics. **52**(2):299–339. doi:10.1103/RevModPhys.52.299.

75. ^Russell DA, Titlow JP, Bemmen YJ (1999). "Acoustic monopoles, dipoles, and quadrupoles: An experiment revisited." Am. J. Phys. **67**(8):660–664. doi:10.1119/1.19349.

76. ^Buckholtz TJ (2016). Models for Physics of the Very Small and Very Large. Atlantis Studies in Mathematics for Engineering and Science. Springer. https://link.springer.com/book/10.2991/978-94-6239-166-6.

77. ^Buckholtz TJ (2016). "Predict particles beyond the standard model; then, narrow gaps between physics theory and data." In: Proceedings of the 9th Conference on Nuclear and Particle Physics (19-23 Oct. 2015 Luxor-Aswan, Egypt). http://www.afaqscientific.com/nuppac15/npc1509.pdf.

78. ^Buckholtz TJ (2022). "New Thoughts, Suggestions, and Results about the Mysterious World ofElementary Particles, Dark Matter, and Dark Energy." In: Comsan MNH, editor. Proceedings, 12thConference on Nuclear

and Particle Physics. Egyptian Nuclear Physics Association. p. 19–34. http://www.afaqscientific.com/nuppac22/npc22016.pdf.

79. ^Feynman RP (2011). The Feynman lectures on physics. Basic Books.
80. ^Quantum-mechanical Systems and Photon Polarization (2018). Vassar College. https://pages.vassar.edu/magnes/files/2018/12/Systems-and-Photon-Polarization.pdf.
81. ^Valbusa Dall'Armi L, Nishizawa A, Ricciardone A, Matarrese S (2023). "Circular Polarization of theAstrophysical Gravitational Wave Background." Physical Review Letters. **131**(4):041401. doi:10.1103/physrevlett.131.041401.
82. ^Satoh M, Kanno S, Soda J (2008). "Circular polarization of primordial gravitational waves in string-inspired inflationary cosmology." Physical Review D. **77**(2):023526. doi:10.1103/physrevd.77.023526.
83. a, b, c, d, e, f, g, h, i, j, kWorkman RL, et al. (2022). "Review of Particle Physics." PTEP. **2022**:083C01. doi:10.1093/ptep/ptac097.
84. a, b, cAaltonen T, Amerio S, Amidei D, Anastassov A, Annovi A, Antos J, Apollinari G, Appel JA, Arisawa T, et al. (2022). "High-precision measurement of the W boson mass with the CDF II detector." Science. **376**(6589):170–176. doi:0.1126/science.abk1781.
85. ^Gonzalez-Sprinberg GA, Vidal J (2017). "Tau magnetic moment." Proceedings of The International Conference On Nanoscience and Technology. **912**(1):012001. doi:10.1088/17426596/912/1/012001.
86. ^Tanseri I, Hagstotz S, Vagnozzi S, Giusarma E, Freese K (2022). "Updated neutrino mass constraints from galaxy clustering and CMB lensing-galaxy cross-correlation measurements." Journal of High Energy Astrophysics. doi:10.1016/j.jheap.2022.07.002.
87. ^Vagnozzi S, Giusarma E, Mena O, Freese K, Gerbino M, Ho S, Lattanzi M (2017). "Unveiling ν secrets with cosmological data: Neutrino masses and mass hierarchy." Phys. Rev. D. **96**:123503. doi:10.1103/PhysRevD.96.123503.
88. a, bMarkevitch M, Gonzalez AH, Clowe D, Vikhlinin A, Forman W, Jones C, Murray S, Tucker W (2004). "Direct Constraints on the Dark Matter Self-Interaction Cross Section from the Merging Galaxy Cluster 1E 0657-56." Astrophysical Journal. **606**(2):819–824. doi:10.1086/383178.
89. ^Banchi L, Caravelli F (2016). "Geometric phases and cyclic isotropic cosmologies." Classical and Quantum Gravity. **33**(10):105003. doi:10.1088/0264-9381/33/10/105003.
90. ^Hertzberg MP (2020). "Structure Formation in the Very Early Universe." Physics Magazine. **13**(26). doi:10.1103/physics.13.16.

91. ^Bucher M, Goldhaber AS, Turok N (1995). "Open universe from inflation." Phys. Rev. D. **52**:3314–3337. doi:10.1103/PhysRevD.52.3314.

92. ^Busca NG, Delubac T, Rich J, Bailey S, Font-Ribera A, Kirkby D, Le Goff J-M, Pieri MM, Slosar A, Aubourg E, et al. (2013). "Baryon acoustic oscillations in the Lya forest of BOSS quasars." Astronomy and Astrophysics. **552**(A96). doi:10.1051/0004-6361/201220724.

93. ^Perlmutter S, Aldering G, Goldhaber G, Knop RA, Nugent P, Castro PG, Deustua S, Fabbro S, Goobar A, Groom, et al. (1999). "Measurements of Ω and Λ from 42 high-redshift supernovae Ω." Astrophysical Journal. **517**(2):565–586. doi:10.1086/307221.

94. ^Riess AG, Filippenko AV, Challis P, Clocchiatti A, Diercks A, Garnavich PM, Gilliland RL, Hogan CJ, Jha S, Kirshner RP, et al. (1998). "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant." Astronomical Journal. **116**(3):1009–1038. doi:10.1086/300499.

95. ^Riess AG, Strolger LG, Tonry J, Casertano S, Ferguson HC, Mobasher B, Challis P, Filippenko AV, Jha S, Li W, et al. (2004). "Type Ia Supernova Discoveries at z > 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution." Astrophysical Journal. **607**(2):665–687. doi:10.1086/383612.

96. ^Wolchover N (2020). "New Wrinkle Added to Cosmology's Hubble Crisis." Quanta Magazine. https://www.quantamagazine.org/new-wrinkle-added-to-cosmologys-hubble-crisis20200226/.

97. ^Freedman WL, Madore BF, Hoyt T, Jang IS, Beaton R, Lee MG, Monson A, Neeley J, Rich J (2020). "Calibration of the Tip of the Red Giant Branch (TRGB)." Astrophysical Journal. **891**(1):57. doi:10.3847/1538-4357/ab7339.

98. ^Poulin V, Smith TL, Karwal T, Kamionkowski M (2019). "Early Dark Energy can Resolve the Hubble Tension." Physical Review Letters. **122**(22):221301. doi:10.1103/physrevlett.122.221301.

99. ^Di Valentino E, Anchordoqui LA, Akarsu O, Ali-Haimoud Y, Amendola L, Arendse N, Asgari M, Ballardini M, Basilakos S, Battistelli E, et al. (2021). "Snowmass2021 - Letter of interest cosmology intertwined II: The hubble constant tension." Astroparticle Physics. **131**:102605. doi:10.1016/j.astropartphys.2021.102605.

100. ^Cyr-Racine F, Ge F, Knox L (2022). "Symmetry of Cosmological Observables, a Mirror World Dark Sector, and the Hubble Constant." Phys. Rev. Lett. **128**:201301. doi:10.1103/PhysRevLett.128.201301.

101. ^Escudero HG, Kuo JL, Keeley RE, Abazajian KN (2022). "Early or phantom dark energy, self-interacting, extra, or massive neutrinos, primordial magnetic fields, or a curved universe: An exploration of possible solutions to the H0 and σ8 problems." Phys. Rev. D. **106**:103517. doi:10.1103/PhysRevD.106.103517.

102. ^Cruz Reyes M, Anderson RI (2023). "A 0.9% calibration of the Galactic Cepheid luminosity scale based on Gaia DR3 data of open clusters and Cepheids." Astronomy and Astrophysics. **672**:A85. doi:10.1051/0004-6361/

202244775.

103. a, b *Wood C (2020). "A New Cosmic Tension: The Universe Might Be Too Thin." Quanta Magazine. https://www.quantamagazine.org/a-new-cosmic-tension-the-universemight-be-too-thin-20200908/.*

104. ^ *Said K, Colless M, Magoulas C, Lucey JR, Hudson MJ (2020). "Joint analysis of 6dFGS and SDSS peculiar velocities for the growth rate of cosmic structure and tests of gravity." Monthly Notices of The Royal Astronomical Society.* **497***(1):1275–1293. doi:10.1093/mnras/staa2032.*

105. ^ *Boruah SS, Hudson MJ, Lavaux G (2020). "Cosmic flows in the nearby Universe:new peculiar velocities from SNe and cosmological constraints." Monthly Notices of The Royal Astronomical Society. doi:10.1093/mnras/staa2485.*

106. ^ *Lauer TR, Postman M, Weaver HA, Spencer JR, Stern SA, Buie MW, et al. (2021). "New Horizons Observations of the CosmicOptical Background." The Astrophysical Journal.* **906***(2):77. doi:10.3847/15384357/abc881.*

107. ^ *Bernal JL, Sato-Polito G, Kamionkowski M (2022). "Cosmic Optical Background Excess, Dark Matter, and Line-Intensity Mapping." Physical Review Letters.* **129***(23):231301. doi:10.1103/physrevlett.129.231301.*

108. ^ *Lauer TR, Postman M, Spencer JR, Weaver HA, Stern SA, Gladstone GR, et al. (2022). "Anomalous Flux in the CosmicOptical Background Detected with New Horizons Observations." The Astrophysical Journal Letters.* **927***(1):L8. doi:10.3847/2041-8213/ac573d.*

109. ^ *Symons T, Zemcov M, Cooray A, Lisse C, Poppe AR (2023). "A Measurement of the Cosmic Optical Background and Diffuse Galactic Light Scaling from the R < 50 au New Horizons-LORRI Data." The Astrophysical Journal.* **945***(1):45. doi:10.3847/15384357/acaa37.*

110. ^ *Bowman JD, Rogers AEE, Monsalve RA, Mozdzen TJ, Mahesh N (2018). "An absorption profile centred at 78 megahertz in the sky-averaged spectrum." Nature.* **555***(7694):67–70. doi:10.1038/nature25792.*

111. ^ *Barkana R (2018). "Possible interaction between baryons and dark-matter particles revealed by the first stars." Nature.* **555***(7694):71–74. doi:10.1038/nature25791.*

112. ^ *Panci P (2019). "21-cm line Anomaly: A brief Status." 33rd Rencontres de Physique de La Vallee d'Aoste. https://cds.cern.ch/record/2688533.*

113. a, b *Behroozi P, Wechsler R, Hearin A, Conroy C (2019). "UniverseMachine: The correlation between galaxy growth and dark matter halo assembly from z = 0-10." Monthly Notices of The Royal Astronomical Society.* **488***(3):3143–3194. doi:10.1093/mnras/stz1182.*

114. ^ *Genzel R, Forster Schreiber NM, Ubler H, Lang P, Naab T, Bender R, et al. (2017). "Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago." Nature.* **543***(7645):397–401. doi:10.1038/nature21685.*

115. ^van Dokkum P, Abraham R, Brodie J, Conroy C, Danieli S, Merritt A, et al. (2016). "A High Stellar Velocity Dispersion and ~100 Globular Clusters for the Ultra-diffuse Galaxy Dragonfly 44." Astrophysical Journal. **828**(1):L6. doi:10.3847/2041-8205/828/1/l6.

116. ^Hall S (2016). "Ghost galaxy is 99.99 per cent dark matter with almost no stars." New Scientist. https://www.newscientist.com/article/2102584-ghost-galaxy-is-99-99-per-centdark-matter-with-almost-no-stars/.

117. ^Mancera Pina PE, Fraternali F, Adams EAK, Marasco A, Oosterloo T, Oman KA, et al. (2019). "Off the Baryonic Tully-Fisher Relation: A Population of Baryon-dominated Ultra-diffuse Galaxies." Astrophysical Journal. **883**(2):L33. doi:10.3847/2041-8213/ab40c7.

118. ^Mancera Pina PE, Fraternali F, Oosterloo T, Adams EAK, Oman KA, Leisman L (2021). "No need for dark matter: resolved kinematics of the ultra-diffuse galaxy AGC 114905." Mon. Not. R. Astron Soc.. doi:10.1093/mnras/stab3491.

119. ^Guo Q, Hu H, Zheng Z, Liao S, Du W, Mao S, et al. (2019). "Further evidence for a population of dark-matter-deficient dwarf galaxies." Nature Astronomy. **4**(3):246–251. doi:10.1038/s41550-019-0930-9.

120. ^van Dokkum P, Danieli S, Abraham R, Conroy C, Romanowsky AJ (2019). "A Second Galaxy Missing Dark Matter in the NGC 1052 Group." Astrophysical Journal. **874**(1):L5. doi:10.3847/2041-8213/ab0d92.

121. ^Webb KA, Villaume A, Laine S, Romanowsky AJ, Balogh M, van Dokkum P, et al. (2022). "Still at odds with conventional galaxy evolution: the star formation history of ultradiffuse galaxy Dragonfly44." Monthly Notices of the Royal Astronomical Society. **516**(3):3318–3341. doi:10.1093/mnras/stac2417.

122. ^Herrera-Camus R, Forster Schreiber NM, Price SH, Ubler H, Bolatto AD, Davies RL, et al. (2022). "Kiloparsec view of a typical star-forming galaxy when the Universe was ∼1 Gyr old." Astronomy and Astrophysics. **665**:L8. doi:10.1051/0004-6361/202142562.

123. ^Comeron S, Trujillo I, Cappellari M, Buitrago F, Garduno LE, Zaragoza-Cardiel J, et al. (2023). "The massive relic galaxy NGC 1277 is dark matter deficient. From dynamical models of integral-field stellar kinematics out to five effective radii." doi:10.48550/ARXIV.2303.11360.

124. ^Day C (2021). "A primordial merger of galactic building blocks." Physics Today. **2021**(1):0614a. doi:10.1063/PT.6.1.20210614a.

125. ^Tarumi Y, Yoshida N, Frebel A (2021). "Formation of an Extended Stellar Halo around an Ultra-faint Dwarf Galaxy Following One of the Earliest Mergers from Galactic Building Blocks." The Astrophysical Journal Letters. **914**(1):L10. doi:10.3847/2041-8213/ac024e.

126. ^Asencio E, Banik I, Mieske S, Venhola A, Kroupa P, Zhao H (2022). "The distribution and morphologies of Fornax Cluster dwarf galaxies suggest they lack dark matter." Mon Not R Astron Soc. doi:10.1093/mnras/stac1

765.

127. ^Meneghetti M, Davoli G, Bergamini P, Rosati P, Natarajan P, Giocoli C, et al. (2020). "An excess of small-scale gravitational lenses observed in galaxy clusters." Science. **369**(6509):1347–1351. doi:10.1126/science.aax5164.

128. ^Temming M (2020). "Dark matter clumps in galaxy clusters bend light surprisingly well." Science News. https://www.sciencenews.org/article/dark-matter-clumps-galaxy-clustersbend-light-surprisingly-well.

129. ^Simon JD, Geha M (2007). "The Kinematics of the Ultra-faint Milky Way Satellites: Solving the Missing Satellite Problem." Astrophys. J.. **670**(1):313–331. doi:10.1086/521816.

130. ^a, b van Dokkum P, Shen Z, Keim MA, Trujillo-Gomez S, Danieli S, Chowdhury DD, et al. (2022). "A trail of dark-matter-free galaxies from a bullet-dwarf collision." Nature. **605**(7910):435–439. doi:10.1038/s41586-022-04665-6.

131. ^Jimenez-Vicente J, Mediavilla E, Kochanek CS, Munoz JA (2015). "Dark Matter Mass Fraction in Lens Galaxies: New Estimates from Microlensing." Astrophysical Journal. **799**(2):149. doi:10.1088/0004-637x/799/2/149.

132. ^Jimenez-Vicente J, Mediavilla E, Munoz JA, Kochanek CS (2012). "A Robust Determination of the Size of Quasar Accretion Disks Using Gravitational Microlensing." Astrophysical Journal. **751**(2):106. doi:10.1088/0004-637x/751/2/106.

133. ^Clavin W (2020). "Rotating Galaxies Galore." Caltech News. https://www.caltech.edu/about/news/rotating-galaxies-galore.

134. ^LeFèvre O, Béthermin M, Faisst A, Jones GC, Capak P, Cassata P, et al. (2020). "The ALPINE-ALMA [CII] survey." Astronomy and Astrophysics. **643**:A1.

135. ^Lokas EL, Mamon GA (2003). "Dark matter distribution in the Coma cluster from galaxy kinematics: breaking the mass-anisotropy degeneracy." Monthly Notices of The Royal Astronomical Society. **343**(2):401–412. doi:10.1046/j.1365-8711.2003.06684.x.

136. ^Rasia E, Tormen G, Moscardini L (2004). "A dynamical model for the distribution of dark matter and gas in galaxy clusters." Monthly Notices of The Royal Astronomical Society. **351**(1):237–252. doi:10.1111/j.1365-2966.2004.07775.x.

137. ^Rudnick L (2019). "The Stormy Life of Galaxy Clusters: astro version." Preprint. doi:10.48550/arXiv.1901.09448.

138. ^Rudnick L (2019). "The stormy life of galaxy clusters." Physics Today. doi:10.1063/pt.3.4112.

139. ^a, b Abazov VM, Abbott B, Abolins M, Acharya BS, Adams M, Adams T, et al. (2005). "Search for right-handed W bosons in top quark decay." Physical Review D. **72**:011104. doi:10.1103/PhysRevD.72.011104.

140. ^Langacker P, Sankar SU (1989). "Bounds on the mass of W sub R and the W sub L - W sub R mixing angle. zeta. in general SU (2) sub L times SU (2) sub R times U (1) models." Physical Review D. **40**(5):1569–1585. doi:10.1103/PhysRevD.40.1569.

141. ^Gurung L, Babij TJ, Hogan SD, Cassidy DB (2020). "Precision Microwave Spectroscopy of the Positronium n = 2 Fine Structure." Physical Review Letters. **125**:073002. doi:10.1103/PhysRevLett.125.073002.

142. ^Holten M, Bayha L, Subramanian K, Heintze C, Preiss PM, Jochim S (2021). "Observation of Pauli Crystals." Phys. Rev. Lett.. **126**:020401. doi:10.1103/PhysRevLett.126.020401.

143. a, bBuote DA, Barth AJ (2019). "The Extremely High Dark Matter Halo Concentration of the Relic Compact Elliptical Galaxy Mrk 1216." Astrophysical Journal. **877**(2):91. doi:10.3847/1538-4357/ab1008.

144. ^Forrest B, Annunziatella M, Wilson G, Marchesini D, Muzzin A, Cooper MC, et al. (2020). "An Extremely Massive Quiescent Galaxy at z = 3.493: Evidence of Insufficiently Rapid Quenching Mechanisms in Theoretical Models." Astrophysical Journal. **890**(1):L1. doi:10.3847/20418213/ab5b9f.

145. a, bWhitaker KE, Williams CC, Mowla L, Spilker JS, Toft S, Narayanan D, et al. (2021). "Quenching of star formation from a lack of inflowing gas to galaxies." Nature. **597**(7877):485–488. doi:10.1038/s41586-021-03806-7.

146. ^Bonaca A, Hogg DW, Price-Whelan AM, Conroy C (2019). "The Spur and the Gap in GD-1: Dynamical Evidence for a Dark Substructure in the Milky Way Halo." Astrophysical Journal. **880**(1):38. doi:10.3847/1538-4357/ab2873.

147. a, bEhrenstein D (2019). "Mapping Dark Matter in the Milky Way." Physics Magazine. **12**(51). https://physics.aps.org/articles/v12/51.

148. ^Necib L, Lisanti M, Belokurov V (2019). "Inferred Evidence for Dark Matter KinematicSubstructure with SDSS–Gaia." Astrophysical Journal. **874**(1):3. doi:10.3847/15384357/ab095b.

149. ^Belokurov V, Deason AJ, Koposov SE, Catelan M, Erkal D, Drake AJ, et al. (2018). "Unmixing the Galactic halo with RRLyrae tagging." Monthly Notices of the Royal Astronomical Society. **477**(2):1472–1483. doi:10.1093/mnras/sty615.

150. a, bCalcagni G (2020). "Next Step in Gravity and Cosmology: Fundamental Theory or Data-Driven Models?" Frontiers in Astronomy and Space Sciences. **7**. doi:10.3389/fspas.2020.00052.

151. ^Cui ZF, Binosi D, Roberts CD, Schmidt SM (2021). "Pion charge radius from pion+electron elastic scattering data." Physics Letters B. **822**:136631. doi:10.1016/j.physletb.2021.136631.

152. ^Olive KA (2014). "Review of Particle Physics." Chinese Physics C. **38**(9):090001. doi:10.1088/1674-1137/38/9/090001.

153. ^ *Anonymous (2022). "Digital Library of Mathematical Functions." National Institute of Standards and Technology. https://dlmf.nist.gov/.*

154. ^ *Weisstein E (2020). "Delta Function." Wolfram MathWorld web page. http://mathworld.wolfram.com/DeltaFunction.html.*

155. ^ *Amiet J-P, Weigert S (2002). "Commensurate harmonic oscillators: Classical symmetries." Journal of Mathematical Physics.* **43***(8):4110–4126. doi:10.1063/1.1488672.*

156. a, b *Langacker P (2012). "Grand unification." Scholarpedia.* **7***(10):11419. doi:10.4249/scholarpedia.11419.*

157. ^ *Liboff RL (1984). "The correspondence principle revisited." Physics Today.* **37***(2):50–55. doi:10.1063/1.2916084.*

158. ^ *Polonyi J (2006). "Quantum-classical crossover in electrodynamics." Phys. Rev. D.* **74***:065014. doi:10.1103/PhysRevD.74.065014.*

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