

Research Article

Life as Form: Biomimetic Transport Networks in Complementary Active Matter

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In this work, I characterize the out-of-equilibrium behavior of active oscillator ensembles governed by chiral symmetry breaking. Operating under a complementary interaction scheme, these systems generate a structural competition between spatial motility and phase coordination, driving endogenous dynamical frustration. Through purely local rules, the collective self-organizes into multi-branch routing topologies characterized by optimal geometric junctions and autonomous self-repair capabilities. Statistical signatures reveal a non-equilibrium phase-space confinement rooted in weak ergodicity breaking, where scale-free fluctuations are sustained far from criticality. The system departs from traditional disordered frameworks, such as spin-glasses, by actively harnessing its glassy complexity: the active medium exploits internal frustrations to guide its own structural remodeling and maintain operational closure. These findings establish a decentralized organizational paradigm where a frustrated stability landscape is transformed into a functional, autopoietic computational substrate, providing a novel theoretical realization of biomimetic transport networks.

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1. Introduction

The historical divergence in theoretical biology can be envisioned by the tension between *life as substance* and *life as form*. The mechanist-reductionist paradigm prioritizes life as substance, anchoring biological identity to the underlying biochemistry, molecular substrates, and anatomical boundaries. Within this materialist view, the system is nothing more than the sum of its physical components. Conversely, the systemic paradigm formulated by, among others, F. Capra, P. L. Luisi, and the Santiago school

conceptualizes *life as form* ^{[1][2]}, where biological organization is a global (systemic) property that endogenously emerges from local interactions. Here, the defining feature of a living organization is not the specific physical matter that embodies it, but the dynamic network of relations—the autopoietic pattern of organization, from the Greek *auto-* “self” and *poiesis* “creation”—that maintains its continuity and operational closure ^[2]. In this context, *continuity* implies the persistent preservation of the system’s global identity and structural integrity over time, achieved through the continuous dynamic reconfiguration of its constitutive nodes. *Operational closure* dictates that the system’s processes are organizationally circular: the system’s internal dynamics generate the very components and boundary conditions required to sustain and regenerate those same dynamics, without depending on external instructive inputs.

This approach permits reasoning and modeling in abstract, agential terms ^[3]. Consequently, the emergence of biological form can be framed as a distributed *cognitive* process ^[4], where cognition is not defined as the processing of external inputs, but as the active maintenance of operational closure through relational network dynamics.

Such a view resonates with C. Langton’s foundational philosophy of Artificial Life ^{[5][6]}. Langton posited that the ultimate goals of biology require the abstraction of the dynamic principles of life away from its specific carbon-based organic chemistry. By focusing on “life-as-it-could-be” rather than “life-as-we-know-it”, this approach allows for the creation of “soft” mathematical and cybernetic instantiations of life. By engineering artificial, agential structures that possess life-like organizational properties—such as operational closure, autonomous behavior, and self-repair—one can systematically investigate the laws of biological form and emergent cognition within an abstract medium.

Historically, the most successful computational implementations of this systemic approach have relied on cellular automata (CA) platforms and the powerful framework of Self-Organized Criticality (SOC) ^[7]. Undeniably, SOC represents one of the most significant revolutions in statistical mechanics over the last decades, providing an elegant exemplification for how scale-free, complex behavior can endogenously emerge without fine-tuned external parameters. However, despite their theoretical elegance, these paradigms face intrinsic limitations when moving toward autonomous cognitive networks. Traditional CA and SOC models remain bound to discrete, rigid lattice structures and algorithmic update rules, lacking the continuous, physically embedded feedback loop between structural form and energetic flow. Moreover, operating precisely at a critical point poses severe evolutionary and functional hazards for any agential network. Criticality is inherently noisy and volatile; because susceptibility diverges, even

microscopic fluctuations can trigger system-wide, catastrophic avalanches. In contrast, biological organization prioritize survival through functional redundancy, structural protection, and active homeostasis. Furthermore, SOC dynamics are essentially memoryless and reactive, failing to provide the long-lived, plastic memory states required for autonomous informational processing.

On the other hand, an assessed path lies in the language of non-equilibrium synchronization, which provides a universal framework for understanding how biological systems dynamically coordinate across scales [8][9][10]. In living systems, mutual synchronization is not merely a passive emergent pattern, but a foundational mechanism for functional coordination across multiple scales, spanning from the circadian pacing of metabolic pathways to the rhythmic firing of neural assemblies and the collective dynamics of multicellular structures. Through phase-locking and dynamical coupling, biological entities manage to balance global coherence with local flexibility, establishing robust channels for information routing and energy distribution away from thermodynamic equilibrium. Within this context, the standard paradigm for modeling non-equilibrium synchronization is the Kuramoto model [11], which, along with its extensive variants [12], captures the transition from disorder to collective coherence.

Building upon this non-equilibrium synchronization paradigm, my research focuses on a framework pairing complementary interactions between two subpopulations of motile oscillators with a continuous, Kuramoto-like frequency distribution. These ensembles obey a distinctive *complementary* interaction scheme that establishes a direct competition between spatial motion and phase-orientation coordination: oscillators from the same subpopulation repel each other positionally but attract in phase, whereas cross-population interactions prompt spatial attraction paired with phase repulsion. This architecture embeds a structural *frustration* into the system. In non-equilibrium statistical mechanics, such frustration arising from competing local constraints is widely recognized as the primary driving force and physical foundation underlying the emergence of multi-scale complexity, effectively preventing trivial relaxation and forcing the active substrate into open-ended dynamical exploration [13].

The complementary framework, when combined with a Kuramoto-like frequency distribution, demonstrates the emergence of a wide variety of self-organized dynamic patterns [14]. However, while sufficient for spatial mobility and patterning, such continuous diversity could not generate stable routing structures. To achieve functional transport networks, I shifted the frequency architecture: by splitting the two populations through a frequency detuning (becoming the control parameter Δ), each ensemble acquires a distinct, intrinsic chirality [15].

As developed in [16], this chirally split complementary coupling drives a robust, dissipationless phase-momentum locking that guides the deterministic emergence of long-lived, transport networks. These networks are *topological* collective structures, meaning they are protected by discrete invariants and are therefore robust against local disruptions.

Moreover, these networks exhibit a significant escalation in both statistical and morphological complexity as the system size scales up [16][17]. The present study focuses on these relatively large-scale systems, aiming at deploying modern statistical methods to investigate their multi-scale complexity and unveil their biomimetic properties.

The manuscript is organized as follows: Section 2 recalls the core mathematical framework along with the representational strategies adopted for tracking multiscale dynamics. Section 3 presents the main numerical results, including the characterization of the transport networks self-organization, the localized perturbation tests for structural resilience, and a comparative analysis between a stable and a collapsing network. Section 4 contains a spatial recurrence quantification analysis, evaluating the fractal dimension of the network and information-theoretic signatures such as Shannon entropy. Section 5 introduces the Internal Clock Phase (ICP) metric to quantify the structural aging of the active medium. Finally, Section 6 discusses the biological implications of these findings within the cybernetic paradigms of autopoiesis and operational closure.

2. The Model

I consider a system of N chiral motile phase oscillators operating in open 2D space. The dynamics of the continuous spatial position vector $\mathbf{x}_i(t) \in \mathbb{R}^2$ and the internal phase clock $\phi_i(t) \in \mathbb{S}^1$ of the i -th unit are governed by the following coupled nonlinear differential equations:

$$\dot{\mathbf{x}}_i = \sum_{j=1}^N \nabla_{\mathbf{x}_i} W(|\mathbf{x}_i - \mathbf{x}_j|) \cos(\phi_i - \phi_j), \quad (1)$$

$$\dot{\phi}_i = \gamma_i \Delta + \sum_{j=1}^N \gamma_i \gamma_j W(|\mathbf{x}_i - \mathbf{x}_j|) \sin(\phi_j - \phi_i), \quad (2)$$

where $\Delta \in \mathbb{R}^+$ is the circular anisotropy acting as the non-equilibrium external driving parameter (making the system *active*). The discrete coefficient $\gamma_i = \pm 1$ denotes the phase-chirality of the unit. Following the convention of [16], $\gamma_i = +1$ identifies a counter-clockwise (CCW) rotating oscillator (represented as red), whereas $\gamma_i = -1$ denotes a clockwise (CW) oscillator (represented as blue). The two

chirally-split families interact according to the complementarity interaction scheme sketched in Fig. 1, i.e. when units share the same color, they repel each other positionally but attract orientationally; conversely, interactions between units of different color prompt spatial attraction paired with orientational repulsion.

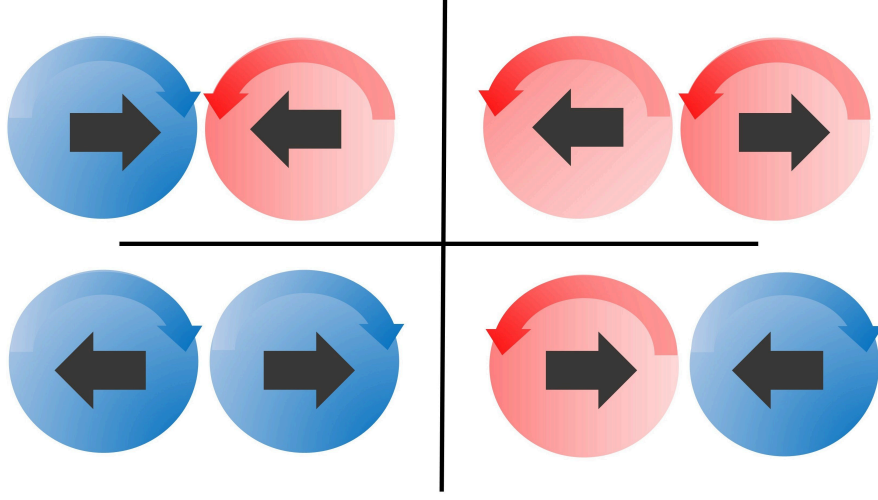


Figure 1. Sketch of the Complementary Interaction Scheme

The spatial coupling is mediated by a symmetric Gaussian potential $W(r) = e^{-r^2}$, where $r = |\mathbf{x}_i - \mathbf{x}_j|$ represents the standard Euclidean distance between particle pairs in the open plane. The characteristic interaction length scale is set to unity without loss of generality.

The framework represents a “dry” active matter system, wherein agent interactions are dictated by direct local couplings rather than being mediated by a surrounding fluid medium, sharing similarities with the classical Vicsek model [\[18\]](#) and the swarmalators [\[19\]](#).

To characterize the global evolution of this many-body system, I exploit the intrinsic symmetries of the governing dynamical equations. In a balanced population ($N_{Red} = N_{Blue}$), both the total linear momentum and the global phase velocity field are conserved quantities, yielding vanishing first moments:

$$\sum_{i=1}^N \dot{\mathbf{x}}_i(t) = \mathbf{0}, \text{ and } \sum_{i=1}^N \dot{\phi}_i(t) = 0. \quad (3)$$

Thus, following standard practices in statistical mechanics, the macroscopic state and its fluctuations are best captured by the second moments of these velocity averages, which act as response functions or susceptibilities. The global kinetic temperature $T(t)$, defined as the second moment of the spatial velocities:

$$T(t) = \sum_{i=1}^N |\dot{\mathbf{x}}_i(t)|^2, \quad (4)$$

which serves as a direct proxy for the system's motility. The internal synchronization landscape is mapped via the global phase temperature $K(t)$, defined as the second moment of the phase velocities:

$$K(t) = \sum_{i=1}^N \dot{\phi}_i(t)^2. \quad (5)$$

The model described by Eqs. (1)-(2) has been numerically analyzed for relatively small systems ($N \lesssim 100$) in [\[16\]](#) identifying a novel form of collective transport called “superexcitability”, organized in vortices of phase-momentum locked counterpropagating flows. Scaling up the particle population ($N \gtrsim 200$) makes these vortices proliferate into long-range dynamic superstructures, multi-branch transport networks [\[15\]\[17\]](#), which complexity increases with N .

This work returns to the large-system regime. My objective is to leverage these insights to perform a deeper statistical and conceptual characterization of the emergent transport collective structures.

A brief comment concerns the representational strategy adopted here. In previous small-scale investigations [\[16\]](#), tracking the coupled phase-displacement dynamics simultaneously via chirality-resolved individual variables (positions, orientations, and color Red or Blue) was viable and intuitive. However, scaling up the population to $N = 1000$ renders such visualization impracticable due to the massive separation of timescales inherent to the large-scale active medium. While individual phases rotate on a rapid microscopic scale, the macroscopic transport network evolves over much slower periods, and yet abruptly interrupted by sudden, intermittent structural bursts. Therefore, resolving the rapid microscopic oscillations while simultaneously spanning the slow macroscopic evolution would generate animations of massive length and size. Thus I opt for a dual representational framework: the spatial occupancy maps and movies are dedicated to capturing the slow morphological growth of the transport channels, disregarding phase dynamics; whereas the collective phase dynamics are tracked by the phase susceptibility $K(t)$ and its derived Internal Clock Phase (*ICP*, introduced in Section 5).

3. Results

The results are based on numerical simulations of Eqs. (1) and (2). In all simulations, I initialize the system from random uniform spatial positions within a squared domain of side length $L = \sqrt{N}$ (initial density $\rho = 1$) while I draw the initial phases from a uniform distribution in the interval $[0, 2\pi]$. The system operates under open boundary conditions in an unconstrained plane, meaning that agent dynamics are governed exclusively by their reciprocal interactions.

I performed the numerical integration using the MATLAB ode113 variable-order solver, with absolute and relative error tolerances set to 10^{-7} and 10^{-6} , respectively. I validated the numerical accuracy of the integration by verifying that both the global center of mass and phase remained invariant (Eqs. 3) within the prescribed precision limits throughout the simulation run.

I consider a population of $N = 1000$ agents, symmetrically partitioned into 500 counter-clockwise (red, $\gamma_i = +1$) and 500 clockwise (blue, $\gamma_i = -1$) oscillators. By tuning the circular anisotropy Δ within the superexcitable regime, I observe (consistently with [\[16\]](#)) that the macroscopic dynamics exhibit a bistable landscape. In approximately 50% of the independent realizations, the active medium relaxes toward a homogeneous absorbing state, characterized by an immediate decay of the kinetic temperature to zero ($T \rightarrow 0$). In the remaining 50% of the trials, the system self-organizes into a long-lived, self-sustaining transport network. Upon nucleation, the network undergoes an initial formation transient where the primary routing channels are established; however, the subsequent duration of this self-sustaining regime is highly variable. Even after achieving macroscale organization, the system can abruptly undergo sudden network death through an attractor crisis, collapsing into the absorbing state as previously documented in [\[17\]](#). The morphological features and operational lifetimes of these emergent topologies are thus highly sensitive to the random initial conditions, exhibiting pronounced statistical variability across independent simulation runs. In the following, I characterize this self-sustaining regime by analyzing a representative long-living evolutionary trajectory, hereafter denoted as Network A.

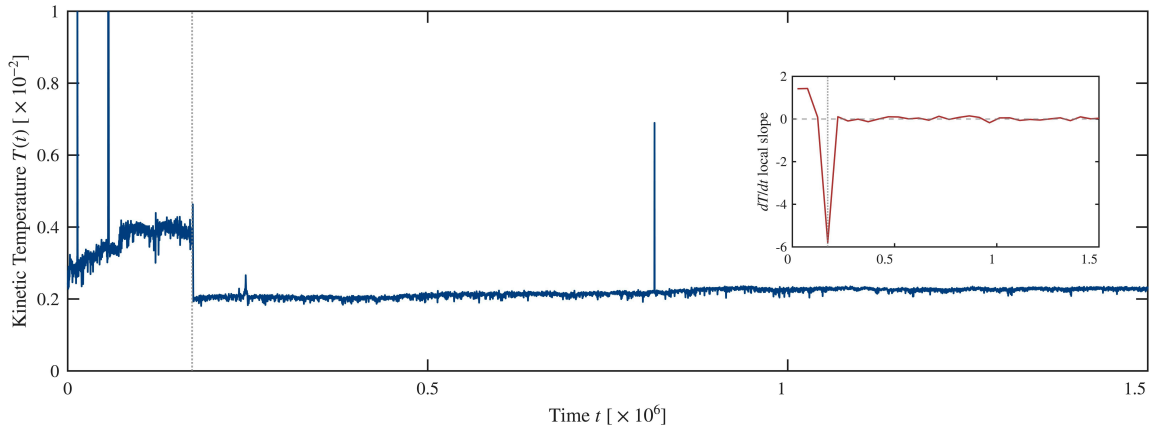


Figure 2. Temporal evolution of the global kinetic temperature $T(t)$ (solid blue line) over a simulation window up to $t = 1.5 \times 10^6$. The vertical dotted gray line marks the main attractor transition point. The inset displays the local time-derivative dT/dt (solid red line) computed via a moving-window linear fit integrated over intervals of 5000 time steps.

As illustrated in Fig. 2, the temporal evolution of the global kinetic temperature $T(t)$ for Network A exhibits a non-monotonic progression. The dynamics are initially characterized by pronounced kinetic fluctuations, which is subsequently terminated by a sharp, abrupt event that marks the system's entry into a quasi-stationary homeostatic plateau. Following this transition, the trajectory displays a progressive stabilization of the local drift slope (dT/dt), signaling a gradual regularization of the energy dissipation rate over time.

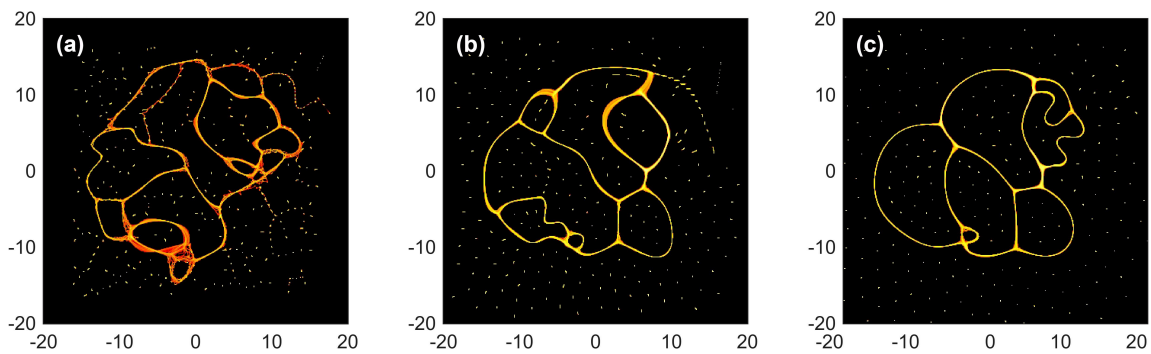


Figure 3. Spatial occupancy maps for Network A ($N = 1000$, $\Delta = 0.02$). Panels show the cumulative density aggregated over distinct temporal windows: (a) from $t = 1$ to $t = 5000$, (b) from $t = 1.8 \times 10^5$ to $t = 2.2 \times 10^5$, and (c) from $t = 6.8 \times 10^5$ to $t = 7.2 \times 10^5$. The color intensity indicates the level of agent presence, where brighter regions correspond to higher spatial occupancy and motility.

The Network A is visible in the spatial configurations of the active medium, aggregated within three specific temporal windows. As shown in Fig. 3, the time-averaged spatial occupancy maps reveal a morphological evolution. During the initial phase (Fig. 3a), the routing infrastructure appears diffuse and highly dynamic; the transport channels are broad and poorly defined, surrounded by a prominent background of uncoordinated, wandering agents. During the intermediate and mature windows (Fig. 3b,c), a spatial condensation of the flow patterns (thick bright lines) takes place. The channels progressively narrow into sharp, structurally defined transport pathways. Subsequently, the network reaches a mature stage where the routing branches become essentially stabilized, and the overall morphology exhibits minor structural variations, as shown in Fig. 4.

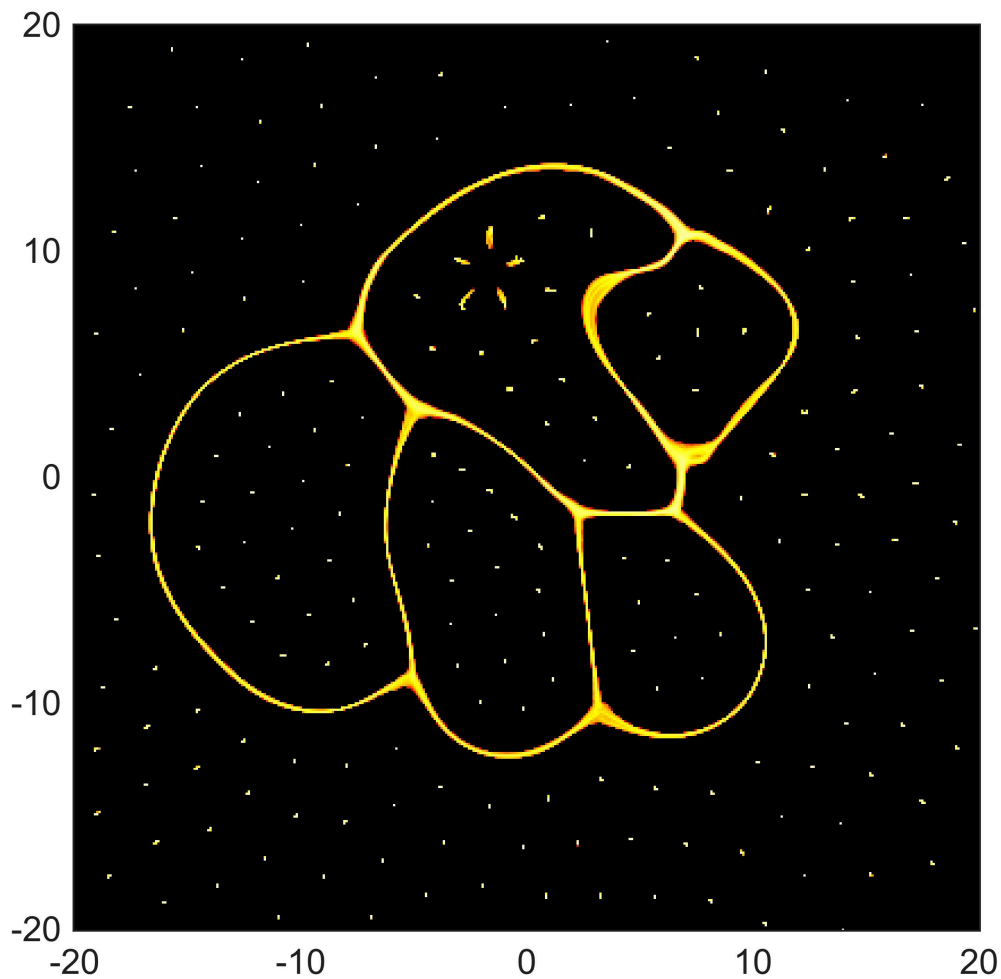


Figure 4. Spatial occupancy map tracking in the mature evolutionary stage of Network A. The panel shows the cumulative agent density aggregated over the late homeostatic interval, integrating the spatial occupancy in a small interval around $t = 1.5 \times 10^6$.

To better appreciate the dynamical reconfiguration of these pathways, this spatial maturation can be fully visualized through an animated movie available in the Supplementary Information ([Movie S1](#)). The video reproduces the evolutionary stages capturing the real-time creation, pruning, and structural stabilization of the transport channels. The movie shows that the network never truly ceases to remodel itself at a local scale, displaying geometric adjustments at the junctions and small, transient waves of kinetic frustration; it exhibits no further signatures of severe restructuring, maintaining its integrity up to $t = 1.5 \times 10^6$, at which point the numerical integration was deliberately concluded.

Over extended temporal windows the network relaxes toward a transport topology dominated by threefold *Y*-junctions (see Fig. 5), with regular 120° inter-branch angles (angular dispersion $\sigma < 15^\circ$). This geometric convergence constitutes a hallmark of Plateau's optimal configurations [\[20\]](#), which are universally recognized for minimizing transport resistance in distributed branching networks. Analogous optimization principles govern a vast repertoire of biological transport structures, spanning from the self-similar architecture of leaf venation patterns [\[21\]](#) and the functional scaling of cortical microvascular capillary beds [\[22\]](#), to the decentralized exploratory routing and metabolic optimization of slime molds like *Physarum Polycephalum* [\[23\]](#). Indeed, as thoroughly formalized by West [\[24\]](#), such optimized branching topologies represent a universal evolutionary strategy across living systems to minimize energy dissipation and internal resistance during mass transport.

Moreover, as highlighted by the velocity-resolved map in Fig. 5, while individual channels sustain different velocities, these velocity magnitudes dynamically match and balance at the branching nodes to guarantee flux continuity.

The above considerations demonstrates that the intermittent kinetic bursts do not represent a process of stochastic wandering across the stability landscape. Instead, they unveil a decentralized organizational strategy through which the complementary active matter stabilizes its routing infrastructure, optimizing global transport efficiency exclusively through local interactions.

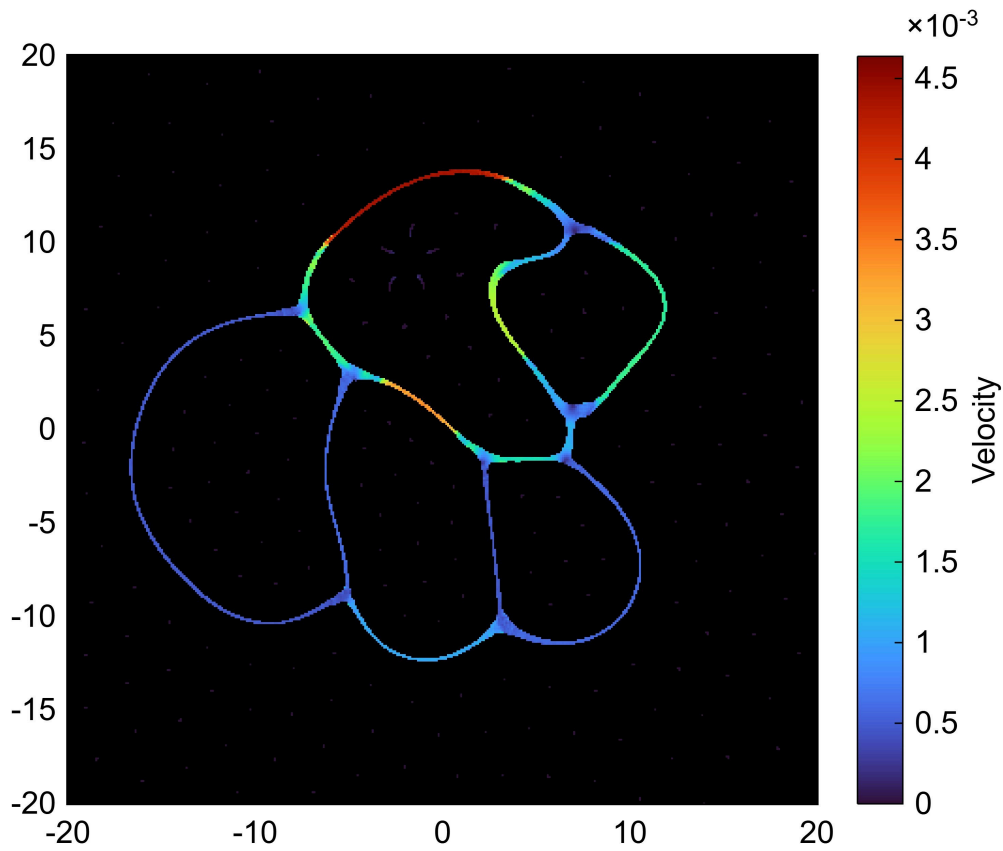


Figure 5. Velocity-resolved spatial occupancy map ($N = 1000$, $\Delta = 0.02$) for Network A. The panel shows the mean velocity field of the active medium aggregated over a short temporal window around $t = 1.5 \times 10^6$, with the colorbar indicating velocity magnitude.

To evaluate the structural resilience of the emergent transport network against localized disturbances, I implemented a perturbation test during the homeostatic plateau phase of Network A. I selected a mature Y-junction within the network and introduced a perturbation within a circular region of radius $R = 1$ centered on the junction core. Within this boundary, the operational state of all residing agents was disrupted by simultaneously randomizing their internal clock phases uniformly within the interval $[0, 2\pi]$, and shifting their spatial positions via a random displacement vector characterized by a uniformly distributed magnitude between 0 and 1 and a random angle in $[0, 2\pi]$.

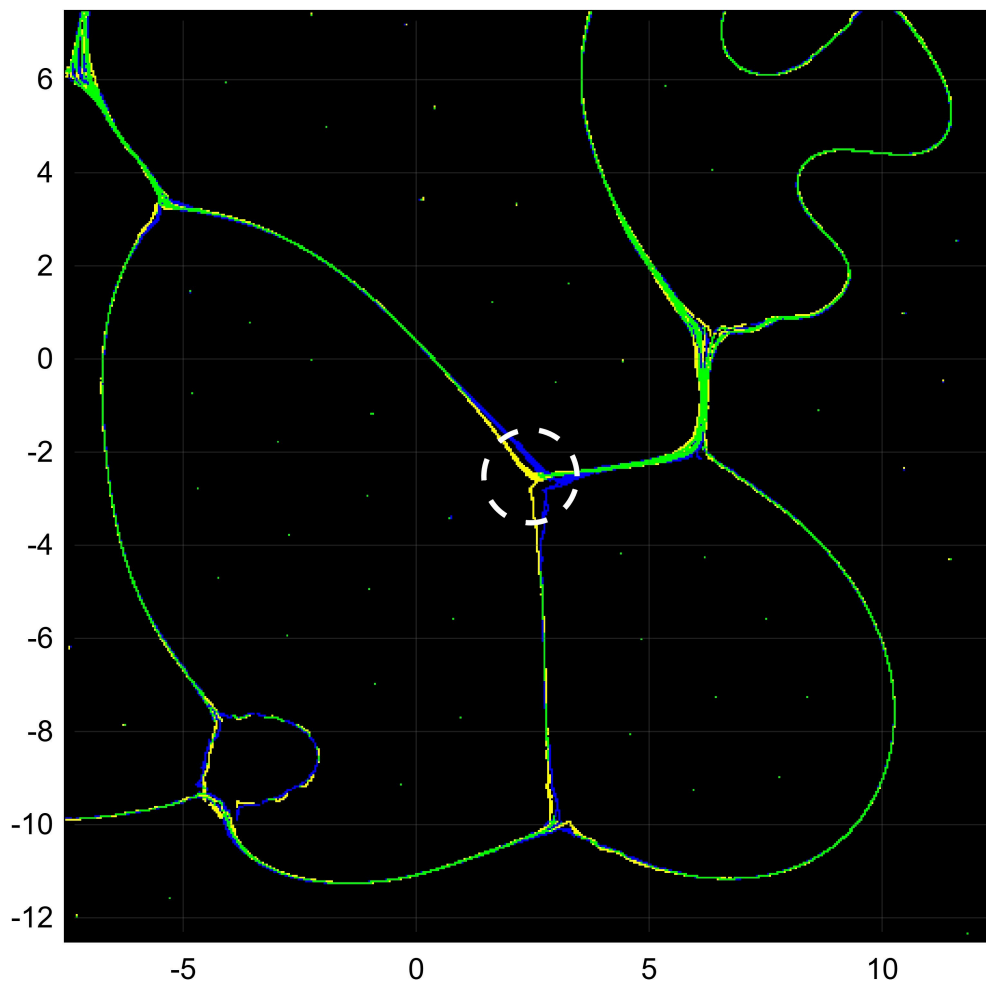


Figure 6. Localized self-repair and structural resilience of transport Network A following a localized disturbance. A sudden perturbation is applied at $t = 6.57 \times 10^5$ within the region bounded by the white dashed circle. The color-coded mapping illustrates the topological re-equilibration before and after the disturbance: yellow tracks the original unperturbed channels, blue represents the re-assembled transport pathways post-perturbation, and green highlights the regions of spatial overlap where the routing geometry remains invariant (yellow + blue = green).

The disrupted Y-junction reforms almost instantaneously, undergoing only a minimal spatial translation, as illustrated in Fig. 6. The structural shock remains confined to the perturbation zone, propagating only marginally to the closest neighboring junctions.

Despite its robustness to perturbations, the network can undergo spontaneous “death” via an attractor crisis if its itinerant trajectory encounters a low-dimensional chaotic saddle. This phenomenon,

anticipated in [17], is illustrated in Fig. 7, which presents an independent simulation run (named Network B) initialized under identical conditions and parameter configurations as the previous case, differing in the stochastic realization of their initial conditions. The system initially self-organizes into a transport network that exhibits geometric characteristics analogous to the stable case (Network A), despite converging toward a different spatial morphology. However, during its itinerant exploration across the frustrated landscape, the network fails to achieve long term stabilization. Instead, it undergoes a series of prominent kinetic energy bursts, eventually stumbling into a chaotic saddle that abruptly drives the active matter toward the absorbing state ($T \rightarrow 0$) at $t \approx 8.45 \times 10^5$.

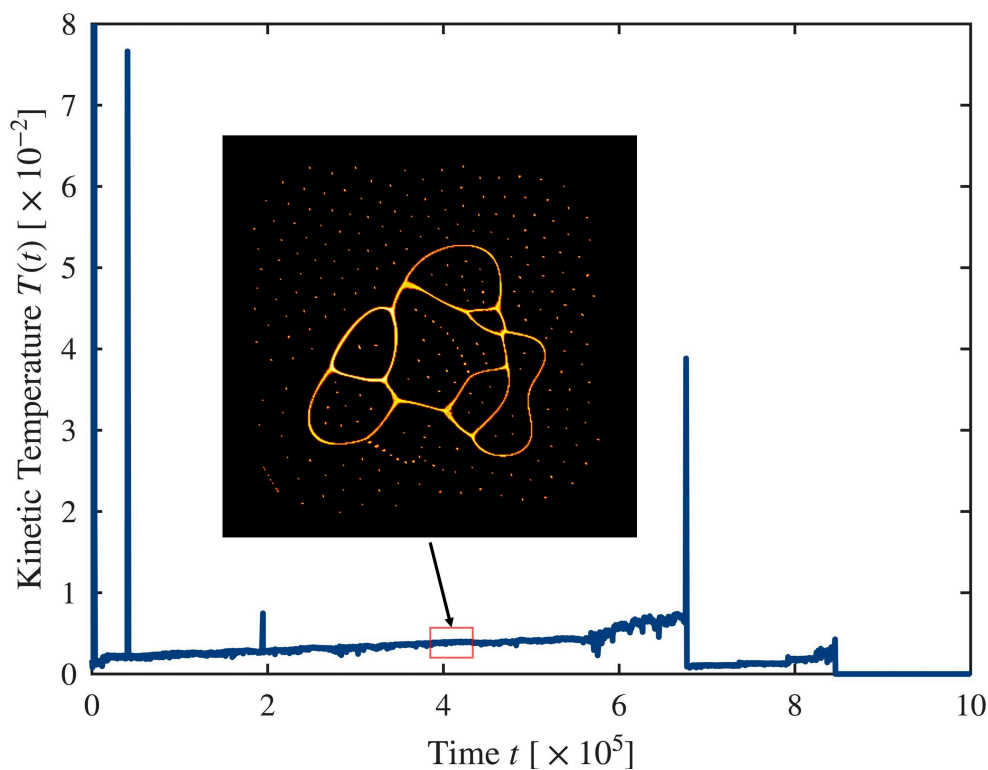


Figure 7. Evolutionary trajectory of the kinetic temperature $T(t)$ leading to Network B collapse via attractor crisis ($N = 1000$, $\Delta = 0.02$). The inset illustrates the spatial configuration of the transient network before the final breakdown at $t \approx 8.45 \times 10^5$.

4. Spatial Recurrence Statistics

As established in the previous section, the transport network evolves continuously through small- and medium-scale remodeling events, punctuated by rare, large-scale transitions. To capture the underlying

statistics of these structural reconfigurations, I perform a spatial recurrence quantification analysis on the long-lived Network A.

Morphological reconfigurations across the time series are then evaluated by computing the all-to-all cross-distance matrix $D_{i,j}$, defined as:

$$D_{i,j} = \|\mathbf{M}_i - \mathbf{M}_j\|_F = \sqrt{\sum_k \sum_l (M_{i,kl} - M_{j,kl})^2} \quad (6)$$

where $M_{i,kl}$ denotes the spatial occupancy density at the grid coordinate (k, l) during the temporal window i .

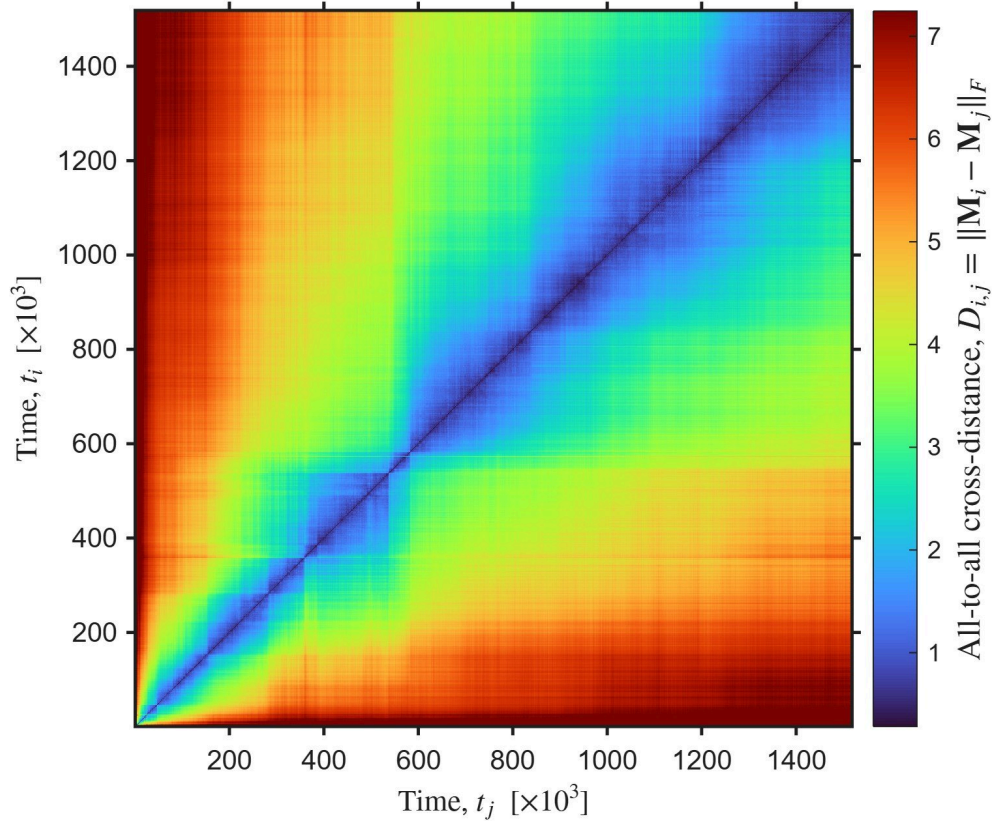


Figure 8. Spatial recurrence plot computed via all-to-all cross-distance of network layouts .
Temporal windows are chosen as $t = 1000$ time steps.

The resulting spatial recurrence matrix $D_{i,j}$ is illustrated in Fig. 8. The large blue blocks aligned along the main diagonal identify long-lived, morphologically stable homeostatic plateaus where the network layout remains structurally rigid. These macroscale blocks exhibit a hierarchy of sub-squares and sub-structures, providing proof of a multi-scale organization of the quasi-attractors wherein the system

recycles specific sub-graphs of its transport channels while shifting others. The sharp 90° corners that truncate these squares coincide with the kinetic temperature remodeling bursts, representing deterministic tipping points where localized kinetic frustration fluidizes specific compartments to steer the system toward adjacent chaotic saddles. Moreover, the diagonal blue blocks expand as time progresses. This growth indicates a progressive lengthening of the network's structural memory, signaling that morphological correlation percolates across increasingly larger temporal windows as the system matures.

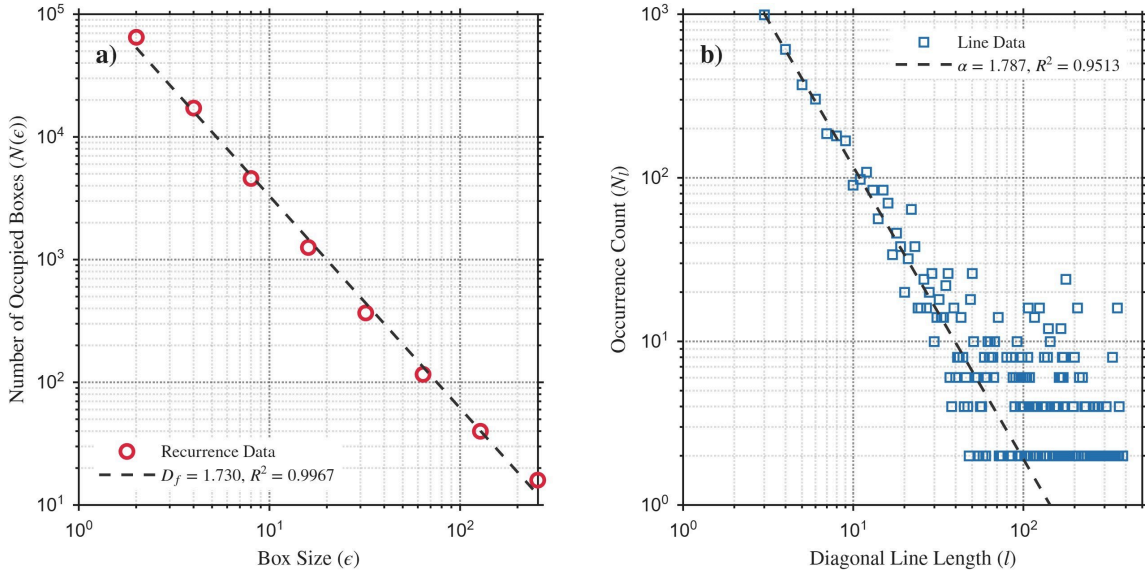


Figure 9. Complexity metrics of the spatial recurrence matrix. (a) Two-dimensional box-counting scaling regime; the linear regression yields a fractional fractal dimension of $D_f = 1.73$. (b) Log-log distribution of coherent diagonal recurrence lengths; the short-to-medium scale domain obeys a power-law scaling with a critical exponent of $\alpha = 1.787$. $R^2 \in [0, 1]$ denotes the coefficient of determination measuring the goodness of fit, with $R^2 = 1$ indicating perfect linear alignment in the log-log scale.

The complexity of this structural nesting is quantified by computing the Shannon entropy of the recurrence diagonal lines. In a recurrence plot, a diagonal line parallel to the main diagonal represents a temporal segment during which the system's morphological evolution replicates a past trajectory after a specific time lag. The probability $p(l)$ of detecting a diagonal segment of exact length l is defined as:

$$p(l) = \frac{P(l)}{\sum_{k=l_{\min}}^{L_{\max}} P(k)} \quad (7)$$

where $l_{\min}$ represents the minimum line length threshold used to filter out microscopic, uncorrelated phase jitter, while $L_{\max}$ is the maximum length of continuous diagonal recurrence segments detected in the recurrence matrix (excluding the main diagonal of identity). The deterministic entropy, which measures the diversity and uncertainty of these recurrent lifetimes, is then evaluated as:

$$H_{\text{DET}} = - \sum_{l=l_{\min}}^{L_{\max}} p(l) \log_2 p(l) \quad (8)$$

The encountered entropy value ($H_{\text{DET}} \approx 6$ bits) demonstrates that the collective dynamics occupies a non-trivial critical regime between periodic predictability (where $H_{\text{DET}} \rightarrow 0$ because a single macroscopic line dominates the spectrum) and memoryless stochastic randomness (where $H_{\text{DET}} \rightarrow 0$ because random phase jitter suppresses long correlations, collapsing the distribution into a single deterministic bin where $p(l_{\min}) \approx 1$). This information-theoretic metric indicates that the network successfully structures its exploration across an effective repertoire of $2^{H_{\text{DET}}} \approx 64$ distinct quasi-attractors. This provides evidence of a structured temporal grammar, wherein the network dynamically partitions its phase space into a diverse set of quasi-attractors, while continuously transitioning across intermediate chaotic saddles.

To geometrically characterize the multi-scale geometry of the phase-space trajectory, I first perform a two-dimensional box-counting analysis on the spatial recurrence matrix. Specifically, the recurrence matrix is partitioned by a grid of square boxes of linear size ϵ , and the number of boxes $N(\epsilon)$ containing at least one recurrent state is counted across a wide range of scales, and reported in Fig. 9(a). The capacity dimension is extracted from the scaling relation $N(\epsilon) \propto \epsilon^{-D_f}$ via linear regression in a log-log plot. The scaling behavior yields a non-integer fractal dimension of $D_f \approx 1.7$, demonstrating that the recurrent states are distributed along a self-similar, Cantor-like structure. This non-integer value, bounded below the embedding dimension of 2, confirms that the system's trajectories do not fill the temporal recurrence plane uniformly. Instead, the recurrences are clustered and intermittent, providing quantitative evidence of a fractured phase space where the active medium remains non-ergodically trapped within a hierarchical structure of quasi-attractors.

Moreover, when mapping the distribution of coherent diagonal lengths onto a log-log coordinate frame (Fig. 9(b)), it obeys a power-law scaling $N_l \propto l^{-\alpha}$ with an exponent of $\alpha \approx 1.8$. This algebraic decay indicates that the temporal structure of the remodeling bursts is approximately scale-free and non-Poissonian, meaning the network shifts between morphological configurations via a self-similar cascade

of modular adjustments. Taken together, these two complementary analyses unveil the multi-scale fractal nature of the system's phase-space trajectory.

By repeating independent simulation trials under identical parameter configurations, I observe that the network can self-organize into morphologically distinct spatial layouts; nevertheless, these varied topologies consistently converge toward nearly identical statistical properties. Even in cases where the network undergoes an attractor crisis and dies, provided it has survived long enough to accumulate a statistically significant time series, it exhibits similar scaling laws and fractal exponents. This invariance demonstrates that the multi-scale temporal structure and fractional dimension are robust, universal signatures of the underlying non-equilibrium topological trapping, regardless of the network's asymptotic long-term survival.

When incrementing the circular anisotropy driving parameter Δ , the observed collective behavior remains substantially repeatable while exhibiting a more frequent nucleation of parasitic clusters, which destabilize the transport network by depleting the energy available for dissipation—a phenomenology that aligns with previous findings in smaller scale systems ^[16]. This parasitic effect progressively degrades structural cohesion until a state of generalized spatial disorder is reached at $\Delta \approx 0.5$.

Finally, a comprehensive statistical scaling analysis of the recurrence and fractal exponents for large system sizes ($N = 1000$) across a continuous sweep of Δ remains computationally challenging, and I defer this systematic exploration to future investigations.

5. Internal Clock and Aging

In disordered out-of-equilibrium systems, aging is inherently non-linear; it is characterized by a dramatic slowing down of the internal relaxation times as the system explores a highly rugged energy or phase-space landscape ^[25]. Analogously, in living organisms, metabolic or functional age diverges from chronological time, progressing through convective developmental stages before settling into stabilized, energy-efficient homeostatic regimes punctuated by health crises ^[26].

In this subsection, I formalize the progression of the medium's internal clock dynamics. Guided by Prigogine's paradigm of an endogenous "internal time" and "internal age" characterizing irreversible systems ^[27], I argue that the cumulative impact of non-equilibrium fluctuations can be mapped onto the trajectory of the phase susceptibility $K(t)$. To this end, I define the *Internal Clock Phase (ICP)* at time t as

$$ICP(t) = \int_0^t \sqrt{K(\tau)} d\tau, \quad (9)$$

where K is the global phase temperature (5). The $ICP(t)$ behaves as an entropic clock tracking the progressive aging and the network stability.

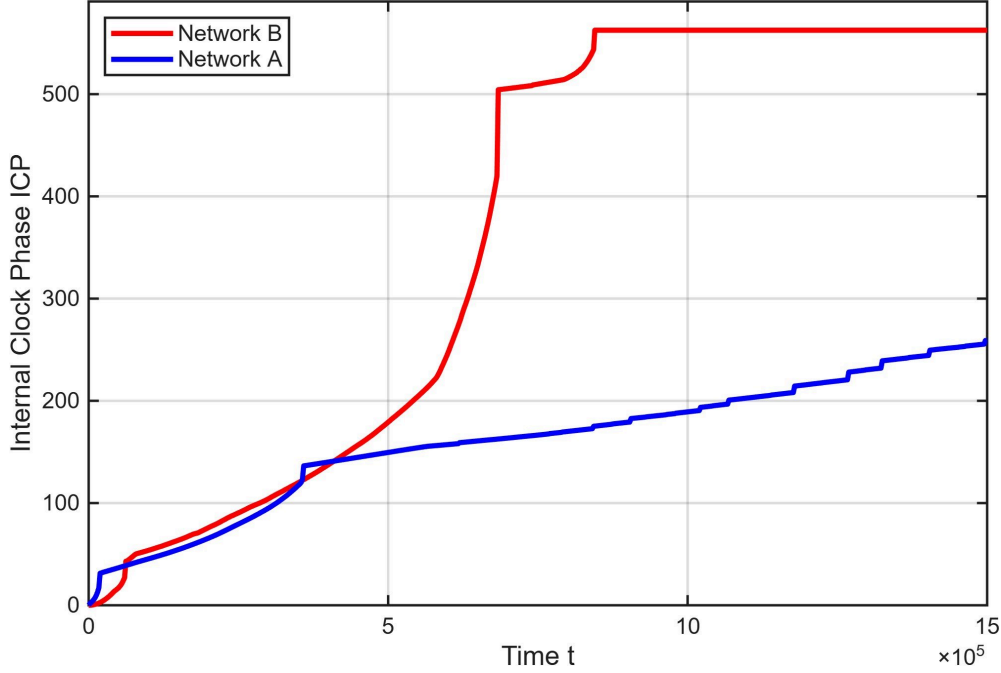


Figure 10. Evolutionary trajectory of the Internal Clock Phase ($ICP(t)$) for the stable Network A and the collapsing Network B. Both systems share identical parameters ($N = 1000$, $\Delta = 0.02$) and differ in the stochastic realization of their initial conditions. Network B ICP remains clamped at $t \approx 8.4 \times 10^5$ corresponding to its collapse into the absorbing state.

To reveal how morphological stability dictates the production of internal time, Fig. 10 contrasts the long-term evolution of $ICP(t)$ across two independent realizations sharing identical parameterizations, Networks A and B previously discussed. In the early stage of morphogenesis, both configurations undergo an accelerated generation of internal time, reflecting the volatile phase space exploration.

However, their long-term developmental trajectories diverge drastically. Network A successfully establishes an optimal topology characterized by regular 120° triple junctions and enters an efficient homeostatic regime. Within this regime, the ICP accumulates linearly, punctuated by irregular jumps that reflect localized structural adjustments and geometric optimizations.

Conversely, Network B fails to stabilize its transport infrastructure, continuously transitioning between different quasi-attractors until it is ultimately caught within a definitive crisis, where it freezes into the absorbing state ($T \rightarrow 0$).

This comparison demonstrates that the $ICP(t)$ is a diagnostic indicator of the network's structural health and stability. Major topological crises are preceded—and thus identifiable—by a distinct, non-linear acceleration in the ICP accumulation rate.

As a statistical analysis, I characterize the statistics of Network A ICP jumps by isolating their inter-event waiting times (T_{ICP}) and corresponding jump heights (H_{ICP}), as illustrated in Fig. 11. Both observables exhibit power-law scaling regimes, $P(T_{ICP}) \propto T_{ICP}^{-\alpha}$ and $P(H_{ICP}) \propto H_{ICP}^{-\beta}$, with critical exponents converging toward $\alpha \approx \beta \approx 1.5$. This dual $3/2$ scaling acts as a confirmation of the fractal, scale-free nature of the remodeling process, proving that the transport network evolves through a self-similar cascade of structural adjustments.

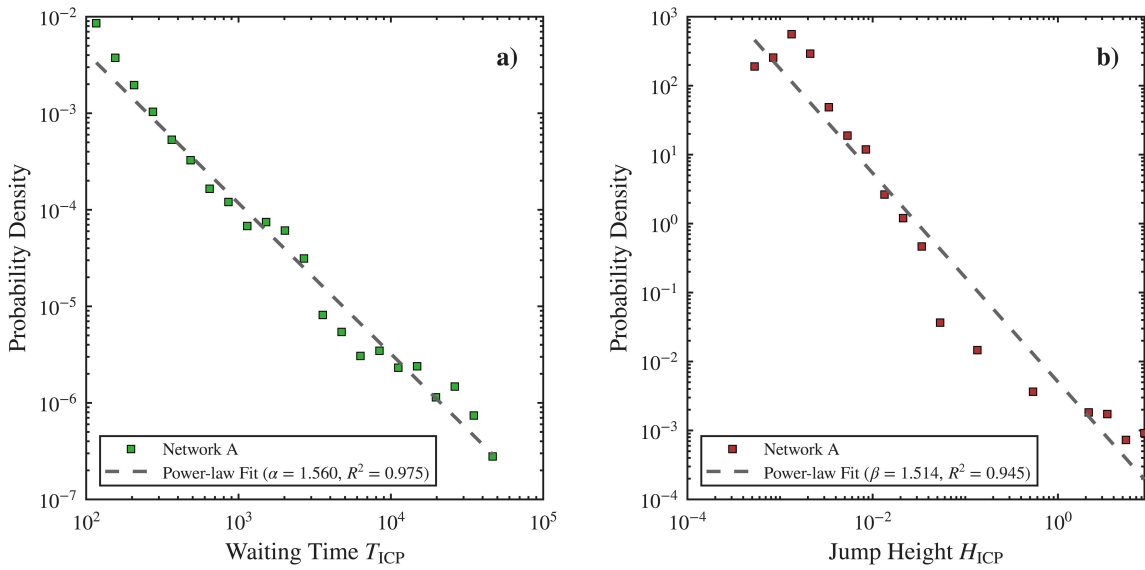


Figure 11. Statistical distribution of the Internal Clock Phase (ICP) jump metrics computed for Network A.

Panel (a) shows the log-log probability density function of the inter-jump waiting times T_{ICP} , along with the power-law fit yielding the exponent α and the corresponding R^2 value. Panel (b) shows the log-log probability density function of the integrated jump heights, along with the power-law fit yielding the exponent β and the corresponding R^2 value.

6. Discussion

This work characterizes the out-of-equilibrium collective dynamics of large complementary active oscillator ensembles, building upon foundations established in earlier studies. While prior investigations identified a deterministic macroscopic transition to structured transport topologies, the present study unravels the underlying statistical mechanics and cybernetic principles of these emergent networks. Specifically, the key findings demonstrate that these emergent networks form an autopoietic architecture that actively sustains operational closure and autonomous self-repair through decentralized local couplings.

This phenomenology marks a substantial departure from established benchmarks in out-of-equilibrium statistical physics, such as classic reaction-diffusion systems [28], continuum models of “wet” active nematics [29], or classical swarmalator frameworks [19]. In these models, local fluctuations yield diffusive, short-lived coordination, generic active turbulence, or static geometric patterns. Information in those settings is rapidly lost to memoryless, non-equilibrium mixing, or trapped within rigid spatial structures.

In contrast, this framework establishes a bi-directional feedback loop between individual internal kinetic states and global configurations, driving the system to self-organization. At the collective level, the emergent dynamics are governed by a tension between two competing tendencies: a structural optimization toward geometric Plateau configurations (120° Y-junctions), and an unconstrained exploration driven by attractor hopping. Technically, this phenomenon is underpinned by a state of weak ergodicity breaking, wherein the active dynamics exhibit chaotic itinerancy across a complex phase space. While this itinerant exploration serves as an endogenous evolutionary engine that allows the system to escape local minima, it is inherently hazardous, as the system can stumble into a terminal, low-dimensional chaotic saddle that disrupts the active transport network into a frozen, absorbing state.

In a cybernetic sense, the system exhibits operational closure: as the network successfully optimizes its transport infrastructure, it progressively “carves out” its own thermodynamic stability. By consolidating fluid corridors into concrete, defined flow lines, the system diminishes the probability of encountering the chaotic saddle. This demonstrates that morphological optimization acts as an active, emergent strategy to sustain network existence over time.

These networks offer a new physical embodiment of Prigogine’s paradigm of dissipative structures [30]. The observed *temporal sedimentation*—where early energetic kinetic bursts progressively dampen as fluid

corridors consolidate into defined flow lines—maps onto Prigogine’s formulation of an *internal time*, which treats aging and structural remodeling as intrinsic, irreversible properties endogenously embedded within the system’s statistical state [\[27\]\[31\]](#).

In my framework, this internal time is mapped onto the trajectory of the Internal Clock Phase (*ICP*), which tracks the system’s structural morphology through a distinct timeline of punctuated equilibrium. While the *ICP* accumulates, it is continuously interrupted by discrete, heavy-tailed bursts of remodeling jumps. The scale-free power-law statistics governing these jumps ($\alpha \approx \beta \approx 1.5$) share foundations with the paradigm of brain criticality popularized by D. Chialvo and co-workers [\[32\]\[33\]](#). In cortical networks, neuronal avalanches propagate as a critical branching process where an active unit triggers, on average, exactly one downstream neighbor, yielding a characteristic mean-field size exponent of $3/2$.

In the present formulation, the equivalent branching process manifests as a cascade of attractor jumps within the fractured phase space. When localized kinetic frustrations fluidize specific network compartments, the low-dimensional chaotic saddles act as topological branching nodes where the system must dynamically arbitrate which sub-graphs of its transport channels to recycle and which to prune. The convergence of *ICP* jump metrics toward this identical $3/2$ scaling provides evidence that the network matures through a self-similar branching of modular, geometric adjustments. The *ICP* trajectory also serves as an early-warning diagnostic indicator of network death: major topological crises are preceded by a violent non-linear acceleration in the *ICP* accumulation rate, which spikes immediately before flatlining as the system freezes into the absorbing dead state.

This non-equilibrium multi-scale grammar is further confirmed by the spatial recurrence analysis of the network layout. The hierarchical, nested block architecture of the cross-distance matrix yields a fractional capacity dimension ($D_f = 1.73$) and a heavy-tailed power-law distribution of coherent recurrence lengths, resulting in an intermediate deterministic Shannon entropy value, confirming that the system structures its temporal evolution across an effective repertoire of distinct quasi-attractors. In classical statistical mechanics, such scale-free signatures and intermediate entropy values—situated far from the trivial extremes of crystallized order and stochastic disorder—are traditionally interpreted as the signature of complex systems operating at the edge of a phase transition via Self-Organized Criticality (SOC).

However, because the proposed system operates far below the synchronization boundary across a wide range of parameter configurations, these properties must instead be interpreted through the lens of weak ergodicity breaking rather than critical point tuning. The non-integer fractal dimension validates that

the active trajectories remain non-ergodically confined within a fractured, low-dimensional support instead of filling the phase space uniformly, resulting in the observed heavy-tailed distribution of predictability windows at short-to-medium scales.

This behavior shares similarities with the slow, non-equilibrium relaxation observed in spin-glasses, which are standard paradigms for exploration within highly frustrated energy landscapes [34]. However, traditional spin-glasses lack any endogenous self-organizing mechanism, meaning their wandering across energy valleys results in a blind, unguided exploration that fails to yield coherent macrostructures or process functional information. In contrast, the framework introduced here demonstrates how an active medium can harness this glassy complexity, guiding its own chaotic itinerancy toward structural optimization and effectively transforming a frustrated landscape into a functional computational substrate.

Supplementary Material

Movie S1: Real-time evolution, structural pruning, and maturation of Network A ($N = 1000$, $\Delta = 0.02$). The active medium self-organizes into an optimized transport infrastructure dominated by stable 120° threefold Y-junctions. The video file is hosted on Zenodo and accessible via the version-invariant DOI: [10.5281/zenodo.22177746](https://doi.org/10.5281/zenodo.22177746).

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Ethics

Not applicable. This manuscript describes purely theoretical and computational research utilizing numerical simulations of synthetic agent-based models. No human participants or animals were involved in this study.

Consent for Publication

Not applicable. This manuscript does not contain any individual person's data, images, or videos.

Data Availability

The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request. The real-time spatial dynamics are publicly available as video material.

Materials

Not applicable. No physical materials or biological reagents were generated or used in this study.

Code

The custom MATLAB code used for the numerical integration, spatial recurrence analysis, and metrics evaluation is available from the corresponding author upon request.

Authors Contribution

Alessandro Scirè is the sole author of this work, responsible for the conceptualization, mathematical modeling, numerical simulations, formal statistical analysis, and writing of the manuscript.

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References

1. [△]Capra F, Luisi PL (2014). *The Systems View of Life: A Unifying Vision*. Cambridge: Cambridge University Press.
2. [△][▽]Maturana HR, Varela FJ (1980). *Autopoiesis and Cognition: The Realization of the Living*. Dordrecht: D. Reidel Publishing Company.
3. [△]Bedau MA (1997). "Weak Emergence." In: Tomberlin J, editor. *Philosophical Perspectives*, Vol 11. Mind, Causation, and World. Malden: Blackwell Publishers, pp. 375–399.
4. [△]Varela FJ, Thompson E, Rosch E (1991). *The Embodied Mind: Cognitive Science and Human Experience*. Cambridge: MIT Press.

5. ^aLangton CG (1989). "Artificial Life." In: Langton CG, editor. *Artificial Life: The Proceedings of an Interdisciplinary Workshop on the Synthesis and Simulation of Living Systems*, Vol VI. Redwood City: Addison-Wesley, pp. 1–47.
6. ^aLangton CG (1992). "Introduction." In: Langton CG, Taylor C, Farmer JD, Rasmussen S, editors. *Artificial Life II*, Vol X. Redwood City: Addison-Wesley, pp. 1–20.
7. ^aBak P, Tang C, Wiesenfeld K (1987). "Self-Organized Criticality: An Explanation of the $1/f$ Noise." *Phys Rev Lett*. **59**:381–384.
8. ^aStrogatz S (2003). *Sync: How Order Emerges from Chaos in the Universe, Nature, and Daily Life*. New York: Hyperion.
9. ^aWinfree AT (1967). "Biological Rhythms and the Behavior of Populations of Coupled Oscillators." *J Theor Biol*. **16**(1):15–42.
10. ^aPikovsky A, Rosenblum M, Kurths J (2001). *Synchronization: A Universal Concept in Nonlinear Sciences*. Cambridge: Cambridge University Press.
11. ^aKuramoto Y (1975). "Self-Entrainment of a Population of Coupled Non-Linear Oscillators." In: Araki H, editor. *International Symposium on Mathematical Problems in Theoretical Physics. Lecture Notes in Physics*, Vol 39. Berlin: Springer, pp. 420–422.
12. ^aAcebrón JA, Bonilla LL, Vicente CJP, Ritort F, Spigler R (2005). "The Kuramoto Model: A Simple Paradigm for Synchronization Phenomena." *Rev Mod Phys*. **77**:137–185.
13. ^aWolf YI, Katsnelson MI, Koonin EV (2018). "Physical Foundations of Biological Complexity." *Proc Natl Acad Sci USA*. **115**(37):E8678–E8687.
14. ^aScirè A, Annovazzi-Lodi V (2017). "Self-Organization in a Diversity Induced Thermodynamics." *PLOS ONE*. **12**(12):e0188753.
15. ^{a, b}Scirè A, Annovazzi-Lodi V (2023). "The Emergence of Dynamic Networks from Many Coupled Polar Oscillators: A Paradigm for Artificial Life." *Theory Biosci*. **142**(2):145–158.
16. ^{a, b, c, d, e, f, g}Scirè A (2026). "Superexcitability in Chiral Oscillators with Spatial Degrees of Freedom." *Phys Rev E*. **113**(5):054201.
17. ^{a, b, c, d}Scirè A (2024). "Emergence and Criticality in Spatiotemporal Synchronization: The Complementarity Model." *Artif Life*. **30**(4):412–428.
18. ^aVicsek T, Czirók A, Ben-Jacob E, Cohen I, Shochet O (1995). "Novel Type of Phase Transition in a System of Self-Driven Particles." *Phys Rev Lett*. **75**(6):1226–1229.
19. ^{a, b}O'Keefe KP, Hong H, Strogatz SH (2017). "Oscillators that Sync and Swarm." *Nat Commun*. **8**(1):1504.

20. [△]Ball P (1999). *The Self-Made Tapestry: Pattern Formation in Nature*. Oxford: Oxford University Press.
21. [△]Roth-Nebelsick A, Uhl D, Mosbrugger V, Kerp H (2001). "Evolution and Function of Leaf Venation Architecture: A Review." *Ann Bot.* **87**(5):553–566.
22. [△]Cassot F, Lauwers F, Fouard C, Prohaska S, Lauwers-Cances V (2006). "A Novel 3D Computer-Assisted Method for a Quantitative Study of Microvascular Networks of the Human Cerebral Cortex." *Microcirculation.* **13**(1):1–18.
23. [△]Tero A, Takagi S, Saigusa T, Ito K, Bebber DP, Fricker MD, Yumiki K, Kobayashi R, Nakagaki T (2010). "Rule s for Biologically Inspired Adaptive Network Design." *Science.* **327**(5964):439–442.
24. [△]West G (2017). *Scale: The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life in Organisms, Cities, Economies, and Companies*. New York: Penguin Press.
25. [△]Bouchaud JP (1992). "Weak Ergodicity Breaking and Aging in Disordered Systems." *J Phys I France.* **2**(9):1705–1713.
26. [△]Bich L, Arnellos A (2016). "Defining Autonomy in Structural Biology and Biosemiotics." *Biosemiotics.* **9**:151–169.
27. [△]Prigogine I (1980). *From Being to Becoming: Time and Complexity in the Physical Sciences*. San Francisco: WH Freeman & Co.
28. [△]Volpert V, Petrovskii S (2024). "Reaction-Diffusion Waves in Biology: New Trends, Recent Developments, and Applications." *Phys Life Rev.* **49**:120–145.
29. [△]Doostmohammadi A, Igné-Mullol J, Glowmin F, Bowick MJ, Marenduzzo D, Sagués F, Yeomans JM (2018). "Active Nematics." *Nat Commun.* **9**(1):3246.
30. [△]Nicolis G, Prigogine I (1977). *Self-Organization in Non-Equilibrium Systems*. New York: Wiley.
31. [△]Misra B, Prigogine I, Courbage M (1979). "On the Foundations of Irreversible Thermodynamics." *Proc Natl Acad Sci USA.* **76**(8):3607–3611.
32. [△]Chialvo DR (2010). "Emergent Complex Neural Dynamics." *Nat Phys.* **6**(10):744–750. doi:[10.1038/nphys1803](https://doi.org/10.1038/nphys1803).
33. [△]Beggs JM, Plenz D (2003). "Neuronal Avalanches in Neocortical Circuits." *J Neurosci.* **23**(35):11167–11177. doi:[10.1523/JNEUROSCI.23-35-11167.2003](https://doi.org/10.1523/JNEUROSCI.23-35-11167.2003).
34. [△]Parisi G (1983). "Order Parameter for Spin-Glasses." *Phys Rev Lett.* **50**(24):1946–1948.

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