Dingle's "Clock Paradox" Short Disproof

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Abstract

In this paper we provide a concise refutation to the fringe arguments brought up by Herbert Dingle against special relativity.

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Introduction

In this appendix to "Science at the Crossroads" on page 230, Herbert Dingle writes [1]:

"Thus, between events E0 and E1, A advances by t_1' *and B by* $t_1' = at_1$ *by (1). Therefore*

$$\frac{rate\_of\_A}{rate\_of\_B} = \frac{t_1}{t_1'} = \frac{t_1}{at_1} = \frac{1}{a} > 1 \quad (3)$$

...

Thus, between events E0 and E2, B advances by t_2' *and A by* $t_2 = at_2'$ *by (2). Therefore*

$$\frac{rate\_of\_A}{rate\_of\_B} = \frac{t_2}{t_2'} = \frac{at_2'}{t_2'} = a < 1 \quad (4)$$

Equations (3) and (4) are contradictory: hence the theory requiring them must be false."

In the next section we will explain the errors in Dingle's thinking and we will provide a simple resolution.

Resolution

Assume that there is observer **A** located in frame S at location x. Observer **A** has a clock that ticks with period T. Observer **B** is located in frame S' at location x'. **B** has a clock that ticks with period T'. S' moves with speed v with respect to S. The Lorentz transform from S to S' is:

$$t' = \gamma(v)(t - \frac{vx}{c^2}) \qquad (1)$$

Therefore, clock **A** period is calculated by observer **B** (from his location x') to have the period:

$$T_B^A = \gamma((t+T) - \frac{vx}{c^2}) - \gamma(t - \frac{vx}{c^2}) = \gamma T \qquad (2)$$

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The Lorentz transformation from S' to S is:

$$t = \gamma(v)(t' + \frac{vx'}{c^2}) \tag{3}$$

Therefore, clock **B** period is calculated by observer **A** (from his location x) to have the period:

$$T_A^B = \gamma((t' + T') + \frac{vx'}{c^2}) - \gamma(t' + \frac{vx'}{c^2}) = \gamma T' \tag{4}$$

Observer **A** continues to measure his own clock to tick with period T. Observer **B** continues to measure his own clock to tick with period T'. Now, what happens if T=T ' (the clock periods are identical)? Then:

$$T_B^A = T_A^B = \gamma T \tag{5}$$

What if the Observers Exchanged Signals in Order to Compare Clock Periods?

A practical way of comparing the clock periods would be for the two observers to send signals from one to another in order to "notify" the other observer about their respective clock periods. In this case, because the observers are in motion with respect to each other expressions (2) and (4) need to be adjusted for the (relativistic) Doppler effect [2]:

$$T_B^A = \gamma T(1 + \frac{v}{c}\cos\phi) \tag{6}$$

$$T_A^B = \gamma T'(1 + \frac{v}{c}\cos\phi') \tag{7}$$

In the above formulas, ϕ is the angle between the line connecting **A** and **B** and the velocity **v** between **A** and **B** as measured by observer **A** and ϕ' is the angle between the line connecting **A** and **B** and the velocity **v** between **A** and **B** as measured by observer **B**. Other than the insertion of the extra multiplying factor nothing changes.

Where is the Fallacy in Dingle's Reasoning?

The Lorentz transformation (1) implies $t' = \gamma t$ **at x = 0** and its inverse transformation (3) implies $t = \gamma t'$ **at x′ = 0**. In his book Dingle alleged that these two facts are mutually contradictory. He failed to realize that these two ratios apply to two **different** conditions, x = 0 and x′ = 0 respectively, hence Dingle's error.

Conclusion

We provided a very concise disproof of Dingle's "clock paradox".

References

[1] H. Dingle, “Science At the Crossroads”, 1972

[2[A. Einstein, “On the Electrodynamics of Moving Bodies”, Ann. der Phys., 1905