

# Superheavy Nuclei X: $1400 \leq A < 1500$ Systems

Joseph Bevelacqua

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## Abstract

Decay properties of nuclei are calculated in the mass region  $1400 \leq A < 1500$ . The calculations are performed using the adjusted Rost interaction which suggests that a new island of stability exists for  $Z = 410$  and in neighboring systems. Enhanced stability occurs in the  $Z=410$   $N=1068$  system.

## 1.0 Introduction

The investigation of the stability of superheavy nuclei has been a continuing area of active experimental and theoretical interest<sup>1-31</sup>. Neutron shells at  $N = 184$ ,  $228$ , and  $308$  have been previously investigated<sup>3, 5</sup> and stability at  $N = 406$  has been suggested<sup>3</sup>, and further investigated<sup>21</sup>. In a previous paper<sup>21</sup>, calculations for  $570 \leq A \leq 620$  superheavy nuclei evaluated stability near the  $N = 406$  neutron shell and found that an island of stability existed at  $Z = 204$ . A weaker shell closure was also found at  $Z = 210$  and  $N = 406$ . Ref. 22 investigated  $620 < A < 700$  systems which suggested an additional island of stability near  $N = 432$  for a small range of  $A$  values associated with the  $Z = 204$  shell. Binding energy calculations of  $700 \leq A < 800$  nuclei indicated that an island of stability may exist near the  $N = 504$  shell for a small range of  $A$  values associated with the  $Z = 226$  shell<sup>23</sup>. Compared to the aforementioned mass regions, calculations in  $800 \leq A < 900$  systems exhibit a decrease in overall binding energy<sup>25</sup>. Within the  $800 \leq A < 900$  mass region, enhanced stability is associated with the  $Z = 274$  shell<sup>25</sup>. Stability at  $Z = 282$  is predicted with the closure of the  $3f_{7/2}$  shell in  $900 \leq A < 1000$  systems, but it is weaker than in the aforementioned mass regions<sup>26</sup>. In terms of the number of bound nuclei, overall stability in the  $1000 \leq A < 1100$  mass region<sup>27</sup> is weaker than in  $800 \leq A < 1000$  systems<sup>25,26</sup>. However, calculations suggest that a new island of stability exists for  $Z = 310-318$  systems<sup>27</sup>. Stability continues to weaken in the  $1100 \leq A < 1200$  mass region<sup>28</sup>. Significantly enhanced stability was noted in the  $1200 \leq A < 1300$  mass region<sup>30</sup> with eight systems having half-lives greater than  $10^{10}$  yr. Stability is considerably degraded in the  $1300 \leq A < 1400$  mass region<sup>31</sup>.

This paper describes calculations for  $1400 \leq A < 1500$  superheavy nuclei and finds that 77 even-even nuclear systems theoretically exist within this mass range. The stability of  $1400 \leq A < 1500$  systems is evaluated by calculating single-particle neutron and proton levels using a methodology previously used to investigate  $A = 298 - 472$  doubly-closed shell nuclei<sup>5</sup> and nuclear systems in the  $570 \leq A \leq 620$ <sup>21</sup>,  $620 < A < 700$ <sup>22</sup>,  $700 \leq A < 800$ <sup>23</sup>,  $800 \leq A < 900$ <sup>25</sup>,  $900 \leq A < 1000$ <sup>26</sup>,  $1000 \leq A < 1100$ <sup>27</sup>,  $1100 \leq A < 1200$ <sup>28</sup>,

$1200 \leq A < 1300$ <sup>30</sup>, and  $1300 \leq A < 1400$ <sup>31</sup> mass regions. The calculations presented herein provide an opportunity to investigate a mass region that has received minimal theoretical investigation. Moreover, these calculations provide insight into binding energy systematics and nuclear stability beyond the mass regions explored by the calculations of Refs. 3 and 5 and the neighboring  $570 \leq A < 1400$  mass region<sup>21-31</sup>.

The use of single-particle energy levels to evaluate nuclear stability is appropriate since extrapolations to the superheavy mass regions are speculative. Using a more sophisticated method is not warranted in view of the uncertainties encountered in these calculations. Methods that are more sophisticated are appropriate when data are available to examine fine model details and interaction characteristics. As was demonstrated in Refs. 3 and 5, single-particle energy level calculations are entirely appropriate for initial calculations into a superheavy mass region where there is no experimental data to guide the calculations. Moreover, theoretical calculations are currently the only way to investigate the  $1400 \leq A < 1500$  mass region because an experimental investigation is not currently feasible.

Alpha decay, beta decay, positron decay, electron capture, and spontaneous fission half-lives are calculated to determine the stability of these superheavy systems. The stability in the  $1400 \leq A < 1500$  mass region is dominated by alpha decay and beta decay. These half-lives are derived from the calculated single-particle level spectrum. The single-particle level energies are sensitive to the model potential<sup>3, 5, 21-31</sup>. This paper also addresses model weaknesses and possible experimental methods to investigate  $1400 \leq A < 1500$  systems.

## 2.0 Computational Methodology

Since the method for calculating single-particle energies in a spherically symmetric potential is well established<sup>3,5,21-31</sup>, only salient features are provided. Details of the methodology were provided in Ref. 21, which extended the approach of Petrovich et al.<sup>5</sup> Specific details of the numerical method, model, and convergence criteria are provided in Refs. 2, 5, 21-34.

### 2.1 Theoretical Model

The model describing the nucleon plus nuclear core system represents an application of the standard method of Lukasiak and Sobiczewski<sup>3</sup> and Petrovich et al.<sup>5</sup>

The calculational method used to generate a single-particle level spectrum determines the binding energy  $E_{NLSJ}$  of a particle in the field of a spherical nuclear core by solving the radial Schrödinger Equation

$$\left[ \frac{\hbar^2}{2\mu} \left( \frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} \right) - E_{NLSJ} - V_{LSJ}(r) \right] U_{NLSJ}(r) = 0(1)$$

where  $r$  is the radial coordinate defining the relative motion of the nuclear core and the particle;  $V_{LSJ}(r)$  is the model interaction;  $E_{NLSJ}$  is the core plus particle binding energy;  $U_{NLSJ}(r)$  is the radial wave function; and  $L$ ,  $S$ , and  $J$  are the orbital, spin, and total angular momentum quantum numbers, respectively.  $N$  is the radial quantum number and  $\mu$  is the reduced mass. Additional details of the model and associated interactions are provided in Refs. 2, 5 and 21-34.

## 2.2 Determination of Q Values and Half-Lives

The reader is strongly cautioned not to interpret the calculated half-lives as representing a definitive value. As noted in subsequent discussion, the half-lives represent relative values, and the largest values suggest regions of stability relative to other systems whose properties are calculated with the same interaction.

The Q value for alpha decay and the alpha and beta decay half-lives of  $1400 \leq A < 1500$  superheavy nuclei with effective half-lives  $\geq 1$  s are listed in Table 1. The alpha decay energies are calculated using the relationship based on Ref. 1.

$$Q_{\alpha} = 28.3 \text{ MeV} - 2S_n - 2S_p(2)$$

where  $S_n$  and  $S_p$  are the binding energies of the last occupied neutron and proton single-particle energy levels, respectively. Alpha half-lives ( $T_{1/2}^{\alpha}$ ) were estimated from  $Q_{\alpha}$  using standard relationships provided in Ref. 3.

The beta decay half-lives ( $T_{1/2}^{\beta}$ ) are determined following the log ft methodology of Wong<sup>1</sup>. Allowed (first-forbidden) transition half-lives were derived using the values of  $\log ft = 5$  (8). Given the uncertainties in the calculated single-particle level energies, second and higher forbidden transitions were not determined. The beta half-life values in Table 1 listed as *stable* are either beta particle stable or decay by these higher order forbidden transitions.

## 3.0 Nuclear Interaction

Nuclear stability with respect to alpha decay, beta decay, positron decay, electron capture, and spontaneous fission is addressed using the method previously published by the author<sup>21-31</sup> and coworkers<sup>5</sup> that is similar to the approach of Ref. 3. The single-particle level spectrum is generated using a Woods-Saxon potential with parameters optimized to permit extrapolation into the superheavy region<sup>2,24,29</sup>.

Uncertainties in the nuclear interaction for  $A \geq 1200$  superheavy nuclei preclude absolute theoretical predictions of nuclear properties including single-particle energies, half-lives and Q-values. However, a model potential can be developed to predict trends in these properties and suggest islands of stability in  $A \geq 1200$  nuclei<sup>29</sup>.

A specific interaction for investigating  $A \geq 1200$  systems was developed in Ref. 29. A potential applicable to  $A \geq 1200$  systems must be constructed in a manner that is consistent with the general uncertainties in the nuclear interaction. Ref. 29 reviewed a representative sample of these uncertainties in order to guide the determination of the strength of an interaction applicable for use in  $A \geq 1200$  systems. The adjusted Rost interaction for use in  $A \geq 1200$  systems is based on calculations and associated uncertainties that span a wide range of nuclear systems including structure and single-particle level calculations in (1) light nuclei, (2) nuclei throughout the periodic table based on over 4000 data values incorporating pp and np scattering in the range of 0 – 350 MeV, (3) the lead region, (4)  $A = 400 - 500$  systems, and (5) nuclear matter. Based on the calculations summarized in Ref. 29, an uncertainty in the potential strength of 10% was judged to be

reasonable.

To account for the 10% potential strength uncertainty in calculating the properties of  $A \geq 1200$  systems, this paper uses the adjusted Rost interaction<sup>29</sup>:

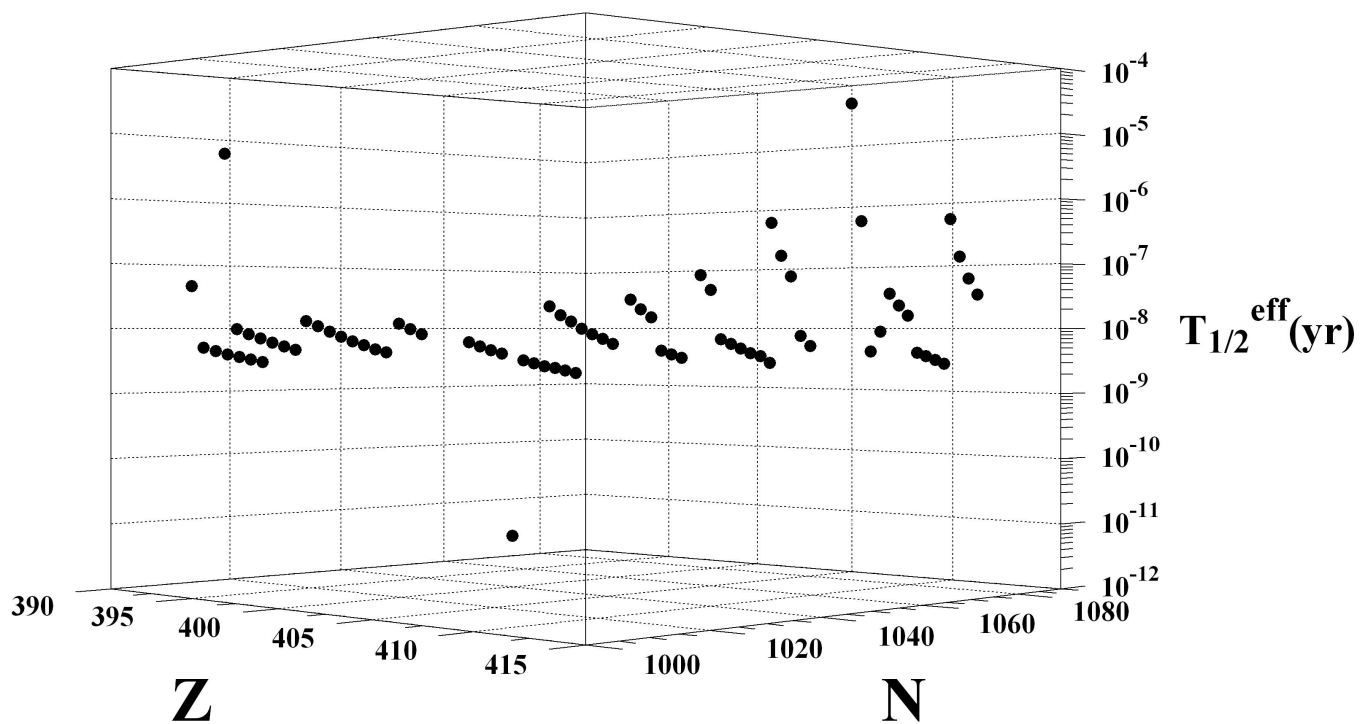
$$V = 51.6\lambda \left[ 1 \pm 0.73 \frac{N-Z}{A} \right] \text{MeV} \quad (3)$$

with  $\lambda = 1.10$  and the unmodified pairing interaction of Blomqvist and Wahlborn<sup>34</sup> to investigate the bounding characteristics of  $A \geq 1200$  superheavy nuclear systems. The adjusted Rost interaction accommodates the range of interaction strengths that were evaluated in Ref. 29.

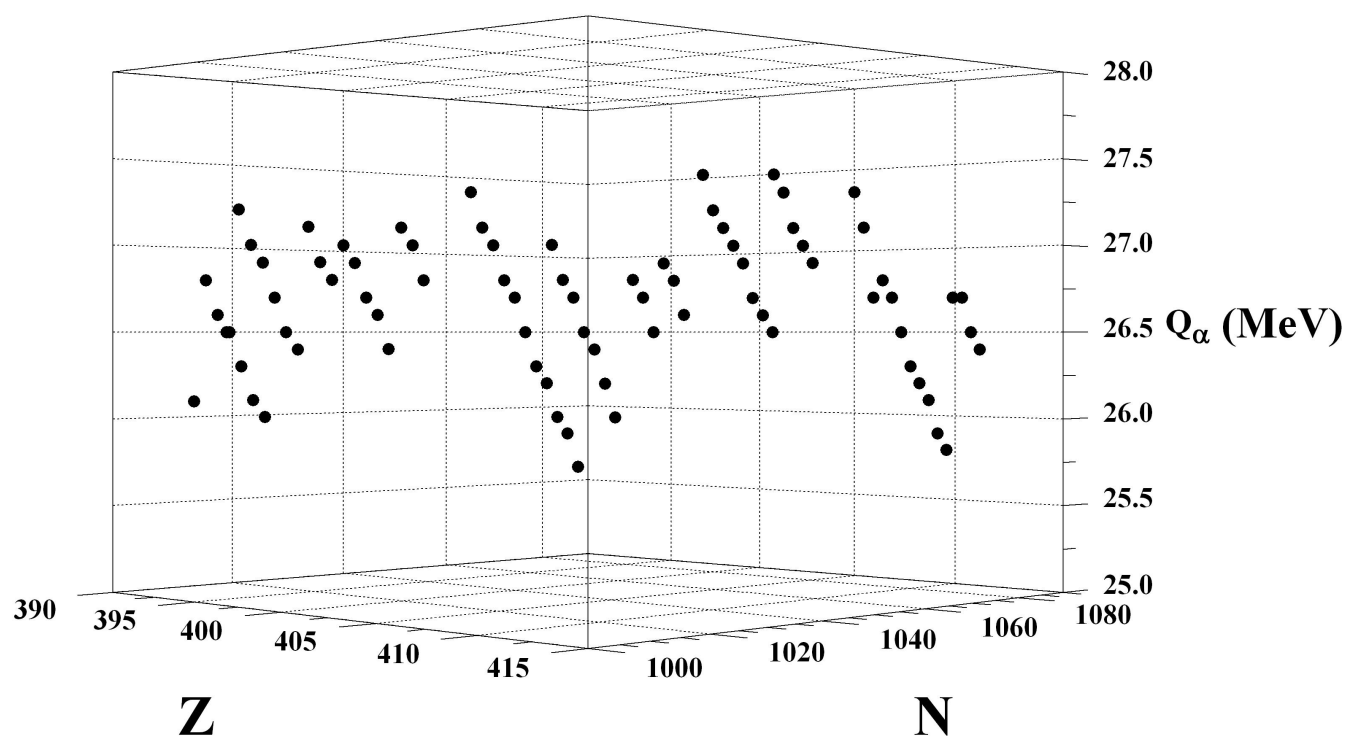
## 4.0 Results and Discussion

The calculations presented in this paper are based on the adjusted Rost interaction<sup>29</sup> which has a potential strength that is 10% stronger than the Rost interaction<sup>2</sup> used in Refs. 5 and 21-23. Accordingly, direct comparison of half-lives with the  $A = 298 - 472$ ,<sup>5</sup>  $570 \leq A \leq 620$ <sup>21</sup>,  $620 < A < 700$ <sup>22</sup>, and  $700 \leq A < 800$ <sup>23</sup> mass regions is not appropriate because they were based on the unmodified Rost interaction<sup>2</sup>. Similarly, comparison to calculations based on the modified Rost interaction<sup>24</sup> used in  $800 \leq A < 1200$ <sup>25-28</sup> systems is also not appropriate since the interactions are not the same. However, comparisons to existing nuclear and  $1200 \leq A < 1400$  systems are outlined in subsequent discussion.

The effective half-life (Eq. 4) for nuclei with  $1400 \leq A < 1500$  is plotted in Fig. 1. The alpha decay  $Q$  value ( $Q_\alpha$ ), and beta ( $T_{1/2}^\beta$ ) and alpha ( $T_{1/2}^\alpha$ ) decay half-lives for the most stable  $1400 \leq A < 1500$  systems are provided in Table 1.  $Q_\alpha$  values for nuclei with  $1400 \leq A < 1500$  are plotted in Fig. 2.



**Fig. 1.** Three-dimensional plot of the effective half-life ( $T_{1/2}^{\text{eff}}$ ) as a function of  $N$  and  $Z$  for  $1400 \leq A < 1500$  nuclear systems.



**Fig. 2. Three-dimensional plot of the  $Q_\alpha$  values as a function of N and Z for  $1400 \leq A < 1500$  nuclear systems.**

**Table 1** Calculated properties for  
 $1400 \leq A < 1500$  nuclei with effective  
half-lives  $\geq 1$  s

<b>Nucleus</b>		<b><math>T_{1/2}^{\beta}</math></b>	<b><math>Q_{\alpha}</math> (MeV)</b>	<b><math>T_{1/2}^{\alpha}</math> (yr)</b>
<b>Z</b>	<b>N</b>			
392	1008	1.4 s <sup>a</sup>	26.1	1.1x10 <sup>8</sup>
394	1008	2.4 min <sup>a</sup>	26.5	3.7x10 <sup>7</sup>
406	1054	2.1 s <sup>b</sup>	27.4	2.7x10 <sup>7</sup>
406	1056	1.2 s <sup>b</sup>	27.2	5.9x10 <sup>7</sup>
408	1060	13 s <sup>b</sup>	27.4	4.0x10 <sup>7</sup>
408	1062	4.1 s <sup>b</sup>	27.3	8.8x10 <sup>7</sup>
408	1064	2.0 s <sup>b</sup>	27.1	1.9x10 <sup>8</sup>
410	1068	14 min <sup>b</sup>	27.3	2.0x10 <sup>8</sup>
410	1070	14 s <sup>b</sup>	27.1	4.7x10 <sup>8</sup>
410	1076	1.1 s <sup>b</sup>	26.7	5.9x10 <sup>9</sup>

**Table 1 (Continued)**

412	1080	16 s <sup>b</sup>	26.7	9.7x10 <sup>9</sup>
412	1082	4.1 s <sup>b</sup>	26.7	1.2x10 <sup>10</sup>
412	1084	1.9 s <sup>b</sup>	26.5	2.7x10 <sup>10</sup>
412	1086	1.1 s <sup>b</sup>	26.4	6.0x10 <sup>10</sup>
<sup>a</sup> First forbidden 4i <sub>13/2</sub> (n) to 2j <sub>15/2</sub> (p) beta decay.				
<sup>b</sup> First forbidden 4i <sub>11/2</sub> (n) to 2j <sub>15/2</sub> (p) beta decay.				

All  $1400 \leq A < 1500$  systems decay through alpha emission. Beta decays occur in all bound  $1400 \leq A < 1500$  systems through the transitions addressed in subsequent discussion.

In general, it is expected that any bound superheavy nucleus will be strongly influenced by its shell structure. Based on previous calculations<sup>3, 5, 21-31</sup>, a bound superheavy nucleus is formed from the extra binding energy from closed-shell effects. The importance of these shell effects are noted in subsequent discussion.

The  $1400 \leq A < 1500$  calculations suggest a new island of stability may exist at  $Z = 410$  and in neighboring systems. Maximum stability occurs in the  $Z=410$   $N=1068$  nucleus, which has partially filled 1r<sub>27/2</sub> neutron and 2j<sub>15/2</sub> proton shells.

As noted in Table 1, effective half-lives  $\geq 1$  s occur in a small subset of the 77 bound nuclei within  $1400 \leq A < 1500$  systems. These enhanced stability regions occur at  $Z = 392$  ( $N = 1008$ ),  $Z = 394$  ( $N = 1008$ ),  $Z = 406$  ( $N = 1054$  and  $1056$ ),  $Z = 408$  ( $N = 1060$ ,  $1062$ , and  $1064$ ),  $Z = 410$  ( $N = 1068$ ,  $1070$ , and  $1076$ ), and  $Z = 412$  ( $N = 1080$ ,  $1082$ ,  $1084$ , and  $1086$ ). As used in this paper, an effective half-life includes the combined effect of the alpha and beta decay modes:

$$T_{\frac{1}{2}}^{eff} = \frac{T_{\frac{1}{2}}^{\alpha} T_{\frac{1}{2}}^{\beta}}{T_{\frac{1}{2}}^{\alpha} + T_{\frac{1}{2}}^{\beta}} (4)$$

Most of the  $1400 \leq A < 1500$  systems summarized in Fig. 1 have effective half-lives less than a second. The calculated half-lives of most  $1400 \leq A < 1500$  nuclei are shorter than the observed half-lives in  $Z = 114 - 118$  systems<sup>35</sup>. The longest-lived systems summarized in Table 1 represent a small subset of the 77 bound  $1400 \leq A < 1500$  nuclei.

Spontaneous fission stability is expected to be enhanced near doubly-closed shells. Detailed calculations of the fission half-lives of  $1400 \leq A < 1500$  nuclei have not been attempted. However, estimates using the Wentzel-Kramers-Brillouin (WKB) approximation methodology and the phenomenological parameter values of Ref. 3 suggest fission half-lives near closed shells are much greater than the effective decay half-lives. However, a more refined calculation is required to establish definitive spontaneous fission half-lives.

The results of the calculations suggest that for a given  $A$  value,  $S_p$  tends to decrease and  $S_n$  tends to increase as  $Z$  increases. This usually results in increasing  $Q_{\alpha}$  values as  $Z$  increases for a fixed  $A$  value. The beta decay systematics are more complex and depend on the occupancy of specific single-particle levels, single-particle level quantum numbers, and single-particle energy level values that permit an allowed or forbidden transition to occur. The specific trends in alpha and beta stability are addressed in the subsequent discussion of nuclear stability.

A few general items are noted and are consistent with the trends noted in Refs. 21-31. For a given  $A$  value, alpha decay half-lives tend to decrease and beta decay half-lives tend to increase as  $Z$  increases. For a fixed  $Z$ , alpha decay half-lives tend to increase and beta decay half-lives tend to decrease as  $A$  increases.

In general, decays in the  $1400 \leq A < 1500$  systems occur through both alpha and beta pathways. The specific beta decay mode varies in the bound  $1400 \leq A < 1500$  systems and is noted in subsequent discussion.

The discussion of specific nuclear systems focuses on the nuclides summarized in Table 1. These nuclei have the longest half-lives of the 77 even-even systems found to theoretically exist within the  $1400 \leq A < 1500$  mass region.

Most of the calculated  $1400 \leq A < 1500$  half-lives are shorter than the longest-lived  $Z = 114 - 118$  nuclei<sup>35</sup>.  $Z = 394$   $N = 1008$  (2.4 min) and  $Z = 410$   $N = 1068$  (14 min) are exceptions. Within the  $Z = 114 - 118$  region, the longest-lived nucleus is  $^{285}\text{Cn}$  that has a half-life of about 34 s<sup>35</sup>.

#### 4.1 $1400 \leq A < 1420$ Systems

In the  $1400 \leq A < 1420$  mass region, beta decays occur through first forbidden  $4i_{13/2}(n)$  to  $2j_{15/2}(p)$  and  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  beta decay transitions. Most systems in the  $1400 \leq A < 1420$  mass region have



effective half-lives that are in the range of 0.1 – 1 s.

The most stable system in the  $1400 \leq A < 1420$  mass region is  $Z = 394$   $N = 1008$  that has a beta decay half-life of 2.4 min and an alpha decay half-life of  $3.7 \times 10^7$  yr.  $Z = 394$   $N = 1008$  decays by a first forbidden  $4i_{13/2}(n)$  to  $2j_{15/2}(p)$  beta decay transition. It has a filled  $1t_{31/2}$  neutron shell and partially filled  $2j_{15/2}$  proton shell. This mass region has two systems with an effective half-life  $\geq 1$  s that are summarized in Table 1.

## 4.2 $1420 \leq A < 1440$ Systems

The  $1420 \leq A < 1440$  mass region has a lower level of stability than the  $1400 \leq A < 1420$  systems. No  $1420 \leq A < 1440$  systems have an effective half-life  $\geq 1$  s.

The  $1420 \leq A < 1440$  systems have effective half-lives in the range of 0.1 – 0.4 s. In the  $1420 \leq A < 1440$  mass region, beta decays occur through first forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  and  $2n_{21/2}(n)$  to  $1m_{19/2}(p)$  beta decay transitions.

$Z = 398$   $N = 1026$  is the most stable  $1420 \leq A < 1440$  system. This system has partially filled  $5g_{9/2}$  neutron and  $1m_{19/2}$  proton shells.  $Z = 398$   $N = 1026$  has a beta decay half-life of 0.38 s and its limiting beta decay is a first forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  beta decay transition. It has an alpha decay half-life of  $5.1 \times 10^6$  yr.

## 4.3 $1440 \leq A < 1460$ Systems

Calculations suggest that the  $1440 \leq A < 1460$  mass region has similar stability to the  $1420 \leq A < 1440$  systems. No  $1440 \leq A < 1460$  systems have an effective half-life  $\geq 1$  s.

Most  $1440 \leq A < 1460$  systems have effective half-lives in the range of a 0.1 – 0.9 s. In the  $1440 \leq A < 1460$  mass region, most beta decays occur through first forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  and  $2n_{21/2}(n)$  to  $1m_{19/2}(p)$  beta decay transitions. First-forbidden  $6d_{5/2}(n)$  to  $4p_{1/2}(p)$  and allowed  $2n_{21/2}(n)$  to  $1n_{23/2}(p)$  beta decay transitions also occur.

$Z = 404$   $N = 1048$  is the most stable  $1440 \leq A < 1460$  system. This system has partially filled  $2n_{21/2}$  neutron and  $2j_{15/2}$  proton shells. It decays through a first forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  beta decay transition with a half-life of 0.88 s.  $Z = 404$   $N = 1048$  has an alpha decay half-life of  $2.2 \times 10^8$  y.

## 4.4 $1460 \leq A < 1480$ Systems

The  $1460 \leq A < 1480$  mass region produces the greatest level of stability in  $1400 \leq A < 1500$  systems with effective half-lives between about 0.1 s and 14 min. In the  $1460 \leq A < 1480$  mass region, beta decays occur predominantly through first-forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  and  $6d_{5/2}(n)$  to  $4p_{1/2}(p)$  beta decay transitions. First-forbidden  $7s_{1/2}(n)$  to  $4p_{1/2}(p)$  beta decay transitions also occur. The  $1460 \leq A < 1480$  mass region has six systems with an effective half-life  $\geq 1$  s that are summarized in Table 1.

$Z = 410$   $N = 1068$  is the most stable  $1400 \leq A < 1500$  system. This system has partially filled  $1r_{27/2}$  neutron and  $2j_{15/2}$  proton shells. It has an alpha (beta) decay of  $2.0 \times 10^8$  y (14 min). Its beta decay occurs through a first-forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  transition.

## 4.5 $1480 \leq A < 1500$ Systems

Calculations suggest that the  $1480 \leq A < 1500$  mass region produces a similar overall level of stability as the  $1460 \leq A < 1480$  systems. Six nuclei in this mass region have half-lives  $\geq 1$  s.

Most systems have effective half-lives in the range of a 0.1 – 16 s. In the  $1480 \leq A < 1500$  mass region, most beta decays occur through first-forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  and  $6d_{5/2}(n)$  to  $4p_{1/2}(p)$  transitions. First forbidden  $6d_{5/2}(n)$  to  $4p_{3/2}(p)$  beta decay transitions also occur.

$Z = 412$   $N = 1080$  is the most stable  $1480 \leq A < 1500$  system. This system has partially filled  $5g_{7/2}$  neutron and  $2j_{15/2}$  proton shells.  $Z = 412$   $N = 1080$  has an alpha decay half-life of  $9.7 \times 10^9$  y. Its beta decay half-life is 16 s.  $Z = 412$   $N = 1080$  beta decays through a first-forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  transition.

## 4.6 Shell Closure

$Z = 410$   $N = 1068$  is the most stable system in the  $1400 \leq A < 1500$  mass region. This system has partially filled  $1r_{27/2}$  neutron and  $2j_{15/2}$  proton shells. The  $Z = 410$   $N = 1068$   $2j_{15/2}$  ( $1r_{27/2}$ ) proton (neutron) shell closure gaps are about 0.20 MeV (0.06 MeV). These gaps are defined by the energy difference between the last occupied proton (neutron) level energy and the energy level that lies above it. For protons, the  $2j_{15/2}$  is the last bound proton level. The  $7s_{1/2}$  level lies above  $1r_{27/2}$  neutron level. These gaps are considerably smaller than the  $Z = 382$   $N = 962$  gaps noted in the most stable  $1300 \leq A < 1400$  system<sup>31</sup>.

The  $Z = 382$   $N = 962$  single-particle level structure illustrates closure of the  $Z = 382$  proton shell as well as closure of the neutron shell at  $N = 962$ . This system has  $4p_{3/2}$  ( $3k_{15/2}$ ) proton (neutron) shell closure gaps of 0.34 MeV (0.55 MeV). These gaps are defined by the energy difference in the closed shell proton (neutron) level energy and the  $4p_{1/2}$  ( $4i_{13/2}$ ) level that lies above it<sup>31</sup>.

The most stable nucleus in  $1200 \leq A < 1300$  system<sup>30</sup> is  $Z = 354$   $N = 872$ . The  $Z = 354$   $N = 872$   $2i_{11/2}$  ( $5f_{5/2}$ ) proton (neutron) shell closure gap is 0.40 MeV (0.058 MeV)<sup>30</sup>. These gaps are defined by the energy difference in the closed shell proton (neutron) level energy and the  $4p_{3/2}$  ( $6p_{3/2}$ ) level that lies above it<sup>30</sup>.

A comparison to other systems should only be made for calculations using the same model interaction. The adjusted Rost interaction<sup>29</sup> was developed for  $A \geq 1200$  systems. Therefore, only a comparison of the  $1400 \leq A < 1500$  systems summarized in this paper to  $1200 \leq A < 1400$  system<sup>30,31</sup> is appropriate.

More specific comments regarding the relative stability to previously investigated  $A = 298 - 472$  doubly-closed shell nuclei<sup>5</sup> and nuclear systems in the  $570 \leq A \leq 620$ <sup>21</sup>,  $620 < A < 700$ <sup>22</sup>,  $700 \leq A < 800$ <sup>23</sup>,  $800 \leq A < 900$ <sup>25</sup>,  $900 \leq A < 1000$ <sup>26</sup>,  $1000 \leq A < 1100$ <sup>27</sup> and  $1100 \leq A < 1200$ <sup>28</sup> mass regions are not appropriate since these calculations used either the unmodified Rost interaction<sup>2</sup> or the modified Rost interaction<sup>24</sup>. As noted previously, the  $1400 \leq A < 1500$  calculations only provide regions of possible stability and should only be compared with calculations that utilize the adjusted Rost interaction<sup>29</sup>.

Table 1 summarizes the current list of most stable  $1400 \leq A < 1500$  systems that utilize the adjusted Rost interaction<sup>29</sup>. As a matter of comparison,  $Z = 354$   $N = 872$  with an effective half-life of  $4.8 \times 10^{12}$  yr<sup>30</sup> is the most stable system determined to date using the adjusted Rost interaction.

## 5.0 Model Weaknesses

The adjusted Rost interaction<sup>29</sup> is extrapolated from  $Z \leq 82$  data without the benefit of experimental benchmarks in the  $1400 \leq A < 1500$  mass region. Although this is a necessity due to the lack of experimental data, it must be acknowledged as a weakness in the present approach. This weakness will be applicable for any current theoretical investigation in the  $1400 \leq A < 1500$  mass region.

In Ref. 30, there were eight  $1200 \leq A < 1300$  systems with effective half-lives  $>10^{10}$  y which is on the order of the current estimate for the age of the Universe ( $\approx 1.4 \times 10^{10}$  y). No such systems are present in the  $1400 \leq A < 1500$  mass region. Table 1 notes that the model predicts 14 nuclear systems with effective half-lives  $\geq 1$  s. The proposed model does not account for the possibility that as the nucleus numbers A, N, and Z become larger, new, more rapid decay modes could exist. These decay modes would then be more likely to dominate all decay processes of these superheavy systems. This is a significant weakness of the proposed extension of the theory beyond its origin via connection to known isotopes.

Another weakness of the approach outlined in this paper is treating all evaluated nuclei as spherically symmetric systems. Many of these systems are likely deformed, and these deformations should be included in subsequent investigations. These calculations have been initiated. However, it seems unlikely that any given A(N, Z) nuclear system will have a deformed structure that is more stable than the spherically symmetric configuration utilized in the model outlined in Section 2.0.

These limitations preclude absolute determinations of single-particle energies, Q values, and half-lives. The model does facilitate a comparison of the relative stability of nuclear systems and identification of possible islands of stability. These limitations are unavoidable given the lack of experimental data and uncertainties in the model and supporting nuclear interaction<sup>29</sup>. The adjusted Rost interaction formulated in Ref. 29 accounted for the uncertainties in the potential strength noted in studies of a wide range of nuclear systems.

The aforementioned weaknesses are difficult to assess, but the model prediction of  $Z = 410$   $N = 1068$  stability can be partially assessed by comparing the (A, Z) values of this system to the predictions of Adler's relationship<sup>36,37</sup> that provides the most stable nucleus Z value for a given A:

$$Z = \frac{0.487A}{1 + K} \quad (5)$$

where  $K = A^{2/3} / 166$ . This relationship suggests that the  $Z = 410$   $N = 1068$  ( $A = 1478$ ) system should be most stable for a Z value of 404 which is about 1.5% smaller than the  $Z = 410$  result obtained by the spherical model outlined in this paper. Although qualitative, the reasonable comparison between the model and predictions of the Adler relationship of Eq. 5 serves to place a portion of the model weakness issues into perspective.

## 6.0 Experimental Verification

$Z = 114$  to  $118$  superheavy nuclei have been created through fusion reactions between  $^{48}\text{Ca}$  beams and actinide targets<sup>19</sup>. Creation of elements with  $Z > 118$  likely requires projectiles with  $Z > 20$ . These

investigations have yet to be successful. Creating  $A \geq 1400$  systems is significantly more complex than the near term challenge of synthesizing  $Z > 118$  nuclei.

Conventional binary collision processes involving heavy ions beams are not currently capable of reaching the  $1400 \leq A < 1500$  mass region. For example,  $^{285}\text{Cn}$  has a half-life of about  $34 \text{ s}^{34}$ . Even if it were possible to perform a  $^{285}\text{Cn} + ^{285}\text{Cn}$  collision, it would not produce the lightest system considered in this paper. Experimental investigation of the  $1400 \leq A < 1500$  mass region requires a novel approach. For example, simultaneously colliding seven  $^{238}\text{U}$  ions theoretically reaches the  $1400 \leq A < 1500$  mass region, but this approach is not yet viable. In the interim, the author hopes that other theoretical work will challenge and refine the conclusions of this paper, and experimentalists will develop accelerator techniques to collide multiple beams or establish other approaches to reach the  $1400 \leq A < 1500$  mass region.

A possible experimental approach is offered by the high alpha particle energies emitted by the postulated  $1400 \leq A < 1500$  systems. The alpha particle energies of these theoretical superheavy nuclei are more than 100% larger than the measured  $Z = 114\text{-}118$  values<sup>34</sup>. This substantial increase in alpha particle energies offers a possible avenue for the experimental verification of  $1400 \leq A < 1500$  nuclei.

Compared to  $Z = 114 - 118$  nuclei, the higher alpha particle energies from the  $1400 \leq A < 1500$  nuclei have a longer range in a material medium. This range manifests itself as a longer track length as the alpha particle is attenuated by the medium. Measuring alpha track lengths is a well-established approach in applied physics including the measurement of the  $^{222}\text{Rn}$  air concentration<sup>37,38</sup>. Since the track length is related to the alpha particle energy, it provides a possible method to verify the existence of a  $1400 \leq A < 1500$  superheavy system.

## 7.0 Conclusions

Calculations in the  $1400 \leq A < 1500$  mass region suggest a new island of stability exists at  $Z = 410$  and in neighboring systems. Using the adjusted Rost interaction<sup>29</sup>, 77 even-even nuclear systems are predicted in the  $1400 \leq A < 1500$  mass region.  $Z = 410$   $N = 1068$  is the most stable  $1400 \leq A < 1500$  system. This system has partially filled  $1r_{27/2}$  neutron and  $2j_{15/2}$  proton shells. It has an alpha (beta) decay half-life of  $2.0 \times 10^8 \text{ y}$  (14 min). Its beta decay occurs through a first-forbidden  $4i_{11/2}(n)$  to  $2j_{15/2}(p)$  transition.

There is considerable uncertainty in extrapolating nuclear potentials to the  $1400 \leq A < 1500$  mass region. Therefore, many of the quantitative details regarding half-lives presented in this paper may be incorrect. However, the qualitative results, including the general predictions of the range of  $N$  and  $Z$  combinations associated with stability are expected to be more reliable. It is hoped that this paper will foster more sophisticated investigations of the  $1400 \leq A < 1500$  mass region.

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