Superheavy Nuclei XIII: 1600 ≤ A < 1610 Systems

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Abstract

Decay properties of nuclei are calculated in the mass region 1600 ≤ A < 1610. The calculations are performed using the Rost-1600 interaction. Model calculations suggest that a new islands of stability could exist in the vicinity of the \( Z = 438 \). The most stable system occurs at \((Z, A) = (438, 1602)\).

1.0 Introduction

The investigation of the stability of superheavy nuclei has been a continuing area of active experimental and theoretical interest\(^1\)-\(^3\). Theoretical predictions of stability in superheavy nuclei necessarily require extrapolations of the nuclear interaction\(^2\), and are fraught with uncertainty. Accordingly, only qualitative results are possible. This becomes increasingly valid as the system mass increases.

Table 1 summarizes previous calculations and provides the mass region investigated, the most stable \((A, Z)\) system in that domain, the alpha Q value for the most stable system, its effective half life, and the interaction strength utilized. The \(570 \leq A \leq 800\) systems utilized the unmodified \((\lambda = 1.0)\) Rost interaction\(^2\), \(800 \leq A < 1200\) systems were evaluated using the modified \((\lambda = 1.05)\) Rost interaction\(^24\), and \(1200 \leq A < 1600\) were based on the adjusted \((\lambda = 1.10)\) Rost interaction\(^29\).
This paper describes calculations for \(1600 \leq A < 1610\) superheavy nuclei and finds that 41 even-even nuclear systems theoretically exist within this mass range. These calculations are performed using the Rost-1600 interaction\(^{34}\).

The stability of \(1600 \leq A < 1610\) systems is evaluated by calculating single-particle neutron and proton levels using a methodology previously used to investigate \(A = 298 - 472\) doubly-closed shell nuclei\(^{5}\) and nuclear systems in the \(570 \leq A \leq 620\), \(620 < A < 700\), \(700 \leq A < 800\), \(800 \leq A < 900\), \(900 \leq A < 1000\), \(1000 \leq A < 1100\), \(1100 \leq A < 1200\), \(1200 \leq A < 1300\), \(1300 \leq A < 1400\), \(1400 \leq A < 1500\), and \(1500 \leq A < 1600\) mass regions. The calculations presented herein provide an opportunity to investigate a mass region that has received minimal theoretical investigation. Moreover, these calculations provide insight into binding energy systematics and nuclear stability beyond the mass regions explored by the calculations of Refs. 3 and 5, and in the neighboring \(570 \leq A < 1600\) mass region\(^{21-34}\).

The use of single-particle energy levels to evaluate nuclear stability is appropriate since extrapolations to the superheavy mass regions are speculative. Using a more sophisticated method is not warranted in view of the uncertainties encountered in these calculations. Methods that are more sophisticated are appropriate when data are available to examine fine model details and interaction characteristics. As was demonstrated in Refs. 3 and 5, single-particle energy level calculations are entirely appropriate for initial calculations into a superheavy mass region where there is no experimental data to guide the calculations. Moreover, theoretical calculations are currently the only way to investigate the \(1600 \leq A < 1610\) mass region because an experimental investigation is not currently feasible.

Alpha decay, beta decay, positron decay, electron capture, and spontaneous fission half-lives are calculated to determine the stability of these superheavy systems. The stability in the \(1600 \leq A < 1610\) mass region is dominated by alpha decay and beta decay. These half-lives are derived from the calculated single-particle level spectrum. The single-particle level energies are sensitive to the model potential\(^{3, 5, 21-34}\). This paper also addresses model weaknesses and possible experimental methods to investigate \(1600 \leq A < 1610\) systems.
2.0 Calculational Methodology

Since the method for calculating single-particle energies in a spherically symmetric potential is well established, only salient features are provided. Details of the methodology were provided in Ref. 21, which extended the approach of Petrovich et al. Specific details of the numerical method, model, and convergence criteria are provided in Refs. 2, 5, 21-37.

2.1 Theoretical Model

The model describing the nucleon plus nuclear core system represents an application of the standard method of Lukasiak and Sobiczewski and Petrovich et al. The calculational method used to generate a single-particle level spectrum determines the binding energy $E_{NLSJ}$ of a particle in the field of a spherical nuclear core by solving the radial Schrödinger Equation

$$\hbar^2 \left( \frac{d^2}{dr^2} - \frac{L(L + 1)}{r^2} \right) - E_{NLSJ} - V_{LSJ}(r) U_{NLSJ}(r) = 0(1)$$

where $r$ is the radial coordinate defining the relative motion of the nuclear core and the particle; $V_{LSJ}(r)$ is the model interaction; $E_{NLSJ}$ is the core plus particle binding energy; $U_{NLSJ}(r)$ is the radial wave function; and $L$, $S$, and $J$ are the orbital, spin, and total angular momentum quantum numbers, respectively. $N$ is the radial quantum number and $\mu$ is the reduced mass. Additional details of the model and associated interactions are provided in Refs. 2, 5 and 21-37.

2.2 Determination of Q Values and Half-Lives

The reader is strongly cautioned not to interpret the calculated half-lives as representing a definitive value. As noted in subsequent discussion, the half-lives represent relative values, and the largest values suggest regions of possible stability relative to other systems whose properties are calculated with the same interaction.

The Q value for alpha decay and the alpha and beta decay half-lives of $1600 \leq A < 1610$ superheavy nuclei with effective half-lives $\geq 10^{18}$ yr are listed in Table 2. The alpha decay energies are calculated using the relationship based on Ref. 1.

$$Q_\alpha = 28.3 MeV - 2S_n - 2S_p (2)$$

where $S_n$ and $S_p$ are the binding energies of the last occupied neutron and proton single-particle energy levels, respectively. Alpha half-lives ($T_{1/2}^\alpha$) were estimated from $Q_\alpha$ using standard relationships provided in Ref. 3. The beta decay, positron decay, and electron capture half-lives were determined following the procedures noted in Ref. 3.

Table 2 Rost-1600 Potential $\lambda=1.15$
Calculated Properties for $1600 \leq A < 1610$ Nuclei ($^8$ Beta stable)
<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Z</th>
<th>N</th>
<th>$T_{1/2}^{\beta}$ (yr)</th>
<th>$Q_{\alpha}$ (MeV)</th>
<th>$T_{1/2}^{\alpha}$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>438</td>
<td>1162</td>
<td>23.0</td>
<td>1.1E25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>440</td>
<td>1160</td>
<td>25.0</td>
<td>1.5E19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>436</td>
<td>1166</td>
<td>24.4</td>
<td>1.4E20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>438</td>
<td>1164</td>
<td>21.3</td>
<td>4.8E32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>440</td>
<td>1162</td>
<td>24.9</td>
<td>4.1E19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>438</td>
<td>1166</td>
<td>21.7</td>
<td>2.2E29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 (Continued) Rost-1600
Potential $\lambda=1.15$ Calculated Properties for $1600 \leq A < 1610$ Nuclei ($^\beta$ Beta stable)
The beta decay half-lives ($T_{1/2}^β$) are determined following the log $ft$ methodology of Wong\textsuperscript{1}. Allowed (first-forbidden) transition half-lives were derived using the values of log $ft = 5$ (8). Given the uncertainties in the calculated single-particle level energies, second and higher forbidden transitions were not determined. The beta half-life values in Table 1 listed as *stable* are either beta particle stable or decay by these higher order forbidden transitions.

### 3.0 Nuclear Interaction

Nuclear stability with respect to alpha decay, beta decay, positron decay, electron capture, and spontaneous fission is addressed using the method previously published by the author\textsuperscript{21-34} and coworkers\textsuperscript{5} that is similar to the approach of Ref. 3. The single-particle level spectrum is generated using a Woods-Saxon potential with parameters optimized to permit
extrapolation into the superheavy region\textsuperscript{34}.

Uncertainties in the nuclear interaction for \( A \geq 1600 \) superheavy nuclei preclude absolute theoretical predictions of nuclear properties including single-particle energies, half-lives, and \( Q \)-values. However, a model potential can be developed to predict trends in these properties and suggest islands of stability in \( A \geq 1600 \) nuclei\textsuperscript{34}.

A specific interaction for investigating \( A \geq 1600 \) systems was developed in Ref. 34. Any potential applicable to \( A \geq 1600 \) systems must be constructed in a manner that is consistent with the general uncertainties in the nuclear interaction. Ref. 34 reviewed a representative sample of these uncertainties in order to guide the determination of the strength of an interaction applicable for use in \( A \geq 1600 \) systems. The Rost-1600 interaction for use in \( A \geq 1600 \) systems is based on calculations and associated uncertainties that span a wide range of nuclear systems including structure and single-particle level calculations in (1) light nuclei, (2) nuclei throughout the periodic table based on over 4000 data values incorporating pp and np scattering in the range of 0 – 350 MeV, (3) the lead region, (4) \( A = 400 – 500 \) systems, and (5) nuclear matter. Based on the calculations summarized in Ref. 34, an uncertainty in the potential strength of 15\% was judged to be reasonable.

To account for the 15\% potential strength uncertainty in calculating the properties of \( A \geq 1600 \) systems, this paper uses the adjusted Rost-1600 interaction\textsuperscript{34}

\[
V_0 = 51.6\lambda \left[ 1 \pm 0.73 \left( \frac{N - Z}{A} \right) \right] \quad (3)
\]

with \( \lambda = 1.15 \) and the unmodified pairing interaction of Blomqvist and Wahlborn\textsuperscript{37} to investigate the bounding characteristics of \( A \geq 1600 \) superheavy nuclear systems. The Rost-1600 interaction accommodates the range of interaction strengths that were evaluated in Ref. 34.

4.0 Results and Discussion

The calculations presented in this paper are based on the adjusted Rost-1600 interaction\textsuperscript{34} that has a potential strength that is 15\% stronger that the Rost interaction\textsuperscript{2} used in Refs. 5 and 21-23. Accordingly, direct comparison of half-lives with the \( A = 298 – 472, 570 \leq A \leq 620\textsuperscript{21}, 620 < A < 700\textsuperscript{22}, \) and \( 700 \leq A < 800\textsuperscript{23} \) mass regions is not appropriate because they were based on the unmodified Rost interaction\textsuperscript{2}. Similarly, comparison to calculations based on the modified Rost interaction\textsuperscript{24} used in \( 800 \leq A < 1200\textsuperscript{25-28} \) systems is also not appropriate since the interactions are not the same. In addition, the Rost-1600 model results should not be directly compared with calculations based on the Rost interaction and its variants\textsuperscript{2,24,29} for \( 570 \leq A < 1600 \) systems.

The effective half-life (Eq. 4) for nuclei with \( 1600 \leq A < 1610 \) is plotted in Fig. 1. The alpha decay \( Q \) value (\( Q_\alpha \), and beta \( (T_{1/2}^\beta) \) and alpha \( (T_{1/2}^\alpha) \) decay half-lives for the most stable \( 1600 \leq A < 1610 \) systems are provided in Table 2. \( Q \) values for nuclei with \( 1600 \leq A < 1610 \) systems are plotted in Fig. 2.
Fig. 1. Three-dimensional plot of the effective half-life ($T_{1/2}^{\text{eff}}$) as a function of N and Z for 1600 ≤ A < 1610 nuclear systems. To simplify the plot, the half-lives of the systems summarized in Table 2 are depicted as $10^{-6}$ yr rather than their actual values. Using the actual half-life values would compress most of the figure causing a loss of detail.
Fig. 2. Three-dimensional plot of the $Q_\alpha$ values as a function of N and Z for $1600 \leq A < 1610$ nuclear systems.

All $1600 \leq A < 1610$ systems decay through alpha emission. Beta decays occur in most bound $1600 \leq A < 1610$ systems through the transitions addressed in subsequent discussion. The most stable $1600 \leq A < 1610$ system (438, 1602) is beta stable.

In general, it is expected that any bound superheavy nucleus will be strongly influenced by its shell structure. Based on previous calculations\(^3, 5, 21-34\), a bound superheavy nucleus is formed from the extra binding energy from closed-shell effects. The importance of these shell effects are noted in subsequent discussion.

The $1600 \leq A < 1610$ calculations suggest that a new island of stability could exist in the vicinity of $Z = 438$ (See Table 2). Maximum stability occurs in the doubly-closed in the (438, 1602) nucleus, which has closed $3l_{17/2}$ neutron and $4p_{1/2}$ proton shells.

As noted in Table 2, effective half-lives $\geq 10^8$ yr occur in a subset of the 41 bound nuclei within $1600 \leq A < 1610$ systems. As used in this paper, an effective half-life includes the combined effect of the alpha and beta decay modes:
Most of the 1600 ≤ A < 1610 systems summarized in Fig. 1 have effective half-lives less than 10^{-3} s. The calculated half-lives of most 1600 ≤ A < 1610 nuclei are shorter than the observed half-lives in Z = 114 - 118 systems^{38}. The longest-lived systems summarized in Table 2 represent a subset of the 41 bound 1600 ≤ A < 1610 nuclei.

Spontaneous fission stability is expected to be enhanced near doubly-closed shells. Detailed calculations of the fission half-lives of 1600 ≤ A < 1610 nuclei have not been attempted. However, estimates using the Wentzel-Kramers-Brillouin (WKB) approximation methodology and the phenomenological parameter values of Ref. 3 suggest fission half-lives near closed shells are greater than the effective decay half-lives^{21-34}. However, a more refined calculation is required to establish definitive spontaneous fission half-lives.

The results of the calculations^{21-34} suggest that for a given A value, S_p tends to decrease and S_n tends to increase as Z increases. This usually results in increasing Q_\alpha values as Z increases for a fixed A value. The beta decay systematics are more complex, and depend on the occupancy of specific single-particle levels, single-particle level quantum numbers, and single-particle energy level values that permit an allowed or forbidden transition to occur. The specific trends in alpha and beta stability are addressed in the subsequent discussion of nuclear stability.

A few general items are noted and are consistent with the trends noted in Refs. 21-34. For a given A value, alpha decay half-lives tend to decrease and beta decay half-lives tend to increase as Z increases. For a fixed Z, alpha decay half-lives tend to increase and beta decay half-lives tend to decrease as A increases.

In general, most decays in the 1600 ≤ A < 1610 systems occur through both alpha and beta pathways. The specific beta decay mode varies in the bound 1600 ≤ A < 1610 systems and is noted in subsequent discussion.

Most of the calculated 1600 ≤ A < 1610 half-lives are shorter than the longest-lived Z = 114 – 118 nuclei^{38}. The systems summarized in Table 2 are exceptions. Within the Z = 114 – 118 region, the longest-lived nucleus is $^{285}$Cn that has a half-life of about 34 s^{38}.

In the 1600 ≤ A < 1610 mass region, beta decays occur most frequently throughallowed $5g_{7/2}$ to $3g_{9/2}$ and $2\sigma_{25/2}$ to $1\sigma_{25/2}$ beta decay transitions. Most systems in the 1600 ≤ A < 1610 mass region have effective half-lives that are < 10^{-3} s.

The most stable system in the 1600 ≤ A < 1610 mass region is (438, 1602) that is stable with respect to beta decay, and has an alpha decay half-life of 4.8x10^{32} yr. (438, 1602) is doubly-closed, and has closed 3\sigma_{7/2} neutron and 4\pi_{1/2} proton shells.

### 5.0 Model Weaknesses

The Rost-1600 interaction^{34} is extrapolated from the Rost interactions^{2,24,29} without the benefit of experimental benchmarks in the 1600 ≤ A < 1610 mass region. Although this is a necessity due to the lack of experimental data, it must...
be acknowledged as a weakness in the present approach. This weakness will be applicable for any current theoretical investigation in the 1600 ≤ A < 1610 mass region.

Table 2 notes that the model predicts 12 nuclear systems with effective half-lives ≥ 10^{18} yr. The proposed model does not account for the possibility that as the nucleus A, N, and Z values become larger, new, more rapid decay modes could exist. These decay modes would then be more likely to dominate all decay processes of these superheavy systems. This is a significant weakness of the proposed extension of the theory beyond its origin via connection to known isotopes.

Another weakness of the approach outlined in this paper is treating all evaluated nuclei as spherically symmetric systems. Many of these systems are likely deformed, and these deformations should be included in subsequent investigations. These calculations have been initiated. However, it seems unlikely that any given A(N, Z) nuclear system will have a deformed structure that is more stable than the spherically symmetric configuration utilized in the model outlined in Section 2.0.

These limitations preclude absolute determinations of single-particle energies, Q values, and half-lives. The model does facilitate a comparison of the relative stability of nuclear systems and identification of possible islands of stability. These limitations are unavoidable given the lack of experimental data and uncertainties in the model and supporting nuclear interaction. The Rost-1600 interaction formulated in Ref. 34 accounted for the uncertainties in the potential strength noted in studies of a wide range of nuclear systems.

The aforementioned weaknesses are difficult to assess, but the model prediction of (438, 1602) stability can be partially addressed by comparing the (Z, A) values of this system to the predictions of Adler’s relationship that provides the most stable nucleus Z value for a given A:

\[ Z = \frac{0.487A}{1 + \frac{A^{2/3}}{166}} \] (5)

This relationship suggests that the (438, 1602) system should be most stable for a Z value of 428 which is about 2.3% smaller than the Z = 438 result obtained by the spherical model outlined in this paper. Although qualitative, the reasonable comparison between the model and predictions of the Adler relationship of Eq. 5 serves to place a portion of the model weakness issues into perspective.

6.0 Experimental Verification

Z = 114 to 118 superheavy nuclei have been created through fusion reactions between^{48}Ca beams and actinide targets. Creation of elements with Z > 118 likely requires projectiles with Z > 20. These investigations have yet to be successful. Creating A ≥ 1600 systems is significantly more complex than the near term challenge of synthesizing Z > 118 nuclei.

Conventional binary collision processes involving heavy ions beams are not currently capable of reaching the 1600 ≤ A
< 1610 mass region. For example, $^{285}$Cn has a half-life of about 34 s. Even if it were possible to perform a $^{285}$Cn + $^{285}$Cn collision, it would not produce the lightest system considered in this paper. Experimental investigation of the 1600 ≤ A < 1610 mass region requires a novel approach. For example, simultaneously colliding multiple $^{238}$U ions theoretically reaches the 1600 ≤ A < 1610 mass region, but this approach is not yet viable. In the interim, the author hopes that other theoretical work will challenge and refine the conclusions of this paper, and experimentalists will develop accelerator techniques to collide multiple beams or establish other approaches to reach the 1600 ≤ A < 1610 mass region.

A possible experimental approach is offered by the high alpha particle energies emitted by the postulated 1600 ≤ A < 1610 systems. The alpha particle energies of these theoretical superheavy nuclei are more than 100% larger than the measured Z = 114-118 values. This substantial increase in alpha particle energies offers a possible avenue for the experimental verification of 1600 ≤ A < 1610 nuclei.

Compared to Z = 114 - 118 nuclei, the higher alpha particle energies from the 1600 ≤ A < 1610 nuclei have a longer range in a material medium. This range manifests itself as a longer track length as the alpha particle is attenuated by the medium. Measuring alpha track lengths is a well-established approach in applied physics including the measurement of the $^{222}$Rn air concentration. Since the track length is related to the alpha particle energy, it provides a possible method to verify the existence of a 1600 ≤ A < 1610 superheavy system.

A final possible verification approach is based on the fact that various lead isotopes are the endpoint of known heavy element decay chains. If lead targets were vaporized, and then accelerated in a charged particle accelerator, they could then be separated by mass. Within this mass spectrum could be the remnants of the long-lived parent superheavy nuclei summarized in Table 2. Extreme precision would be required to detect these primordial superheavy trace isotopes. At the very least, an experimental bound could be placed on the existence of superheavy isotopes. This is an interesting possibility for an experimental technique, but sufficient sensitivity would be required. Although the needed sensitivity may not be achieved with existing technology, it is worth further investigation. This problem and the requirements for extreme sensitivity are similar to the challenges involved with ongoing neutrino oscillation experiments that investigate CP violation. Further discussion of this verification methodology is provided in Ref. 42.

7.0 Conclusions

Model calculations suggest that a new island of stability could exist in the vicinity of Z = 438. Using the Rost-1600 interaction, even-even nuclear systems are predicted in the 1600 ≤ A < 1610 mass region. The most stable system in the 1600 ≤ A < 1610 mass region is (438, 1602) that is stable with respect to beta decay, and has an alpha decay half-life of 4.8x10^32 yr. (438, 1602) is doubly-closed, and has closed 3l$_{7/2}$ neutron and 4p$_{1/2}$ proton shells.

There is considerable uncertainty in extrapolating nuclear potentials to the 1600 ≤ A < 1610 mass region. Therefore, many of the quantitative details regarding half-lives presented in this paper may be incorrect. However, the qualitative results, including the general predictions of the range of N and Z combinations associated with stability are expected to be more reliable. It is hoped that this paper will foster more sophisticated investigations of the 1600 ≤ A < 1610 mass region.
8.0 Acknowledgments

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References


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