Review of: "Negativity, zeros and extreme values of several polynomials"

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In this paper, the authors solve a problem (Proposition 1) about the negativity of a polynomial function, G(t), of degree 43, on the interval [0, 1), proposed by a Chao-Ping Chen, and another problem (Proposition 2) about three other polynomials J(t), H(t), and K(t), which are related to the first and second derivatives of the polynomial G(t), and G(t) studied this time on the interval (−∞, ∞).

For the first problem, the authors present two solutions. First the authors observe that the polynomial G(t) is divisible by (t – 1)^2, and since this is a non-negative factor, they focus on proving the negativity of the quotient, which is the polynomial H(t). In the first solution, the authors are using the classic technique, from Calculus, about proving the positivity of a smooth function f(t) on an interval I, by repeatedly differentiating the function until a certain derivative f^{(k)}(t) becomes obviously positive or negative on that interval. That fact makes the previous derivative f^{(k–1)}(t) strictly monotone on that interval, and looking at the signs of f^{(k–1)}(a) and f^{(k–1)}(b), where a and b are the margins of the interval I, they can draw conclusions about the sign of f^{(k–1)}(t) on the interval I. This sign, in turn, provides information about the monotonicity of the previous derivative f^{(k–2)}(t), and so on. This reasoning is repeated until they obtain the desired information about the sign of the original function f(t) on the interval I. The value of k, for the function f(t) := H(t), is k = 29. The second solution uses Descartes' rule of signs and the fact that H(1) < 0 < H(2).

Even though the second solution is much quicker, I am glad that the authors included also the first solution since it shows that sometimes to prove an inequality about smooth functions, one must be tenacious and differentiate until one derivative has a clear constant sign on the interval of study.

In Proposition 2, the authors are using again the Descartes' rule of signs and the Darboux property for continuous functions to draw information about some key features, like total number of zeros, location of zeros, and locations of extreme values for the polynomial functions J(t), G(t), H(t), and K(t). In this proposition, the authors are going beyond the interval [0, 1), studying these polynomial functions of their entire domain of definition,

(−∞, ∞).

The paper is well written, and the solutions are correct. The authors combine classic tools from Calculus and Algebra with technology (to compute large numbers or Wolfram Mathematica to graph), and I think the techniques that they use are specially useful for graduate students preparing for the Ph.D. Analysis exam, and even for mathematicians trying to prove sharp inequalities.