Review of: "A New Family of Solids: The Infinite Kepler-Poinsot Polyhedra"

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This paper, and a previous one by the same author [5], introduce the infinite Kepler-Poinsot polyhedra. They describe an infinite polyhedron with its building element derived from the cubohemioctahedron, and its dual. An “open faces” visualisation of polyhedra has been implemented and used throughout the paper.

The paper makes a rewarding reading. It is interesting and gives a fresh insight to a subject that is classic, under the strictest meaning of the term. However, there are two issues that I think should be addressed to increase the paper’s value to the field.

First, prior work needs to be covered in more depth, and then the introduced infinite Kepler-Poinsot polyhedra should be placed and discussed within the context of the reviewed related work. This is quite important because, as Grünbaum has pointed out, the mathematical literature on polyhedra and regular polyhedra suffers from repeated errors and confusion [3], [4]. One of the reasons seems to be that the class of objects under study is continuously expanding, and while two definitions could agree on the current class under study, and used interchangeably, they might diverge on one of its extensions.

An example of such an extension of the class of polyhedra under study, which led to significant progress in the field, is Grünbaum’s [2]. Crucially, there, Grünbaum gave a precise definition of the proposed class, explaining how it extends previously considered classes. Thus, one was able to tell if an object belongs or does not belong to this newly proposed class of polyhedra, and relatively soon, Dress gave a complete enumeration of the objects of that class [1]. Here, the level of detail and rigour of the exposition would not allow something similar.

A second issue, related to the first but easier to address, is the absence of discussion of the symmetries (isometry groups) of the introduced objects. This is quite important since a common definition of regularity is based on isometries (what Grünbaum in [4] calls the global criterion of regularity vs the local criterion based on face configurations around vertices, which it seems it was adopted here).

[1] Dress, A. W. (1985). A combinatorial theory of Grünbaum's new regular polyhedra, Part II: Complete enumeration. Aequationes mathematicae, 29, 222-243.

[2] Grünbaum, B. (1977). Regular polyhedra—old and new. Aequationes mathematicae, 16(1-2), 1-20.

[3] Grünbaum, B. (2003). Are your polyhedra the same as my polyhedra? Discrete and Computational Geometry: The Goodman-Pollack Festschrift, 461-488.

[4] Grünbaum, B. (2009). An enduring error. Elemente der Mathematik, 64(3), 89-101.

[5] Huylebrouck, D. (2017). A New Regular (Compound) Polyhedron (of Infinite Kepler–Poinsot Type). The American Mathematical Monthly, 124(3), 265-268.