Review of: "Exact Solutions of Kantoweski-Sachs spacetimes in the Framework of Creation-Field Cosmology"

Armando Andrés Roque Estrada[1]

1 Universidad de Guanajuato

Potential competing interests: No potential competing interests to declare.

The manuscript proposes some analytical solutions for a Kantowski-Sachs metric in the context of the General-Relativity (GR). In my opinion, the present version must be reviewed in detail by the author (or equivalently not continue as an online version until the author reviews). Next, we comment on some aspects of the manuscript:

- The author refers to the name of the space-time solution (or equivalently metric) as Kantoweski-Sachs. However, as can be seen in Ref. [1] and as it is commonly known is Kantowski-Sachs (KS). I recommend to the author that in Eq. (5) add the KS metric solution's reference.
- In the first and second paragraphs, the author gives some ideas about the Big Bang model and points out that the \Lambda-CDM model is not capable of reproducing everything observed. However, they do not provide references to it, I recommend adding some references. On the other hand, the author comments in the third paragraph on the advantages of the Hoyle and Narlikar (HN) model with respect to \Lambda-CDM model. But the \Lambda-CDM model is a collection of models, i.e., General Relativity + Standard Model of Particles (and extensions of this) + Friedmann–Lemaître–Robertson–Walker metric+ …. I suppose that HN introduces a modification in some sector of the \Lambda-CDM model, for example, could be in the gravitational sector, in the matter sector, or in the background spacetime solution. I suppose that it is in the matter sector with the addition of a new field, about this, I recommend to the author that comment on the properties of this C-field. Until my understanding of this manuscript, this field would be the usual spinless, non-massive scalar field.
- To close the introduction the author comments about the KS solution, however, they never cite the original work, Ref.[1], no manuscripts that follow a similar approach that they used in this manuscript (some examples that I found are Refs.[2] [3][4]). I believe it would be opportune to point out the novelty of this work with respect to the existing ones.
- The author writes the next sentence: ‘’Therefore, we consider, in this paper, a more general C-field function depending on two variables, i.e C=C(t,r) aiming to provide a better explanation of the puzzles arising in cosmology without having to invoke the assumptions made to reconcile our observation with General Relativity". I think that this sentence can confuse since GR is not a Cosmological model, it is just only an ‘’ingredient" inside of the Lambda-CDM model, and the inclusion of (matter) scalar field only extends the matter sector of the Lambda-CDM model, and it is this which could be reconciled with the observations.
- In several parts of the manuscript, the author mixes Latin and Greek indices, writing inconsistencies as for example Eq. (3) where on the left side we have Greek indices while on the right side only Latin indices. I recommend to the author used (and rewrite the wrong expressions) the usual notation where the Latin indices are used for spatial components

only, and takes on values 1, 2, or 3, while the Greek is used for space and time components, and takes on values 0, 1, 2, or 3.

- I don't understand the equation (4), the indices are wrong and the written does not make sense. If the author refers to the conservation of the energy $\nabla^{\mu}T_{\mu\nu} = 0$, he can check that in this case using the scalar field equation, the result is the same as for the case where we have only baryonic matter, i.e.,

$$\nabla^{\mu}\left(T^{(m)}_{\mu\nu} + T^{(C)}_{\mu\nu}\right) = \nabla^{\mu}T^{(m)}_{\mu\nu} = 0.$$

- The last equality written in equation (6) is wrong because this result corresponds to $T^{(m)\mu}{}_{\nu} = \mathrm{diag}(\rho, -p, -p, -p)$ and not to T^{m}_{ij} as is written. As a recommendation the author to keep in mind to always use the same notation in the text, note as used T^{m}_{ij} and $T^{(m)}_{ij}$. Additionally, the author needs to say the dependences of the functions ρ, p.
- I strongly recommend to the author define the convention used in the text. It no was clear to me when I tried to reproduce the system (7-9). See for example the discussion in Ref.[5] .
- During the review of the manuscript, the system (7-9) could not be reproduced, which prevented me from validating the results presented. I recommend the author review the possible existence of typographical errors. I would like to point out some details about this system:

1. From equation (7) notice that the energy density has a negative signature, and the usual notation is with a positive density. However, this may be due to the convention used. Hence the importance of the author specifying it.
2. The radial derivative of the C-field that appears in the system (7-9) I think that it should be divided by the r-r metric component.

1. If the author checks the 0-1 (or equivalently 1-0) component of the field equation he arrives to that the product of the radial and temporal derivate of the C-field is equal to zero. This means that one of the two will be zero. In this case, as the used metric is homogeneous is logy that this is the radial. This means that the radial field derivate could be removed from the system (7-9) and this will reduce the reported in Ref.[3] for example. Now, the author can verify that this does not happen. This can be due to the used notation, or typos in the text. However, the system obtained by me during the checking (and which presents some differences from 7-9) is reduced as I expected to the reported in Ref. [3]. For this case I used Wald's notation for Ricci, Einstein, etc., and the signature of the metric, as well as the definitions given in this manuscript.

The subsequent results reported by the author were not verified due to the mentioned discrepancies in deducing the system (7-9). The author needs to make revisions to the manuscript to address the comments and make it more polished and rigorous.

I thank the journal for the opportunity to review this manuscript and I would be happy to review a future version of this.

References

1. [a, b]*R. Kantowski, R. K. Sachs. (1966).* *Some Spatially Homogeneous Anisotropic Relativistic Cosmological Models*. *doi:10.1063/1.1704952.*
2. [^]*Basilis C. Xanthopoulos, Thomas Zannias. (1992).* *Kantowski–Sachs metrics with source: A massless scalar field*. *doi:10.1063/1.529717.*
3. [a, b]*K. S. Adhav, V. G. Mete, A. S. Nimkar, A. M. Pund. (2008).* *Kantowski-Sachs Cosmological Model in General Theory of Relativity.* *Int J Theor Phys, vol. 47 (9), 2314-2318. doi:10.1007/s10773-008-9663-8.*
4. [^]*S.V.B. Gonçalves, B.B. Silva. (2022).* *Kantowski-Sachs cosmological model: a general equation of state*. *doi:10.21203/rs.3.rs-1942379/v1.*
5. [^]*Signs in Einstein's Equation.*