Research Article

Axion-Photon-Mixing Dark Matter Conversion Mediated by Torsion Mass Constrained by the Barbero-Immirzi Parameter

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In the Standard Model (SM) of particle physics, photon-torsion mixing is extended to include the Einstein-Cartan portal to dark-photon-axion-torsion mixing beyond the Standard Model (BSM), mediated by torsion. The Barbero-Immirzi(BI) parameter, of the order of 10^{-31}, is more stringent than those obtained by Aliberti and Lambiase using matter-antimatter asymmetry. This paper presents the coupling of the SM with dark matter (DM) axions, both mediated by torsion. We discuss tordions, the quanta of torsion, and the damping of propagating torsion. It is shown that with both kinds of vectorial torsion masses, equations from Einstein-Cartan-Holst gravity can be derived, which reduce to axionic photon equations where torsion appears only through its mass spectrum. Photon-axion conversions and axion mixing are found to depend on the BI parameter. This study demonstrates that when the spin-0 torsion mass is finite and Proca electrodynamics is not ghost-free, dark axion masses align with spin-0 torsion masses via axion-driven torsion and photon-torsion mixing. Our results provide innovative insights into Proca gravity models and the role of torsion in photon-axion conversion and dark matter dynamics, thereby offering a solid foundation for future research and new theoretical frameworks in quantum gravity.

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I. Introduction

In recent years, there has been a growing interest and activity[1] in the Einstein-Cartan (EC) portal involving fermions and bosons [2] from the Standard Model (SM) of particle physics, as well as Beyond-Standard-Model (BSM) dark matter (DM), one of the universe's mysteries. On the pathway of quantum gravity with torsion, other sources of active research include the investigation of ghosts and tachyon-free scenarios in the propagation of torsion, and tordions[3] the quanta of torsion in Poincaré gauge gravity[4]. Loop quantum gravity, another theory of quantum gravity, has also introduced an important free parameter known as the Barbero-Immirzi (BI) parameter [5]. Barbero-Immirzi parameter is a numerical coefficient appearing in loop quantum gravity (LQG), a nonperturbative theory of quantum gravity. It measures the size of the quantum of area in Planck units. These topics illuminate the complex subject of quantum gravity within alternative theories. Quantum torsion, sometimes referred to as tordions, has been explored by several authors[6] both in connection with propagating torsion waves and independently. From a mathematical perspective, the Holst and Nieh-Yan (NY) terms are topological invariants that involve the BI parameter. In this paper, we have integrated these topics into Kruglov's photon-torsion mixing [7] as a mediation to the Einstein-Cartan (EC) portal for DM. They are represented by

$$\mathcal{H} = R_{ijkl}\epsilon^{ijkl}, \tag{1}$$

where ϵ^{ijkl} (with $i, j, k, l = 0, 1, 2, 3$) is the totally-skew-symmetric Levi-Civita (LÇ) symbol, and R represents the four-dimensional Riemann-Cartan (RC) curvature and torsion tensor. The NY invariant is defined as

$$\mathcal{N}\mathcal{Y} = \partial_i (T_{jkl}\epsilon^{ijkl}) = \partial_i S^i, \tag{2}$$

where T is the Cartan torsion tensor with three indices [8], and S is the totally symmetric part of the torsion tensor. In this work the usual $\sqrt{-g}(g = \det(g_{ik})$ is the metric tensor determinant) is omitted in the above formulas due to its minimal impact on torsion at LHC CERN hadron accelerators where the effect of Riemannian curvature is minimum [4]. Explicitly, the totally skew-symmetric part of the RC curvature tensor in a spacetime manifold endowed with torsion and curvature is given by

$$\mathcal{H} \supset T_k S^k, \tag{3}$$

where the symbol $\supset$ means "contains", indicating that $\mathcal{H}$ might include other terms or factors. In this Holst term, we notice the presence of a mixing term between two types of torsion vectors and pseudo-

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vectors. The RC Ricci tensor is expressed as

$$R = \tilde{R} - 2\nabla_i T^i - \frac{2}{3} T^i T_i + \frac{1}{24} S_i S^i. \quad (4)$$

On the right side of the equation, the first term $\tilde{R}$ represents the Ricci-Riemannian scalar,which accounts solely for the curvature part in Riemannian geometry, excluding any contributions from torsion. The second term signifies the divergence of the torsion vector T^i, with ∇_i denoting the covariant derivative. The third term captures the self-interaction of the torsion vector T^i, highlighting the complexities of torsion dynamics. Finally, the fourth term encapsulates the self-interaction of the pseudovector component of the torsion tensor S_i, adding another layer of interaction within the torsion framework.

In this paper, we utilize the EC theory, a minimal and widely accepted alternative to General Relativity (GR). This theory, also known as Einstein-Cartan-Kibble-Sciama gravity[9], incorporates non-dynamical torsion to address spin-spin four-fermion interactions, which do not propagate in a vacuum. Conversely, its dynamical torsion propagates in the form of torsion waves. Previously, Duncan et al.[10] explored the coupling of axions with torsion by considering the axial pseudo-torsion vector. This coupling has been investigated as DM axion gradients in galaxies[11]. Here, we extend this concept by coupling torsion-induced dark axions with Kruglov's model[7], where propagating torsion interacts with electromagnetic SM photons. This demonstrates the potential for coupling dark photons and examining their abundance[12] through standard massless Maxwell photons. Consequently, EC gravity acts as a portal between SM and BSM dark matter.

Shaposhnikov et al.[1] have studied the EC portal to DM[13][14], which is not predicted by particle physics DM, and have explored its implications in quantum gravity[15]. Using the EC portal, researchers such as C. H. Cao[16] have addressed gauge dark bosons, while Elgammal et al.[17] have investigated gauge dark bosons and fermionic DM. Torsion mediates the coupling between SM and BSM via the EC portal. The presence of the NY term in the action is fundamental for introducing the BI parameter in the propagating torsion in this work. Theoretically, Barker et al.[18] have studied the role of the torsion trace vector in metric-affine gravity, where Weyl non-metricity can also be present. In their work, dynamical torsion introduces ghosts that can only be removed by considering scalar torsion[19]. Recently, Barker and Zell[20] have identified additional challenges in propagating torsion. Here we show that ghosts and tachyons may appear in Cartan-Proca-Holst- Nieh-Yan photon-torsion mixing with respective topological terms. In this Cartan-Proca gravity framework, it is shown that boson quantization can eliminate ghosts and tachyons as the BI parameter approaches infinity. The DM sector can emerge in these theories by obtaining an expression where ghosts and tachyons are removed. From the cosmological dark sector for bouncing cosmology, it is possible to see that the mixed Holst term of both torsion modes may yield axion-torsion as a candidate for DM.

The remainder of this paper is organized as follows: Section 2 tackles the complexities of quantum corrections and ghost problems in axion-driven torsion within the Einstein-Cartan-Proca (ECP) theory, deriving constraints on the BI parameter and examining torsion-electromagnetic field interactions to provide an in-depth analysis of the dynamics and implications in Proca gravity. Section 3 delves into determining the BI parameter in the context of axion-driven torsion within loop quantum gravity, highlighting its role and significance. Section 4 establishes the derivation of torsion-photon coupling from the electromagnetic (EM) equation, demonstrating its dependence on constant torsion divergence and exploring profound implications for dark matter dynamics within the Proca-magnetogenesis framework. Section 5 identifies and eliminates tachyons and ghosts through Fourier transform applications to Proca and torsion propagation equations, resulting in a robust, ghost-free, and tachyon-free model suitable for quantum gravity with torsion. Section 6 illustrates that, although the theory is not initially ghost-free, it becomes so as the BI parameter approaches infinity, aligning with classical EC theory. Section 7 explores the pivotal role of Einstein-Cartan-Holst gravity in photon-axion conversion, particularly considering axion-torsion transmutation. Lastly, Section 8 summarizes our findings and provides a comprehensive outlook on future research directions, emphasizing the potential breakthroughs and advancements our work facilitates.

II. Dynamics of Torsion Propagation in Proca-Nieh-Yan Gravity

In this section, we address the challenges posed by quantum corrections and ghost problems arising from axion-driven torsion, which need to be resolved. To restore the quantum gravity torsion formulation of the Einstein-Cartan-Proca (ECP) theory, even in the absence of quadratic gravity terms, we first express the Lagrangian for this model

$$\mathcal{L}_{ECP} \supset= \frac{1}{2}m_s^2 S^2 - \gamma\frac{m_P^2}{2}(\partial S) - \frac{1}{2}m_P^2(\partial_{[i}S_{j]})^2 + C, \tag{5}$$

where $\frac{1}{2}m_s^2 S^2$ is the mass term for the torsion field S, signifying the self-interaction energy related to mass m_s. The term $-\gamma\frac{m_P^2}{2}(\partial S)$ accounts for the interaction between the field S and its derivatives, modulated by the Immirzi parameter γ, The kinetic term $-\frac{1}{2}m_P^2(\partial_{[i}S_{j]})^2$ represents the dynamic energy of the torsion field S, impacting its propagation in spacetime. The auxiliary function C is given by

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$$C(b,S,T)=b(\partial S)^2-\gamma m_P^2 TS+\frac{1}{2}m_T^2T^2. \tag{6}$$

where γ is the inverse of the IB parameter β, $m_P=10^{19}$ GeV is the Planck mass, m_s and m_T represent the masses associated with torsion, axial, and vector modes, $S=S_i$, $\partial S=\partial_k S^k$ and $(\partial_{[i}S_{j]})^2$ is the square of the gravitational term. The parameter b represents the quantum correction term, with indices $i=0,1,2,3$.

To build a quantum Einstein-Cartan-Proca gravity, and eliminate ghosts, we set $b=0$, which simplifies the Lagrangian to

$$\mathcal{L}_{ECP}\supset\frac{1}{2}m_s^2S^2-\gamma\frac{m_P^2}{2}(\partial S)+A(S,T,T^2), \tag{7}$$

where

$$A(S,T,T^2)=-\frac{1}{2}m_P^2(\partial_{[i}S_{j]})^2-\gamma m_P^2TS+\frac{1}{2}m_T^2T^2. \tag{8}$$

To derive the necessary constraints, we perform the variation of this Lagrangian, incorporating the Holst and Nieh-Yan terms, with respect to δT, resulting in

$$\gamma\frac{m_P^2}{m_T^2}S=T. \tag{9}$$

Substituting this constraint into Equation (4) and varying the Lagrangian with respect to S yields

$$\frac{m_P^4}{\beta^2m_T^2}S-\frac{m_P^2}{2}\Box S-\frac{1}{\beta}m_P^2\partial(\partial S)+m_T^2S=0, \tag{10}$$

utilizing the relation $\beta=1/\gamma$. To solve this torsion wave equation, decoupling it from the dark photon and torsion mixing (addressed in the next section), we assume the ansatz

$$S=S_0\exp[i(\omega t-kz)], \tag{11}$$

assuming the torsion spin-1 wave propagation along the z-direction. Notably, torsion propagates outside matter, unlike non-dynamical original E-C gravity[21]. In contrast to Kruglov [7], no external magnetic field is required. Generalizing Kruglov's work to Einstein-Cartan-Proca gravity, substituting Equation (6) into Equation (5) gives

$$\omega^2+(\gamma-1)k^2-\gamma k\omega-c_0\gamma^2=0, \tag{12}$$

and

$$(\gamma-1)\omega^2+\gamma k\omega-c_0\gamma^2=0, \tag{13}$$

where $c_0 = \frac{m_P^4}{m_T^2}$. These two equations are obtained respectively by taking $i = 1$ and $i = z = 3$. Combining them leads to an algebraic characteristic eigenvalue equation for the IB parameter

$$-4c_0\gamma^2 + \gamma - k^2 + \gamma k^2 = 0. \tag{14}$$

Assuming resonant BI parameter $\gamma_+ = \gamma_-$, implying a vanishing discriminant Δ, the general solution is

$$\gamma = \frac{\omega^2 + k^2 \pm \sqrt{\Delta}}{c_0}, \tag{15}$$

with

$$\Delta = \frac{(\omega^2 + k^2)^2 - 8k4\sqrt{c_0}}{c_0}. \tag{16}$$

Solving for ω^2 and substituting into Equation (11) gives the BI parameter in terms of the torsion wave vector of the torsion wave

$$\gamma = 2\frac{m_T}{m_P^2}k. \tag{17}$$

Taking $k = \frac{1}{\lambda}$ and considering the length scale $l_P = 10^{-33}$ cm, transformed to GeV units, we obtain the denominator is the length scale in the universe where we are measuring

$$\gamma = 2 \times 10^{-32}. \tag{18}$$

This matches the value obtained by Aliberti and Lambiase[22] for matter-anti-matter asymmetry. The following table presents data for the BI parameter across various cosmic scales, converted to GeV units:

Einstein-Cartan-Proca	λ **cm**	**BI(LQG)**
Planck scale	10^{-33}	10^{-32}
Cosmic Dust	10^{-6}	10^{-43}
Nuclei	10^{-8}	10^{-41}

Table 1. BI parameter from distinct cosmic eras in the universe

All computations were performed in GeV units. In the next section, we will calculate the BI parameter based on another physical axial anomaly involving electromagnetic fields and compare the results.

III. BI Parameter Boubds Induced by Dark-Photons and Axio-Torsion Mixing

Motivated by the work of Lazanttani and Mercuri[23] on the BI field as an axion itself and the solution to the strong CP problem via torsion with the Nieh-Yan (NY) term, as well as the work of Karananas[24], we have recently investigated QCD axion-torsion couplings, exploring light torsion as a dark matter candidate. Building on these foundations, this section focuses on determining the LQG BI parameter in the context of axion-driven torsion. By incorporating the NY term into the Lagrangian (21), we derive the following

$$\mathcal{L}_{ECP} \supset \frac{1}{2}m_s^2 S^2 - \gamma\frac{m_P^2}{2}(\partial S) - \gamma m_P^2 TS + D(T,\phi,F^2), \tag{19}$$

where

$$D(T,\phi,F^2) = \frac{1}{2}m_T^2 T^2 + \phi F\tilde{F} - \frac{1}{4}F^2 - \frac{1}{2}m_\phi^2\phi^2. \tag{20}$$

In this Lagrangian, the squared gravity term is absent as $S_{[i,j]}$ vanishes due to Minkowskian nature of BI torsion, being the gradient of the dark axion. Holst and NY terms are present and $b=0$ ensures a ghost-free theory. By substituting S, an axial pseudo-vector torsion, with the gradient of a Nambu-Goldstone pseudo-scalar axion boson ϕ, our Cartan-Proca Lagrangian becomes a pure scalar Lagrangian where the axion is fundamental. Varying this equation with respect to the axion yields

$$\left[1 + \left(\gamma^2 - \gamma\right)\frac{m_P^4}{m_s^2 m_T^2}\right]\Box\phi + m_\phi^2\phi = \mathbf{E}\cdot\mathbf{B}. \tag{21}$$

To simplify matters, we find the BI parameter in the case of an anomalous chiral source term

$$\mathbf{E}\cdot\mathbf{B} = 0, \tag{22}$$

assuming orthogonal electric and magnetic fields in the generated electromagnetic wave. Considering the second term in the brackets as much weaker than one, we derive an upper bound for the BI parameter:

$$\gamma \ll 10^{-52}. \tag{23}$$

This bound is extremely stringent and not in agreement with other previous bounds, although not significantly different. This suggests that, as far as the BI parameter is concerned, we might favor the Proca formulation with the Holst term as discussed in Section 2. Next, we generalize the photon-torsion mixing proposal. In Proca gravity, as presented by Kruglov, the axion-driven torsion formulation naturally introduces the Proca equation for scalar fields, which minimally coincides with the Klein-Gordon equation. Lut us reproduce Kruglov's interaction Lagrangian

$$\mathcal{L}_{int} = \frac{1}{4} g \left(\partial_k S^k\right) F\tilde{F}, \tag{24}$$

where $\tilde{F}$ is the dual of the EM tensor represented by $F = F_{ij}$. Utilizing the axion-driven torsion of Duncan et al.[10], not addressed by Kruglov, this interaction Lagrangian becomes

$$\mathcal{L}_{int} = \frac{1}{4} g \left(\Box \phi\right) F\tilde{F}, \tag{25}$$

with electromagnetic equations become

$$\partial_i F^{ij} - g \partial_i \left(\Box \phi\right) \tilde{F}^{ij} = 0. \tag{26}$$

These equations can be solved, or a dynamo equation can be derived by simply substituting the D'Alembertian wave operator with an axion function. Therefore, obtaining the axion equation for DM requires substituting the 4-gradient Minkowskian operator of axion-driven torsion coupling into the torsion wave equation

$$\Box S_i - \delta \partial_i \left(\partial_k S^k\right) - m^2 S_i = \frac{1}{4} \partial_i \left(F\tilde{F}\right), \tag{27}$$

with constant

$$\delta = \left[1 - \frac{m^2}{m_0^2}\right]. \tag{28}$$

Here m is the spin-1 torsion mass, and m_0 is the spin-0 torsion mode. It is interesting that, for a ghost-free axion- driven torsion Proca gravity formulation, m_0 must approach infinite, making $\delta = 1$, and reducing the equation to

$$\partial_i \left(\Box \phi\right) - \delta \partial_i \left(\Box \phi\right) - m^2 S_i = \frac{1}{4} \partial_i \left(\mathbf{E} \cdot \mathbf{B}\right). \tag{29}$$

Considering the non-chiral or non-axial anomaly of the EM field with orthogonal electric and magnetic fields, we obtain the equation for the axion,

$$\Box \phi + \phi = 0. \tag{30}$$

This shows the spin-0 torsion mass coincides with the axion mass in Proca electrodynamics with photon-torsion-axion mixing. Now, adding the Holst term TS multiplied by the BI parameter into the photon-torsion mixing interaction, we obtain

$$\mathcal{L}_{int} = \frac{1}{4} g(\partial_k S^k) F\tilde{F} + m_P^2 \gamma T_k S^k. \tag{31}$$

Using the same axion torsion-driven and photon-torsion mixing assumption that the torsion vector mass vanishes, we derive

$$\left[\frac{\gamma}{m_P^2} - \frac{1}{m_0^2}\right] \Box\phi + m_\phi^2 \phi = 0. \tag{32}$$

From this equation, the value of the BI parameter ($\beta = \frac{1}{\gamma}$) is estimated as

$$\beta \gg \frac{m_s^2}{m_P^2} = 10^{-52}, \tag{33}$$

yielding

$$\beta \gg 10^{-26}. \tag{34}$$

This aligns with the BI parameter induced by matter-antimatter asymmetry as discussed by Aliberti and Lambiase. To obtain this upper bound for the BI parameter, we assume the spin-0 torsion mass is equal to the axion mass, $m_0^2 = m_s^2$. Now let's explore magnetogenesis by solving the Proca electrodynamic equation

$$\nabla \cdot \mathbf{E} + \frac{g m_\phi^2}{4} \phi [\mathrm{k} \cdot \mathbf{B}] = 0. \tag{35}$$

This is analogous to the Coulomb-Maxwell equation in photon-torsion Proca context. Here, ”Proca equation” refers to massive torsion and axions, not just the Proca massive photon. To derive this last equation, we have used the no-monopole equation, $\mathrm{divB} = 0$. The remaining equations are

$$\partial_t \mathbf{B} = -\nabla \times [\phi \mathbf{E}]. \tag{36}$$

A simple solution for this Faraday equation is given by

$$B^z = B_0 e^{\omega_1 t - k_z z}, \tag{37}$$

where the resonance $\omega_1 = \omega$ was used, and $B_0 = -i\phi_0 k$. This results in an oscillating magnetic field induced by the axion-torsion-driven field in photon-torsion mixing with spin-0 and spin-1 torsions. Note that in these examples, the vector torsion has also been used as discussed by Barker and Zell [18].

IV. Photon-Torsion Coupling in Proca Dark Magnetogenesis Sector

In this section, we demonstrate the possibility of deriving the torsion-photon coupling expression in terms of a constant torsion divergence and wave equation from the EM equation

$$\left[\partial_t^2 - \nabla^2\right]\mathbf{A} - c_0 g\left[\lambda\mathbf{A} + 2\lambda_0\partial_t\mathbf{A}\right] = 0, \tag{38}$$

where

$$\nabla \times \mathbf{A} = \lambda\mathbf{A}. \tag{39}$$

This describes magnetic helicity, applicable to EM fields. The dispersion relation is given by:

$$\omega^2 + 2gic_0\lambda^2\omega + \left(k^2 + c_0 g\lambda^2\right) = 0. \tag{40}$$

We solve this equation under resonance conditions where the discriminant Δ vanishes. From a very low-frequency EM wave or very low-frequency (VLF) suitable for detection in the VLF array, we obtain

$$g = -c_0. \tag{41}$$

Therefore, the photon-torsion coupling is dependent on a constant torsion divergence. In this scenario, only the EM wave survives, and torsion waves are absent in this Proca regime. Next, we explore the potential for DM dynamos in the dark Proca-magnetogenesis sector by solving the above equation in a more general frame where the 4-divergence of the axial torsion vector does not vanish. This more complex solution for non-chiral magnetic fields is described by the field equations

$$\frac{d^2}{d\eta^2}\mathrm{A} - g\lambda\left[\frac{dS_0}{d\eta}(\lambda + im_0) + \frac{d^2S_0}{d\eta^2}\right]\mathbf{A} = 0 \tag{42}$$

where η is the conformal spacetime coordinate. Let's take the ansatz

$$S_0 = \exp[i\omega\eta]. \tag{43}$$

At the early universe, $\eta \to 0$, and we approximate

$$S_0 \approx i\omega\eta, \tag{44}$$

where

$$\omega^2 = g\lambda. \tag{45}$$

and the electromagnetic solution is

$$\mathbf{A} = \mathbf{A}_0\exp(i\omega\eta). \tag{46}$$

Note that to achieve this solution, we use the result that the axion mass and torsion mass approximately coincide. From expression (41), we notice that the magnetic helicity can be expressed in terms of the photon-torsion mixing constant g and the spin-0 mass. Assuming that torsion propagates at UHF of 1 GHz, we have

$$\lambda = \frac{\omega}{m_0 g} \approx 10^{-12}, \tag{47}$$

using a spin-0 mass of 1 TeV and $g = 10^{-24}$ for a massive photon. At this TeV scale, the magnetic helicity is minimal, indicating weak coupling with massive photons, which could be potential dark matter candidates. Note that the magnetic field can be obtained using the relation $B = iKA$. The oscillation frequency of the magnetic field depends on the magnetic helicity torsion-photon coupling g and the spin-0 axion mass. In the next section, we explore hypotheses on the tachyonic Proca equation and its ghosts within the Proca theory framework.

V. Ghosts and Tachyons in Fourier Spectrum of Photon-Torsion Mixing Self-Dual Proca Fields

Previously, Barker and Zell[18] showed that tachyons and ghosts appearing in Proca equations in Riemann-Cartan spacetime could be resolved by applying constraints on torsion. In this section, we extend this analysis by applying the Fourier transform $ik^j \rightarrow \partial^j$ to the Proca and torsion propagation equations, aiming to identify the potential for finding tachyons and ghosts. We begin by expressing these equations with this transform:

$$ik_l F^{lp} + gk_m k_l S^l \epsilon^{mprs} F_{rs} = 0, \tag{48}$$

where the dual of the EM field tensor is

$$\epsilon^{mprs} F_{rs} =\sim F^{mp}. \tag{49}$$

By taking the complex dual of F as $iF_{rs} =\sim F_{rs}$, expression (49) simplifies to

$$iF^{mp} = -gk_l S^l \sim F_{rs}. \tag{50}$$

Applying the self-dual relation, we obtain

$$g^{-1} = -k_l S^l > 0. \tag{51}$$

This implies that if the EM wave vector is light-like, $k^s k_s = -m^2 < 0$, as we shall see below, it is space-like or tachyonic. Meanwhile, the torsion wave expression becomes

$$-(k_s^2 + m_s^2) + \delta S^i (k^l k_l)^s = 0. \tag{52}$$

An important issue in this equation is that k, with indices indicating axial torsion, is not a null vector, unlike the EM wave vector where

$$k^i k_i = 0. \tag{53}$$

From the first set (52), one obtains

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$$k_i F^{ij} = 0. \tag{54}$$

From the conformal flat metric:

$$ds^s = d\eta^2 - d\mathbf{x}^2, \tag{55}$$

the expression (47) reads

$$\left(\partial_\eta{}^2 + m^2{}_s\right) S^i + \delta S^i (k^l k_l)^s = 0. \tag{56}$$

Multiplying the last equation by torsion wave vector k^s, we get

$$[-(k_s^2 + m_s^2) + \delta(k^l k_l)^s]\alpha = 0, \tag{57}$$

where $\alpha = k^s \cdot S$. Thus, we obtain

$$\left(\partial_\eta^2 + m_s^2\right) + \delta(k_s)^2 = 0. \tag{58}$$

There are two cases to consider: the ghost-free case $\delta = 1$

$$\left[\left(-k_s^2 + m_s^2\right) + \left(k^l k_l\right)^s\right]\alpha = 0, \tag{59}$$

which reduces to the vanishing of torsion mass. Next, we examine the case of a finite ghost mass m_0, yielding

$$-k_s^2 + m_s^2) = 0, \tag{60}$$

indicting a relation $\left(k^l k_l\right)^s > 0$, meaning torsion vector propagation is not tachyonic. Therefore, the Proca equation with spin-0 and spin-1 states are tachyonic-free. Assuming it is also ghost-free with an axial anomaly

$$\left(\partial_\eta^2 + m_s^2\right) S^i + k_s^2 S^i = \partial^i \left(F\tilde{F}\right). \tag{61}$$

Using the self-dual identity $iF = \tilde{F}$ into the RHS of the last equation, we find that the source is identical to $F^2 = \left(E^2 - B^2\right)$. Substituting these expressions into the previous formula, we get

$$\left[k_s^2 + m_s^2 + (k_s)^2\right] k_i^i S^i = \left(k_s^i\right)^2 \left(B^2 - E^2\right), \tag{62}$$

which yields

$$m_s^2 k_i^i S^i = \left(k_s^i\right)^2 \left(E^2 - B^2\right). \tag{63}$$

Note that if $k_i^i S^i > 0$, and $B^2 > E^2$, then $k_s^2 > 0$ and the torsion wave vector k_s is time-like. Therefore, Proca-Cartan is tachyonic. It is possible to find a tachyonic-free, ghost-free Proca theory with photon-torsion mixing, which is free of pathologies in quantum gravity with torsion.

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Next, we investigate what happens when m_0 approaches infinity, making Proca-Cartan ghost-free. The existence of tachyons is derived from the equation:

$$\frac{m_0^2 - k_s^2}{m_0^2} k_s S = \left(\frac{m_0}{m_s}\right)^2 \left(k_s^i\right)^2 \left(B^2 - E^2\right). \tag{64}$$

Notice that in this formula, when the Proca is ghost-free or m_0 goes to infinity, the mass coefficient tends to one on the LHS of the equation which becomes

$$k_s S = \left(\frac{m_0}{m_s}\right)^2 \left(k_s^i\right)^2 \left(B^2 - E^2\right), \tag{65}$$

and if $k_s S > 0$ and the magnetic energy density is higher than the electric one, we conclude that $\left(k_s^i\right)^2 > 0$, indicating that the torsion wave vector k_s is time-like. This demonstrates that the model is free of ghosts and tachyons, making it suitable for quantum gravity with torsion.

VI. Boson Quantization and Ghost-free Cartan-Proca Fields by Constraining BI Parameter

Kruglov[7] has shown that boson quantization applied to the propagating torsion equation leads to a ghost mass of spin-0 torsion, causing formulation problems. In the absence of the BI parameter and without Kruglov's photon-torsion mixing, Barker and Zell[18] have already identified this problem in torsion propagation. In this section, we use the Lagrangian with Holst and NY terms to show that the propagator matrix can be expressed in terms of the BI parameter. Although this approach is not inherently ghost-free, in the limit where the BI parameter approaches infinity, the Cartan-Proca equation becomes ghost-free. This phenomenon is well known, even in classical physics, where in this BI limit, Einstein-Cartan-Holst-Nieh-Yan gravity reduces to the original EC theory. Moreover, this is analogous to Kruglov's photon-torsion mixing, where in the limit of spin-0 mass approaches infinity, the ghost of Proca theory with propagating torsion is eliminated.

To demonstrate this, we express the equation in the form

$$-\frac{m_P^2}{\beta^2 m_T^2} S + \Box S + \beta^{-1} \partial (\partial S) - \frac{m_T^2}{m_P^2} S = 0. \tag{66}$$

By computing the $4x4$ matrix M_{ij} used to determine the propagator, we have

$$M_{ik} = (p^2 + m_s^2)\delta_{ik} - 2\beta^{-1} p_i p_k. \tag{67}$$

This straightforward expression involving the four-momenta p highlights that the second term has the wrong sign, indicating that the theory is not free of ghosts. Therefore, the only viable solution is to consider the scenario where the BI parameter approaches infinity, thereby achieving a ghost-free solution.

VII. Photon-Axion Conversion with Background of Torsion Mass Spectrum and FRB Constraints from BI Parameter

In this section, by utilizing Duncan et al.'s axion-torsion transmutation[10], we show that Einstein-Cartan-Holst gravity can investigate the role played by the torsion mass spectrum on photon-axion conversion. When the BI parameter assumes a certain value, the axion-photon conversion equations are free from the torsion spectrum. We propose the following Lagrangian

$$\mathcal{L}_{ECH} \supset \frac{1}{2}(\partial a)^2 + \frac{1}{2}m_s^2 S^2 - \gamma\frac{m_P^2}{2}(\partial a)T + D(T, a, F^2) \tag{68}$$

by following Duncan et al's proposal, the axion $a(z.t)$ is transmuted to torsion using the relation $S = \partial a$. Here, ∂ denotes a suitable partial gradient derivation. The z-direction is chosen to align with the polarization direction. Before explicitly constructing the field equations and their perturbations to analyze the photon-axion-torsion conversion, we must express the auxiliary relations as follows

$$D(T, a, F^2) = \frac{1}{2}m_T^2 T^2 + g_{a\gamma\gamma} aF\tilde{F} - \frac{1}{4}F^2 - \frac{1}{2}m_a^2 a^2, \tag{69}$$

where $g_{a\gamma\gamma}$ represents the axion-photon coupling constant.

In this section, we examine the influence of the Holst term on the torsion mass spectrum during photon-axion conversion within a constant magnetic field, denoted as $\mathrm{B}_0 = (B_0, 0, 0)$. Initially, we demonstrate that varying the axion a and the torsion trace vector T yields significant results: the torsion trace vector T depends on the gradient of a and through appropriate substitutions, only the presence of torsion masses remains in the final field equations to be perturbed. Variation of this equation with respect to the axion results in

$$\Box a - \gamma\partial T m_P^2 + m_a^2 a = g_{a\gamma\gamma}\mathbf{E}\cdot\mathbf{B}. \tag{70}$$

To simplify, we assume the axial torsion mass vanishes in this section. By varying T in the last Lagrangian, we get

$$T = \gamma\frac{m_P^2}{m_T^2}\partial a. \tag{71}$$

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This equation shows that the torsion trace vector depends on the gradient of the axion through a different expression. Substituting this expression into Equation (70), we obtain

$$\Box a - m_{a,\text{eff}}^2 = -\frac{1}{2} g_{a\gamma\gamma} (\frac{m_{a,\text{eff}}}{m_a})^2 \mathbf{E} \cdot \mathbf{B}. \tag{72}$$

where $m_{a,\text{eff}}$ denotes the effective axion mass. To obtain the effective axion mass. which appears in terms of the BI parameter, we assume the LHC result that torsion masses are very high, around 1 TeV. The effective axion torsion mass is given by

$$m_{a,\text{eff}} = m_a \beta \frac{m_T}{m_P}. \tag{73}$$

Now, we propose the following perturbations for axions and EM fields to build the Schrödinger-like matrix for axion-photon mixing and conversion. The perturbations are

$$a(z,t) = 0 + \delta a(z,t) \tag{74}$$

and

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}(z,t). \tag{75}$$

Due to polarization, the constant magnetic field in modulus is denoted by $\mathbf{B}_0 = (B_0, 0, 0)$. The perturbation in the electric field is given by

$$\mathbf{E} = 0 + \delta \mathbf{E}(z,t) \tag{76}$$

which gives rise to the following wave equations

$$\Box \delta A_{||} = g_{a\gamma\gamma} B_0 (\partial_t \delta a) \tag{77}$$

and

$$\Box \delta A_{ort} = 0. \tag{78}$$

Combining this with the axion electrodynamics equation

$$\partial_i F^{ij} + F^{ij} \partial_i a = 0, \tag{79}$$

we derive the perturbative axion equation

$$\Box \delta a + m_{a,eff}^2 \delta a = -g_{a\gamma\gamma} B_0 \frac{m_{a,eff}^2}{} \partial_t . (\delta A_{||}) \tag{80}$$

Choosing the solution ansatz

$$\delta a(z,t) = e^{i\omega(z-t)} \delta a(z) \tag{81}$$

and a similar expression for $\delta A_{||}(z,t)$, we obtain the following Schrödinger-like matrix equation

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$$i\frac{d\psi}{dz} = A\psi, \tag{82}$$

where the transpose of column matriz is $\psi^T = (\delta a(z), \delta A_{||}(z))$, and the matriz A is given by $\begin{pmatrix} \Delta_a & \Delta_{a\gamma\gamma} \\ \Delta_{a\gamma\gamma} & 0 \end{pmatrix}$ where Δ_a and $\Delta_{a\gamma\gamma}$ are, respectively, the axion mass term and the mixing axion-photon term. They are explicitly given by

$$\Delta_a = \frac{m^2_{a,eff}}{2\omega}, \tag{83}$$

and

$$\Delta_{a\gamma\gamma} = -\frac{1}{2} g_{a\gamma\gamma} B_0 (\frac{m_{a,eff}}{m_a})^2. \tag{84}$$

These expressions highlight the contributions of the torsion trace mass and the BI coefficient in photon-axion conversion mixing. The interplay of these factors is crucial for determining the system's dynamics.

This result has significant implications for fast radio bursts (FRBs) in astrophysics[25]. The presence of the torsion trace mass and the BI coefficient can alter the interaction between photons and axions, potentially affecting the propagation and characteristics of FRBs. Understanding these effects could provide new insights into the fundamental physics governing these mysterious cosmic events. Further research in this area may lead to breakthroughs in our comprehension of the universe's dark matter and energy components, as well as the behavior of quantum fields under extreme conditions. The ongoing exploration of these phenomena promises to enrich our knowledge and may open new avenues for scientific discovery.

VIII. Summary and Outlook

This paper explores various classes of Proca gravity that incorporate torsion without metric-affine gravity, where torsion propagates in a vacuum. It demonstrates that, although the presence of ghosts often signifies new physics, when the spin-0 torsion mass is finite and Proca electrodynamics is not ghost-free, dark axion masses align with spin-0 torsion masses through axion-driven torsion and photon-torsion mixing. This study reveals that Proca magnetogenesis with dark axions does not favor dynamo action but rather induces oscillations of magnetic fields via photon-torsion mixing for massless photons.

Recent studies have investigated dynamo and gravitational waves with massive photons in Einstein-Cartan-Holst gravity, showing minimal phenomenological effects due to the extremely weak photon mass in the Proca regime. Furthermore, it is concluded that photon-torsion mixing models with axial anomalies are free of ghosts and tachyons, making them suitable for quantum gravity applications. The paper also suggests that the inclusion of the Holst term in the Proca Lagrangian could lead to BI bounds.

In the future, exploring photon-torsion mixing with quantum corrections, such as those investigated by He et al. , could unveil new facets of torsion dynamics, offering insights into dark matter and fundamental forces while refining our understanding of photon-torsion interactions. Additionally, photon-axion conversion presents intriguing contributions from the torsion mass spectrum. Investigating these could optimize axion-photon conversion conditions, enhancing detection and study of axions and impacting cosmological models.

Overall, our findings provide a strong foundation for further research into the complex relationships between torsion, axions, and photons. Continued exploration in these areas promises to deepen our understanding of quantum gravity and uncover new physics, broadening our comprehension of the universe. As we delve further, we anticipate making significant discoveries that will advance our knowledge and theoretical frameworks.

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